Looking outside Earth
Big players in the developing the world view:

Classical period (Greece/Rome) - 500BC

Philolaus  Aristarchus of Samos

Middle Ages

Ptolemy

Modern era

Aristotle  Eratosthenes

Copernicus  Galileo
Ancient view of the universe (for example, the bible)

Let there be a firmament in the midst of the waters, and let it divide the waters from the waters.

Now the springs of the deep and the floodgates of the heavens had been closed, and the rain had stopped falling from the sky.

And let them be for lights in the firmament of the heaven to give light upon the earth: and it was so.

He shakes the earth from its place and makes its pillars tremble.
The solar system
Aristotle put the Earth at the center of the universe

Several ancient Greeks proposed that the Sun is at the center: Philolaus (470-385BC), Aristarchus of Samos (310-230 BC) Erasthotenes (279-194 BC) third librarian of the Great library of Alexandria
Eratosthenes (279-194 BC) noticed a difference in shadows at the summer solstice in Alexandria and Syene.
1. Measure angle \( \alpha = 7.2^\circ \)

2. What is this distance?

Send some soldiers on a march from Alexandria to Syene

3. Calculate portion of circumference

\[
\text{circumference} = 750 \times \frac{360^\circ}{7.2^\circ} = 37,500 \text{ km}
\]

4. Diameter \( \frac{37,500}{\pi} = 12,700 \text{ km} \)
Spinning

\[ v = 2\pi \frac{6350}{24} = 1662 \text{ km/h} \]

As a reference:

\[ V = 1200 \text{ km/h} \]
What is the size of the Moon?

1. How long does it take for the Moon to traverse the Earth’s radius during a lunar eclipse?

During a lunar eclipse the Moon has to go through the Earth’s shadow (which is about the size of the Earth).
2. Compare this to the time needed for the Moon to traverse its own radius

Aristarchus had already found it was about 3/8 (it’s actually closer to 1/4)

So the 3/8 x 12700 = 4700 km
What is the distance to the Moon?

$\alpha = 0.5$ degrees
Entire circle is $2\pi r$

It is also 360 degrees

$\frac{r_{\text{Moon}}}{2\pi} = \frac{0.5}{360}$

Or $r = r_{\text{Moon}} \frac{360}{0.5} \frac{2\pi}{2\pi}$

With $r_{\text{Moon}} = 3476 \text{ km}$

$r = 400,000 \text{ km}$
\[ \text{distance}_{\text{Earth-Sun}} = \frac{\text{distance}_{\text{Earth-Moon}}}{\cos \alpha} = \frac{400,000}{\cos 89.85} \approx 150,000,000 \text{ km} \]
Speed: $2 \pi \frac{150,000,000}{(365 \times 24)} = 108,000 \text{ km/h}

For comparison: Rocket sled 10,385 km/h (10 times slower)

For comparison: NASA X-43A 12,144 km/h (9 times slower)
Earth 108,000 km/h
Beetle 100 km/h
747 900 km/h
X-43A 12,144 km/h

Somewhat less impressive in comparison....
Size Sun?
The Sun takes up about the same area in the sky as the Moon (accidentally)

\[
diameter = 150 \times 10^6 \text{ km} \times \frac{0.5^\circ}{360^\circ} \times 2\pi \approx 1.3 \times 10^6 \text{ km}
\]
So the Sun is kinda big.....
In the end, the geocentric view prevailed
So why did people think the Earth was at the center?
Of course, everything is centered around us
SELF IMPORTANCE
YOU REALLY AREN'T THE CENTER OF THE UNIVERSE
The most comprehensive system was developed by Ptolomy (90-168)

- The universe existed of celestial spheres
- The Sun was at 1210 Earth radii (off by a factor of about 20)
- The outer sphere of the stars was at 20,000 Earth radii (this is actually the distance to the Sun), but is only 0.000015 lightyear about 7 light minutes
However, the motion of the planets in the sky are far from simple....
This was explained by Ptolemy using so-called epicircles.
So where are we now in time?

Classical period (Greece/Rome)

Middle Ages

Modern era

Aristotle   Eratosthenes   Ptolemy

Copernicus  Galileo

500BC   0   500   1000   1500   2000
Around the time of Ptolomy, the Roman empire was at its greatest extent.

Ptolomy was Greek, like Eratosthenes, but now working in the Roman empire.
This view again held till about 1450-1600...

We keep jumping between the ancient Greeks and the renaissance

So nothing really happened in between...?
So did nothing of scientific interest happen in Rome?

First, the Romans were much more practical than the Greeks.
They were magnificent architects

Colosseum 72-80

Hagia Sophia (Istanbul/Constantinople 6th cent. Mosque from 1453)
aqueducts

Circus Maximus
This is truly a great building from antiquity
Although some might say the pyramids are a greater achievement.

