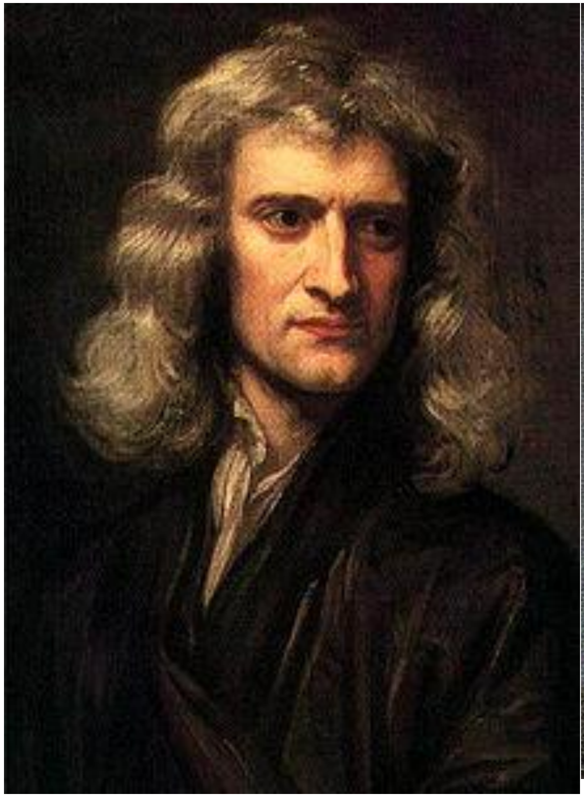
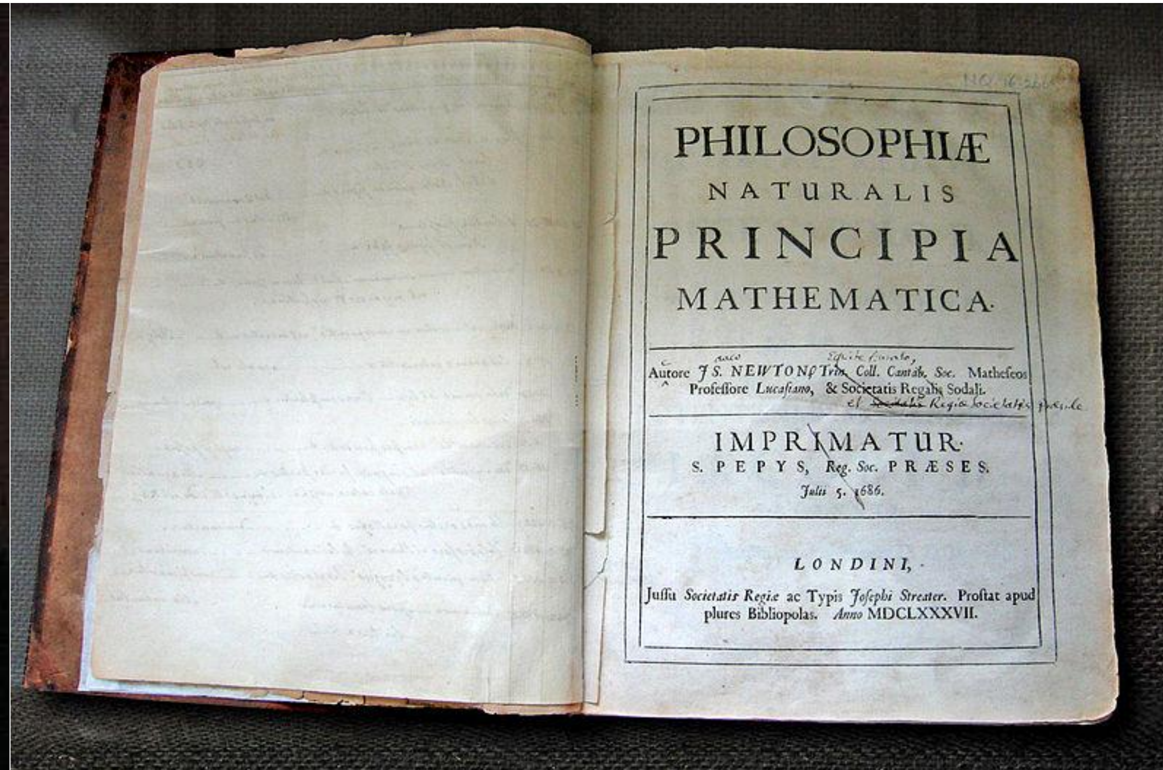
Classical Mechanics

Isaac Newton and the revolution in physics





Philosophiæ Naturalis Principia Mathematica, (Mathematical Principles of Natural Philosophy), or simply the ***Principia*** (1687)

Newton was born on Christmas day, 4 January 1643

Woolsthorpe Manor
Woolsthorpe-by-Colsterworth

How can January 4th be Christmas day?