Well, they are bigger....
But this is not really fair, since the pantheon was built by Romans

But the pyramids were built by aliens
Now the pyramids in Egypt are not the biggest

1. **Pyramid of Djoser**
   Saqqara, Egypt, c.2610 BC, 62.6 m

2. **Bent Pyramid**
   Dahshur, Egypt, c.2600 BC, 101.1 m

3. **Red/North Pyramid**
   Dahshur, Egypt, c.2580 BC, 104 m

4. **Great Pyramid of Giza/Khufu/Cheops**
   Giza, Egypt, c.2560 BC, 138.8 m (was 146.5 m)

5. **Pyramid of Khafre/Chefren**
   Giza, Egypt, c.2530 BC, 136.4 m (was 143.5 m)

6. **Pyramid of Menkaure/Mycerinus**
   Giza, Egypt, c.2500 BC, 65.5 m

7. **Mount Li/Qin Shi Huang mausoleum**
   Xian, China, c.210 BC, 47 m (was 76 m)

8. **Pyramid of the Moon**
   Teotihuacán, Mexico, c.300, 43 m

9. **Pyramid of the Sun**
   Teotihuacán, Mexico, c.100, 75 m

10. **Pyramid of the Moon**
    Cholula, Mexico, c.900, 66 m

11. **Great Pyramid of Cholula**
    Cholula, Mexico, c.900, 66 m

12. **El Castillo**
    Chichen Itza, Mexico, c.1000, 30 m

13. **Transamerica Pyramid**
    San Francisco, United States of America, 1972, 260 m

14. **Louvre Pyramid**
    Paris, France, 1989, 20.6 m

15. **Luxor Las Vegas pyramid**
    Las Vegas, United States of America, 1993, 111 m

16. **The Shard**
    London, United Kingdom, 2012, 309.6 m

17. **Ryugyong Hotel**
    Pyongyang, North Korea, under construction, 330 m
Can’t launch a rocket, but they can certainly build pyramids!
So a civilization possessing interstellar space travel comes to Earth...
To stack stones.....?

They’re not even buildings, they’re mounds....
Also they sucked at it first....

Ok, alien proponents would say that these are not built by aliens... Only the really big ones.
Extra terrestrials

North Koreans
However the pyramids have many more mysteries!

$4B/H = 4 \times \frac{115.2}{146.5} = 3.14$

That is $\pi$
Now, the Egyptians did not know the number \( \pi \)

Also these pyramids are big and impressive, hence aliens!
All right Egyptians, here’s the deal. You are living in the bronze age. That’s cool... Now bronze kind of sucks. Let us show you how to make stainless steel. Then we turn that into nice beams that you can use to make steel frames. Then you can construct awesome buildings, and then you can make...

However, wouldn’t this make more sense?
A pyramid that might actually be useful....
(Several thousands of guest rooms, instead of one tomb...)

And when you’re ready we’ll give you kryptonite
Sure, but what about $\pi$?
Alternative explanation: Egyptians worked in cubits.

Egyptian Royal cubits:
7 palms x 4 fingers = 28 digits
Now some Egyptian architects Moses van der Rohe or Ted Mosesby
(Moses being an Egyptian name...)

Suggest some nice little pyramid

45 degrees is for loosers! Give me something steeper

Statue approximation of Sneferu
angle = \arctan(28/20) = 54.5\ degrees

Me likey
We have a problem! The pyramid is succumbing under its own weight!!
Now let's not panic and reduce the angle a bit. How about 30?

\[
\text{angle = arctan}(28/30) = 43.0 \text{ degrees}
\]
And we end up with.....
(2600 BC)

With measured angles 54.8 and 43.4 degrees.

Not too far off from the 54.5 and 43.0 degrees we obtained....
One generation later:

Make me a pyramid and none of that bent crap that you did to my father!!
Last time they tried 20 and that did not go very well.
Leading to....
(2560 BC)

with a measured angle of 51.82 degrees

Not too far off from the 51.84 degrees
One generation later....

Make me a pyramid and it has to be steeper than my dad’s!!

Inbreeding...?
Daddy issues...?
You want to talk about it?

Khafra/Chephren
Approximate image
We managed 22, but I still have a bad feeling about 20.
Et voila!

Measured angle 53.1 degrees, which is equal to our calculation.
Now what about $\pi$?

$4 \frac{B}{H} = 4 \times \frac{22}{28} = \frac{22}{7} \approx 3.1428$

Which is not too far off from $\pi \approx 3.1415$

But this is a coincidence

Although $\frac{22}{7}$ is also the upper bound for $\pi$ found by Archimedes (287-212 BC)

But maybe...
The moral of the story:

• Do not get fooled by coincidences
• Keep thinking
• Failing to understand something is never a proof of something else!
(just because you don’t understand 3.14 does not proof the existence of aliens)

And yes those pyramids are impressive....
The Romans must have known $\pi$. Largest unreinforced concrete dome 43.3 m.
Try to build that in sand and you see the difference in complexity between a pyramid (solid) and a real building (hollow)
Great passage pyramid of Gizeh

Look at the inner spaces

Pantheon

8.6 m

2 m
And yes: concrete

Lost in 400 AD after the fall of the Roman empire

Only to be reinvented in the early 19th century
What else happened in science?

Well, not too much... Pliny the Elder (23-79) wrote a book on just about everything
Galen (129-200) Roman, but of Greek descent (again...)

Tried to understand the human body in terms of temperament

Recognized the difference between arteries and veins
(slightly wrong interpretation....)
The middle ages

- Classical period (Greece/Rome)
- Middle Ages
- Modern era

- 500 BC
- 0
- 500
- 1000
- 1500
- 2000

- Split of the Roman Empire
- Fall of Constantinople
- Byzantine Empire

- Aristotle
- Copernicus Galileo

Map of Western and Eastern Roman Empires with key events such as the battle of Chalons in 451 and the fall of Constantinople in 1453.
Byzantine empire

Justinian II

Hagia Sophia (holy wisdom)

530

Transmitted classical knowledge to Islamic world and renaissance Italy

555 AD
Rise of the Arab world

Mohammed 570-632

Dome on the Rock, Jerusalem, 690

Alhambra, Granada, Spain 1330-50
Innovations coming from the East

Hindu–Arabic numeral system

Roman numerals

CIII = C + I + I = 100 + 1 + 1 + 1 = 103

MCMLIV = M + CM + L + IV = 1000 + 900 + 50 + 4 = 1954

No zero!

Try to add roman numbers
(Why do clocks use IIII instead of IV? ...)

Arabic numerals moved from India through the Arab world
Appeared in Europe around the year 1000

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>V</td>
<td>5</td>
</tr>
<tr>
<td>X</td>
<td>10</td>
</tr>
<tr>
<td>L</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>500</td>
</tr>
<tr>
<td>M</td>
<td>1,000</td>
</tr>
</tbody>
</table>
Irrational numbers $\pi, e, \sqrt{2}$
Rational number $7/8, 113/435, 90/17$

Let’s see if we can write the down all the numbers between 0 and 1

<table>
<thead>
<tr>
<th></th>
<th>0.123567443567888999441....</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.675433356778889999657....</td>
</tr>
<tr>
<td>3</td>
<td>0.644685765323683566786...</td>
</tr>
<tr>
<td>4</td>
<td>0.887562727776945445657...</td>
</tr>
<tr>
<td>5</td>
<td>0.679497687878975655677....</td>
</tr>
<tr>
<td>6</td>
<td>0.253436457688657577888....</td>
</tr>
<tr>
<td>7</td>
<td>0.347899879754654575777....</td>
</tr>
<tr>
<td>8</td>
<td>0.234436457587878979789.....</td>
</tr>
<tr>
<td>9</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Now what if you think you had them all....
But take the numbers in the circles and add one
In that way the number is never equal to any of the numbers in the list
0.28560796....
And you’re not done yet, because you can do it again, and again, and...
and it goes on and on without ever repeating itself....
Astronomy

Abū ‘Alī al-Ḥasan ibn al-Ḥasan ibn al-Haytham, also known as Alhacen or Alhazen (965 -1040)

Wrote Al-Shukūk ‘alā Batlamyūs, or Doubts Concerning Ptolemy

Disputed Aristotle’s view that the Milky Way was ignition in the atmosphere of emissions from the stars (no parallax)

Born in Basra, location of Sumer, birthplace of Sinbad, location of the Garden of Eden (?)
Early middle ages

- Western civilization barely surviving
- De-urbanization, only pockets of learning are monasteries
- Science not a primary concern of the clergy

Book of Kells, Ireland 800
A somewhat brighter period: Carolingian “Renaissance”

- Charlemagne addressed the problems of illiteracy
- Not all priests could read the Bible (still in Latin...)

Aachen 805
But that didn’t last long

Source of a lot of misery in the coming centuries....
Enter the high Middle Ages

High middle ages

Chartres 1194-1250 - No aliens?
Renewed contact with Byzantine empire and muslims

Maybe not entirely for the right reasons...

Rediscovery of Aristotle, Euclid, Ptolomy, Galen...
Formalization of higher education:
Early on: monastic or cathedral schools

Organized in universities granted by the pope
Bologna, Paris, Salerno, Oxford, Cambridge, Salamanca
Coimbra, Montpellier, Modena

~1200-1300
A priest, a doctor, and a lawyer walk into a bar.
The bartender says, “What is this, a joke?”

An elderly man, 82, just returned from the doctors only to find he
didn't have long to live. So he summoned the three most important
people in his life to tell them of his fate:
1. His Doctor
2. His Priest
3. His Lawyer.

He said, "Well, today I found out I don't have long to live.
So, I have summoned you three here, because you are the most
important people in my life, and I need to ask a favour. Today, I am going
to give each of you an envelope with $50,000 dollars inside.
When I die, I would ask that all three of you throw the money
into my grave."

After the man passed on, the three people happened to run into each other.
The doctor said, "I have to admit I kept $10,000 dollars of his money. He
owed me from lots of medical bills. But, I threw the other $40,000 in like
he requested."

The Priest said, "I have to admit also, I kept $25,000 dollars for the church.
It's all going to a good cause. I did, however, throw the other $25,000
in the grave."