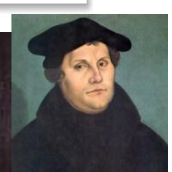
In the 17th century, England was still on the Julian calendar introduced by Julius Caesar in 46 BC

Each year is 365.25 days (a leap year every for years).
(pretty accurate…)

However, each year is 365.242189 days, this gives a difference of (365.25- 365.242189)x1700 years=13 days
(there was some additional fudging around with the calendar…)

To correct this Pope Gregory XIII introduced a new calendar, 1582

Now, England and other protestant countries had lots of reasons not to do what pope wanted, no matter how sensible.



?

Pope Clemens VII

?#@*&%!

Henry VIII

~1530s

So when it finally came to calendar reform in 1752, people lost 11 days of their lives….

William Hogarth:
An Election Entertainment (1755)

“Give us our eleven days”

This clearly demonstrates the need for:

- Accurate measurement of time
- The importance of understanding planetary motion
- *(and a need to work together on some issues…?)*

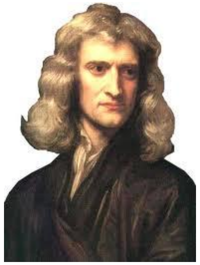
In 1661, Newton enters Trinity college in Cambridge

In 1665, Newton generalized the binomial theorem:

Blaise Pascal

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$(x+y)^{n+1} = \sum_{k=0}^{n+1} \binom{n+1}{k} x^{n+1-k} y^k,$$



Great Plague broke out in London, killing 100,000 people or 20% of the population

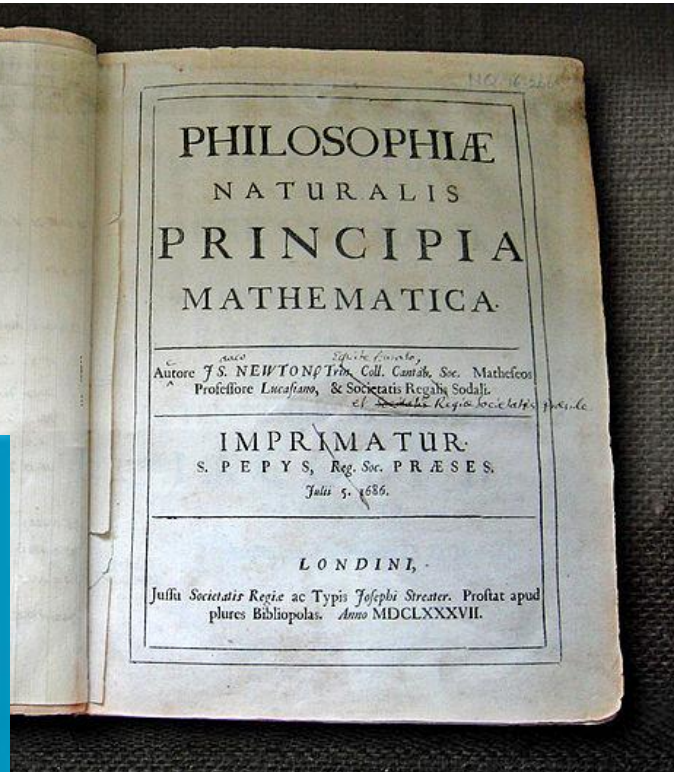
Great Plague 1665

Not the first time….

Black Death, 1348-50

Forcing Newton to return to Woolthorpe

However, those years back at the farm were not an entire waste



Newton

- invented infinitesimal calculus
- gained insights into the laws of planetary motion
- did major work in optics
- We'll come back to that later!

Newton’s first law

Let us start with the first law Newton wrote down:

Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

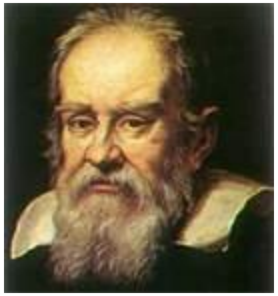
or in plain old English

Law I: Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed,

often abreviated to "a body in motion stays in motion."