Well the Lawyer just couldn't believe what he was hearing! "I am surprised
at you two for taking advantage of him like that. I wrote a cheque
for the full amount and threw it all in!!!
Roger Bacon (1214-1294)

Stressed the need for experimental knowledge
19th century (romanticized interpretation):
First modern scientist, ahead of his time
More modern: not too much different (Grosseteste)
too much emphasizing of Middle Ages as Dark Ages

Work on optics closely followed Ptolomy and Islamic scholars
The Late Middle Ages (1300-1500) had some issues....

- 1315-7 the Great Famine, start of little Ice Age
- 1348-50 the Black Death
- 1453 Fall of Constantinople
Philosophical developments in the late Middle Ages

Occam/Ockham’s razor
(traceable to John Duns Scotus (1265-1308) or even Aristotle and Ptolomy)

*entia non sunt multiplicanda praeter necessitatem* (entities must not be multiplied beyond necessity)

Other things being equal, a simpler explanation is better than a more complex one

In science:
Not really scientific logic, but certainly helpful

---

William of Ockham (1288 – 1348)
We used a similar principle.

We never proved that one is right and the other is wrong, but appealed to the argument: Which solution looks simpler and more likely?
a body moving with constant velocity travels distance and time equal to an accelerated body whose velocity is half the final speed of the accelerated body

Constant velocity: \( x = \frac{1}{2} v t \)

Accelerated body: \( v = at \) or \( a = \frac{v}{t} \)

\( x = \frac{1}{2} a t^2 = \frac{1}{2} \left(\frac{v}{t}\right) t^2 = \frac{1}{2} v t \)

By jolly, that is indeed the same

Thomas Bradwardine (1290-1349) and the Oxford calculators:

William Swineshead

John Dumbleton

William Heytesbury

Approximate image of a medieval Oxford calculator
So that’s Galileo, more than two centuries before Galileo

It diffused to the continent: France, Italy, and other parts

Why don’t we hear about it?

• It’s good, but not good enough
• It lacks algebra, to make it really general
• So in the end, it kind of fizzled...
So here we are at the end of the Middle Ages, and the Earth is still flat....
Ok, most people accepted that the Earth was round
Well, some misconceptions remained….

I wonder if Christopher Columbus used Apple Maps to discover America.

Ha! Piece of cake!
DISCOVERED AMERICA AND GAVE US:

COLUMBUS DAY SALE!
NO SALES TAX*

Sealy
NOW ONLY $899
WAS $1799 Queen Set

Simmons
NOW ONLY $1099 Queen Set

SAVE OVER $800
SAVE OVER $900
Copernicus
And we still don’t have a simple (remember Ockham...) for the weird motion of the atoms
In 1543 (the year he died), Copernicus published his theory that the Earth rotated around the Sun.

De revolutionibus orbium coelestium (On the Revolutions of the Celestial Spheres)

How did we get there?
Why now?
Why circles?
We often simplify the contrast between the Middle Ages and the Renaissance as:

*Memento Mori* = Remember you will die

*Carpe Diem* = Seize the day
Town hall Leuven, Belgium, 1448-1469
Several architects

Palazzo Farnese, Rome, Italy, 1517-1541
Antonio de Sangallo
Naumburg cathedral 1250

Reims 1230

Notre Dame, Paris

Verrocchio 1467-1483

Lombardo 1494-5

Michelangelo 1501-04

Loom 1517

Donatello 1453
Music in the middle ages:

Composer by Anonymous
(most prolific composer ever!)
Music in the renaissance

Giovanni Pierluigi da Palestrina (1525 -1594)

St. Peter’s basilica

Shortest-reigning popes
Urban VII (15–27 September 1590): reigned for 13 calendar days, died before coronation.
Boniface VI (April 896): reigned for 16 calendar days
Celestine IV (25 October – 10 November 1241): reigned for 17 calendar days
Theodore II (December 897): reigned for 20 calendar days
Sisinnius (15 January – 4 February 708): reigned for 21 calendar days
Marcellus II (9 April – 1 May 1555): reigned for 22 calendar days
Damasus II (17 July – 9 August 1048): reigned for 24 calendar days
Pius III (22 September – 18 October 1503): reigned for 27 calendar days
Leo XI (1–27 April 1605): reigned for 27 calendar days
Benedict V (22 May – 23 June 964): reigned for 33 calendar days

After his death, it would be 415 years before another Pope would choose a name with an ordinal number less than IV (John Paul I).
Bramante/Michelangelo 1506-1547

Carlo Maderno 1612

Bernini 1656-1667.
Shows the tremendous wealth and power of the Roman Catholic church!

This will come back and haunt 16th Century scientists
However, just because the people in the renaissance thought that the Middle Ages (in the middle between the classical period and the renaissance) were the dark ages.

In fact, the high Middle Ages were pretty spectacular:
Palace of Westminster 1840-70

Washington National Cathedral 1907-90

Tribune tower 1925

And has been an inspiration throughout the centuries
And the way people in the Renaissance (and now!) look at the ancient Greeks is often wrong!
Which to us (indoctrinated by those renaissance guys) looks often pretty tacky

Really, Augustus...?

Already noticed in the 18th, but ignored/denied, because it didn’t really fit into their view of ancient Rome/Greece (period of enlightenment)

Classic Greek statue of caped Adonis in Metropolis, IL [sic]

Dude!!
No self-respecting Greek would keep this in his palazzo
Some of the conceptions in the renaissance were maybe too idealistic

• It was a time where one was going back to classical ideals
• Focus back on men as opposed to God
• Men created in the image of God

• Let’s compare some Roman portraits with renaissance ones
Maybe Michelangelo could have used some Roman realism
With all this striving for perfection, circles seems to be the only option
Giordano Bruno
Some went even several steps further than Copernicus: One of those was Giordano Bruno (1548 –1600)

The universe is then one, infinite, immobile.... It is not capable of comprehension and therefore is endless and limitless, and to that extent infinite and indeterminable, and consequently immobile

The universe is infinite!
The Sun is just another star....

All the stars are just like the Sun....
Speaking of the Sun. What color is the Sun?
Yup, it is white, close to the color above
Not yellow, red, or whatever....
Of course looking at the Sun, when it is white is something you don’t want to do...
When we can look at it, it’s usually yellow or red.
And speaking of the Moon.... (which we were not...)

What’s wrong with this picture?
The Moon is not transparent!
What else did Giordano Bruno think of? The universe is not finite and divided into different space with different laws

- **Laws of nature**
  - earth, water, wind, and fire

- **Laws of heaven**

- **Aether**
  - Heaven of Fire for Greeks and others

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The illustration shows a model of the universe with different layers, including:

- **Heaven of Heavens**
- **Ocean of Heaven**
- **Firmament with the Stars**
- **Underworld**
- **Primeval Ocean**
- **Pillars of Heaven**
- **Pillars of Earth**
Same laws of nature here.
It is often asserted that Bruno died at the stake for supporting Copernicus’ heliocentric views.
However, the church had a whole laundry list of acts of heresy against him:

- holding opinions contrary to the Catholic faith and speaking against it and its ministers;
- holding opinions contrary to the Catholic faith about the Trinity, divinity of Christ, and Incarnation;
- holding opinions contrary to the Catholic faith pertaining to Jesus as Christ;
- holding opinions contrary to the Catholic faith regarding the virginity of Mary, mother of Jesus;
- holding opinions contrary to the Catholic faith about both Transubstantiation and Mass;
- claiming the existence of a plurality of worlds and their eternity;
- believing in metempsychosisis and in the transmigration of the human soul into brutes, and;
- dealing in magics and divination.

etc., etc., etc., etc., etc., etc., etc., etc.,
Galileo Galilei
However, Galileo got into problems for supporting the heliocentric view.
Galileo discussed the remains of the supernova, stating that the universe does change, contrary to Aristotle’s position.
Previous page: Crab nebula
• Exploded in 1054 AD
• Explosion observed by Indian, Arabic, Chinese, and Japanese astronomers (Hey, Europeans, what were you doing?)

Optical image

X-ray image

25 km

Turns 30 times per second
Remnant of Kepler’s supernova

- Occurred in the Milky-way in 1604
- 20,000 light years from Earth (1.9 $10^{17}$ km)
- Brighter than all stars and planets, except Venus
- Visible during the day for three weeks
- Lucky: Tycho Brahe’s supernova happened in 1572
Galileo studied the parallax with respect to distant stars.

Presence of parallax (the movement of the finger) with respect to the distant object indicates finger is close to your eyes.

Same can be done with stars.
First distance determination using parallax

Motion first observed in 1804 by Giuseppe Piazzi

Calculation of distance from parallax by Friedrich Bessel (1784 –1846)

Determined 10.4 light years, actual distance 11.4 light years = $1.07 \times 10^{14}$ km

61 Cygni is a double star system

Sun (left), 61 Cygni A (bottom) and 61 Cygni B (upper right).

Period 659 years

15th nearest known star system
Galileo did no such thing, but he noticed that there was no parallax. This shit was happening really far away and not in our solar system.
Hans Lippershey 1608

Galileo heard of this in Venice in 1609
Galileo’s telescope 20x magnification

Obviously, he directly turned his telescope to the sky!
He made detailed observations of Jupiter and saw something orbiting the planet.
This is what one can do with a modern telescope

total time 3-4 hours
Galileo’s notes 1610

Modern interpretation
There are mountains on The Moon

The Milky Way consists of separate stars, Not one big cloudy (milky?) substance
Phases of Venus

Galileo’s notes

Modern telescope
Epicircle (not to scale)

Full phase:
Furthest away

Crescent:
Closest

Observations support heliocentric view!
Dialogue Concerning the Two Chief World Systems

Series of discussions between Salviati (Copernican heliocentric view), Sagredo (layman, neutral), and Simplicio (Ptolomean geocentric view).

Got into problem with the church, who found Galileo “vehemently suspect of heresy”
Galileo’s sentence of imprisonment was converted into house arrest

*Eppur si muove*---And yet it moves
Brahe and Kepler
Further additions to the heliocentric system

Tycho Brahe (1546-1601) and Johannes Kepler
Brahe was known for his accurate measurements of planetary motions. We return to that with Johannes Kepler.

Also observed supernova.

Had his own compromise geo/heliocentric model (it kind of sucked...).
However Brahe is also known for:

Loosing part of his nose in a sword duel over a mathematical formula with his third cousin Manderup Parsberg.

Brahe was from a very rich noble family.

Had a pet moose (does such a thing exist) that died by falling down the stairs, drunk....

Brahe died by contracting kidney failure by refusing to go to the bathroom since that would be bad etiquette.

It was like that when I got here.
He was succeeded by Johannes Kepler (1571-1630)

Born 1571

Studies 1589-1594

Math+astronomy

Teacher 1594-1600

Mysterium Cosmographicum

Defense of Copernican system

1600-1612
Banished from Graz for refusing to convert to Catholicism 1600-1609 Prague

Worked for Brahe who died in 1601
Advisor to Rudolf II
(even though Lutheran)
Ok, the major part of the job involved being Rudolf II’s personal astrologer.
He still had some time to do some serious stuff
However, Johannes Kepler noticed failings in the heliocentric model proposed by Copernicus.

Are we starting to see cracks in the harmony and reform of the renaissance?
So Kepler had to search for something else...

This seems to go completely against Occam's razor that simpler (circle) is better.

**OCCAM'S RAZOR**
Sure there are simpler ways to catch that bird, but the complicated ones kick ass.
The renaissance was focused on classical beauty and perfection.
So when it came to switching from geocentric to heliocentric

epicycles explain everything

Circles seem to be the only option
However, the times were changing

That’s what I said

One discovered there was more than being created in the image of God
There’s beauty in just about everything
Renaissance ideas on a dead-end track?

Early renaissance
Gentile Bellini (1429-1507)
The virgin and child enthroned 1475-85

High renaissance
Raphael (1483-1520)
The Sistine Madonna 1513-14

Late renaissance
Parmigianino (1503-1540)
Madonna with the long neck 1540
So after 40 failed egg-shaped orbits....

You’re telling me that those Renaissance dude never tried ellipses? And I am wasting my time with egg-shaped orbits?

I don’t like it!
1609: Kepler formulates his first two laws:

1. The orbit of every planet is an ellipse with the Sun at one of the foci.
How ellipse are the orbits?
2. A line joining a planet and the Sun sweeps out equal areas during equal time intervals

Speed is not constant

Now how do we get there. Newton had a proof, but that was 80 years later (and in addition somewhat complicated...). Kepler used a different mathematical technique: infinitesimals.
So what is happening here? It doesn’t look like much...
Renaissance: it looks like renaissance scientists and artists are doing everything
However, the “modern” disciplines are still quite disconnected
Science in the renaissance

Physics:
• Laws of motion mainly discussed in philosophical/metaphysical terms (what we did when describing a falling object in terms of mathematics was not done in the renaissance)
• Mathematics hardly used in physics
• Trying to escape from Aristotle

Astronomy
• Observation was key
• One was not clearly making the connection between the laws of motion in physics and planetary motion
• Trying to escape from Ptolomy

Mathematics
• Still a rather separate discipline concerned with mainly algebra and geometry (and related to that trigonometry)

Obviously this slide oversimplifies things a little bit
Science in the 17th Century

Mathematics/Physics
• Development of calculus

Physics:
• Laws of motion described using calculus
• Development of classical mechanics based on mathematical principles

Astronomy
• Planetary motion described in terms of laws of physics

Johannes Kepler: astronomy=celestial physics
Now math was mainly geometry and algebra. What does that have to do with planetary motion and physics?
Ok....Back to the ancient Greeks....

We have all heard of Pythagoras rule

The Egyptians before 2000 BC were aware of the 3-4-5 triangle

But probably not of the proof (unless aliens helped them...)

where $3^2 + 4^2 = 9 + 16 = 25 = 5^2$
The Babylonians knew other Pythagorean triangles, such as 5-12-13 and 65-72-97. Such as those found on the clay tablet Plimpton 322 (1790-1750 BC). To the Babylonian we also owe the sexagesimal (you know 60 minutes, 360 degrees).
But the first known proof came from Pythagoras

Area big square
Area small square plus four triangles:

\[(a + b)^2 = c^2 + 2ab \implies a^2 + 2ab + b^2 = c^2 + 2ab\]

\[\implies a^2 + b^2 = c^2\]
Ok, that’s all nice, but let’s get a little bit more serious

The first real attempts at calculating surfaces was done by Archimedes, 287-212 BC (the first great mathematician/physicist?)

Archimedes was born in Syracuse (Sicily, not NY)
He came up with some amazing stuff

Archimedes principle: (I guess he did not make the name)

First scientific streaker
Archimedes screw

And the first death ray

Plus quite a few things in mathematics

Death ray
Finds its victims
Archimedes was killed after a two-year siege of Syracuse by the Romans. His last attributed words were: “Do not disturb my circles”
How to calculate the area of a circle?
In the example, n=6 triangles

The circle is bigger than this hexagon:

Area triangle \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \sqrt{3} = \frac{1}{8} \sqrt{3} \).

Area hexagon \( 12 \times \frac{1}{8} \sqrt{3} = \frac{3}{2} \sqrt{3} \approx 2.59 \).

But smaller than this hexagon:

Area triangle \( \frac{1}{2} \frac{1}{3} \sqrt{3} = \frac{1}{6} \sqrt{3} \).

Area hexagon \( 12 \frac{1}{6} \sqrt{3} = 2 \sqrt{3} \approx 3.46 \).

Therefore
\[ 2.59 \leq \pi \leq 3.46 \]
(It works somewhat better with the circumference)
Well, that’s a pretty bad guess….
$2.59 \leq \pi \leq 3.46$

However, it gets better with more triangles!

Using 96 triangles, Archimedes found

$$3 \frac{10}{71} = 3.1408 < \pi < 3.1428 = 3 \frac{1}{7}$$

$3 \frac{1}{7} = \frac{22}{7}$

That’s exactly that number that we found with the pyramids, showing that Archimedes is an alien….
This method was the only method used to calculate $\pi$ for the next 2,000 years

1596 Ludolph van Ceulen used a $2^{62}$-gon. He calculated $\pi$ up to 35 digits:

3.1415926535897932384626433832795029

First professor of mathematics at Leiden University, founded 1575. Needed after the separation from Spain.
So that’s how Kepler finds the areas

That’s what daughter did in sixth grade...

Area=$\sum$ area$_{\text{square}}$

$\sum$ is the sum over all small squares
Kepler published his results in 1609 in “astronomia nova”

Or somewhat less catchy
New Astronomy, Based upon Causes, 
or Celestial Physics, Treated by Means of Commentaries 
on the Motions of the Star Mars, 
from the Observations of Tycho Brahe

He also proves in Chapter 16, that physics is boring: “If thou art bored with this wearisome method of calculation, take pity on me, who had to go through with at least seventy repetitions of it, at a very great loss of time.”
One more stop

Rudolf II

Matthias (his brother)

1612-1630
And one more law:
The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

\[
\text{Period}^2 / \text{Axis}^3 = \text{constant}
\]

Let us compare Earth and Mars:

**Mars:**
- Axis = 227,939,100 km
- Period = 686.971 days

\[
\text{Period}^2 / \text{Axis}^3 = 3.9849 \times 10^{-20}
\]

**Earth:**
- Axis = 149,598,261 km
- Period = 365.25636 days

\[
\text{Period}^2 / \text{Axis}^3 = 3.9849 \times 10^{-20}
\]

No wonder Kepler’s last book was called Harmonices Mundi.

The Harmony of the world

Even turned into an opera, premiered 1957.
Renaissance defined by ideal shapes:

Limitations to perfection:
• There is only so much you can do with it: no decoration, sober
• Too rational and intellectual
• There is more to life: emotions, drama
• People are not Gods

Time for something new:

Baroque defined by rules: \[ \frac{P^2}{A^3} = \text{constant} \]

In the arts:
• Focus on more direct emotions
• Abundance of details, decorative elements
• Life is a theatre
To be, or not to be, that is the question:
Whether 'tis Nobler in the mind to suffer
The Slings and Arrows of outrageous Fortune,
Or to take Arms against a Sea of troubles,
And by opposing end them: to die, to sleep
No more; and by a sleep, to say we end
.....

Intellectual plays of, for example, Shakespeare are replaced by....

the more direct form of Opera

Jean-Baptiste Lully (1631-87)
Harmony and balance

Botticelli, the birth of Venus, 1486
Drama through Contrapposto or counterpoise

Rubens
The rape of the daughters of Leucippus (1618)
Baroque was a style preferred by the Roman Catholic church around 1600 as a response to the Protestant Reformation. It was emotional, dramatic, and grandiose. It spread throughout most of Europe.
Baroque tends to be busy

You might feel that there are no rules.

The problem is that there are too many rules.
Life is a theatre, so everyone must play their part following the rules.
Louis XIV (Le Roi Soleil) developed a highly codified etiquette, meant to assert his supremacy.
However, if everything is theatre, what about Rembrandt’s calm introspection?
This is Rembrandt as the Apostle Paul
Which leaves us with the following questions:

- Is this really a modern type of self-portrait?
- Why all the dress-up as the Apostle Paul?
- Is this really about Rembrandt himself or is he just playing a role?
- Self-reflection or theatre?
- Are we fooling ourselves and projecting our own 20th Century ideas on this painting?
- Are we falling into the same trap as the Renaissance artists with ancient Greece and Rome?
Back to physics:

If it is no longer about ideal shapes, then what gives us the rules that govern physics?

The answer is simple:
Although Newton’s three laws are nice (we still haven’t done the 3rd law), his real achievement was developing a mathematical basis for mechanics. This was calculus. This had a tremendous impact, not only on mechanics, but on virtually all disciplines in physics.
I know:

- This is a non-calculus course
- Calculus is hard
- We all hate it....
- And there will be no exam question about it

- However, physics revolves around it
Waves: Optics and acoustics

Electricity and magnetism

Classical mechanics
\[ F = m \frac{d^2 x}{dt^2} \]

Relativistic quantum mechanics

Thermodynamics

Entropy of a Gas

\[ S = \text{Entropy} \]
\[ T = \text{Temperature} \]
\[ V = \text{Volume} \]
\[ E = \text{Internal Energy} \]
\[ C_p = \text{Heat Capacity (constant pressure)} \]
\[ C_v = \text{Heat Capacity (constant volume)} \]

1st Law
\[ dQ = dH - V \, dp \]
\[ dQ = C_p \, dT - RT \, dp \]
\[ dS = \frac{C_p}{T} \, dT - R \, dp \]

specific form:
\[ s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \]

2nd Law
\[ S_2 - S_1 = \frac{\Delta Q}{T} \]
differential form:
\[ dS = \frac{dQ}{T} \]

And God said:
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \cdot \mathbf{D} = \rho \nu \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

and then there was light.
However, we (sneakily already did calculus)

\[ v = \frac{\Delta x}{\Delta t} \]

Velocity is the change in position divided by the change in time

However, this caused problems when \( \Delta t \) was very small. But in the end, we worked around these problems

\[ v = \frac{x_{\text{end}} - x_{\text{begin}}}{t_{\text{end}} - t_{\text{begin}}} = \frac{\frac{1}{2}a(t + \Delta t)^2 - \frac{1}{2}at^2}{(t + \Delta t) - t} \]

Now if \( \Delta t \) is very small

\( (t + \Delta t)^2 \approx t^2 + 2t\Delta t \)

And we obtain

\[ v \approx \frac{\frac{1}{2}a(t^2 + 2t\Delta t) - \frac{1}{2}at^2}{\Delta t} = \frac{at\Delta t}{\Delta t} = at \]
Well, that was differentiation!!

From

\[ v = \frac{\Delta x}{\Delta t} \]

We found that

\[ x = \frac{1}{2}at^2 \implies v = \frac{\Delta x}{\Delta t} = at \]

However, if the change is very small, we indicate this as:

\[ \Delta \to d \implies v = \frac{dx}{dt} \]

Pronounce: the velocity \( v \) is the derivative or change of the position \( x \) with respect to time.

Or: the velocity describes the change in position due to a change in time, since \( \Delta x = v \Delta t \)
For the relation between the velocity and the acceleration, we also found that:

\[
\begin{align*}
    a &= \frac{v(t + \Delta t) - x(t)}{t + \Delta t - t} \\
    &= \frac{a(t + \Delta t) - at}{\Delta t} \\
    &= \frac{a\Delta t}{\Delta t} = a
\end{align*}
\]

Or, if the change is very small:

\[
    a = \frac{\Delta v}{\Delta t} \quad \Rightarrow \quad a = \frac{dv}{dt}
\]
Therefore,

\[ v = \frac{dx}{dt} \quad \text{and} \quad a = \frac{dv}{dt} \]

This means we can rewrite Newton’s second law as

\[ F = ma \quad \Rightarrow \quad F = m \frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{F}{m} \]

What if there is no net force working on the object? Then:

\[ m \frac{dv}{dt} = 0 \quad \text{or} \quad \frac{dv}{dt} = 0 \]

Now read it: The change of the velocity \( v \) due to a change in time is zero. This means that the velocity does not change as the time changes. Or:

\[ v = \text{constant} \]
However, 

\[ v = \text{constant} \]

in the absence of a net force, is just Newton’s first law: 
An object in motion stays in motion

So Newton’s first law is just a special case of Newton’s second law!

(Why are we still teaching them separately).
What about integration?

Integration has to do with the calculation of areas

\[ \text{Area} = \sum \text{area}_{\text{small}} \]

\( \sum \) is the summation over all the small areas
The smaller the small areas are, the better the total area is calculated. Preferably, we want the small areas to be infinitesimally small

But what does that have to do with mechanics?
We know that

\[ v = at \]

However, that is exactly the shaded area in the graph!
Let us see if we can get the expression for the position

We just found that \( v = at \)

Long ago, we saw that \( x = \frac{1}{2}at^2 \)

This is again the shaded area in the graph
Usually things are more complicated

\[ x = \text{Area} = \sum_{\text{all rectangles}} \text{area}_{\text{rectangle}} = \sum_t v(t) \Delta t \]

For infinitesimally small rectangles: \( \sum \rightarrow \int \) and \( \Delta t \rightarrow dt \)

This gives \( x = \int v(t) \, dt \)
So, knowledge of differentiation and integration allows one to switch between position, velocity, and acceleration. This works for any dependence on the time. Not just Galileo’s simple cases of constant speed and constant acceleration. Newton was ready to attack any problem!

\[ v = \frac{dx(t)}{dt} \quad \text{and} \quad a = \frac{dv(t)}{dt} \]

\[ x = \int v(t) \, dt \quad \text{and} \quad v = \int a(t) \, dt \]
HARMONICAE
M V N D I

\[ v = \frac{dx(t)}{dt} \]
\[ x = \int v(t) \, dt \]

\[ a = \frac{dv(t)}{dt} \]
\[ v = \int a(t) \, dt \]
Music by Henry Purcell (1659-1695)
Rondeau from Abdelazer 1695

James II

Britten 1913-76