Law 1: a body in motion stays in motion

- Not even Newton's….



and others

- There's no need to write it down, because it's contained in the 2nd law
- That's it? I could have thought of that….

- Why did Newton think it was important to write this down separately?
- Why is this not as trivial as it might sound to you?

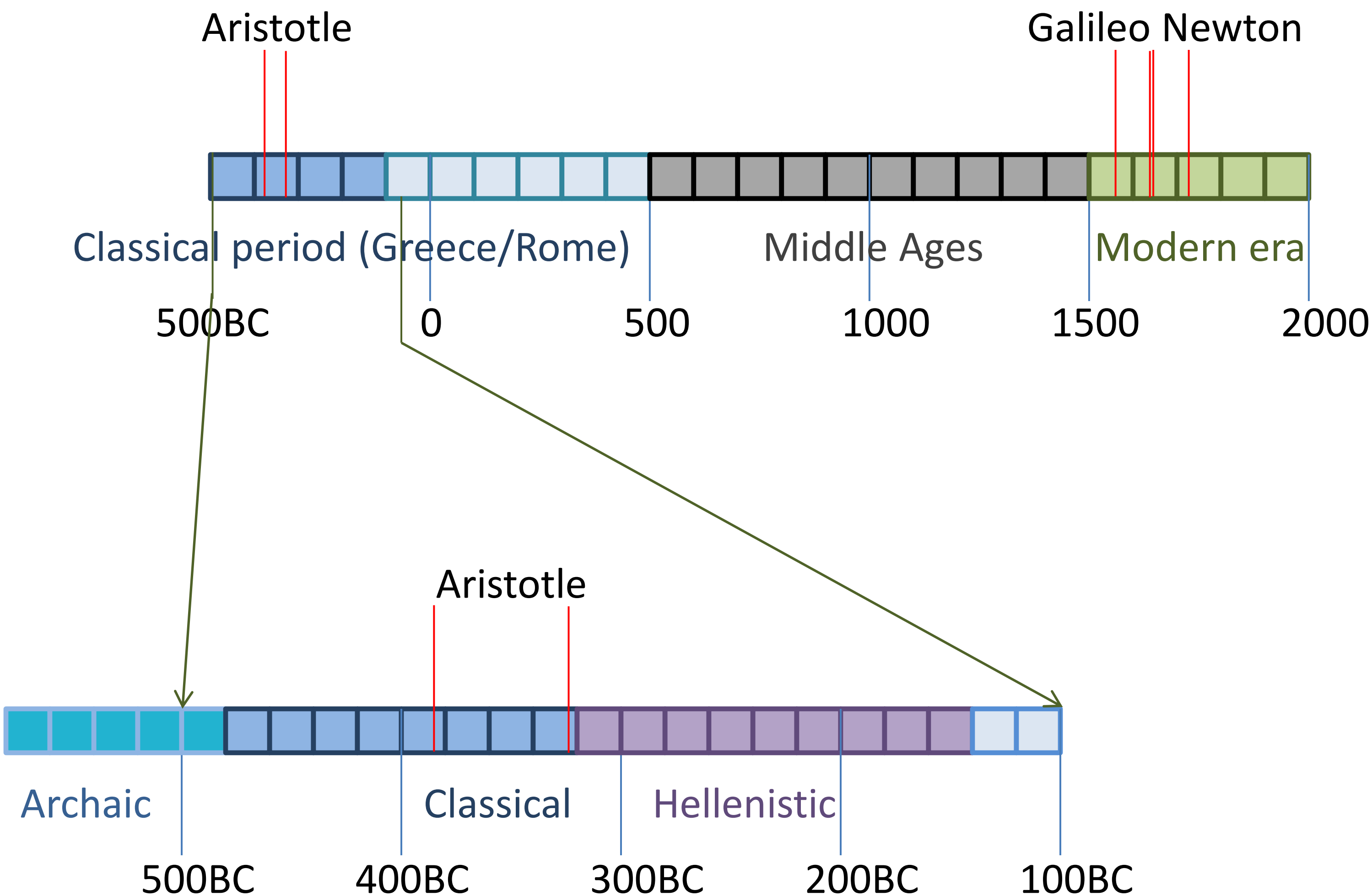
First, you have to figure out that what people were thinking and teaching for 2,000 years was wrong.

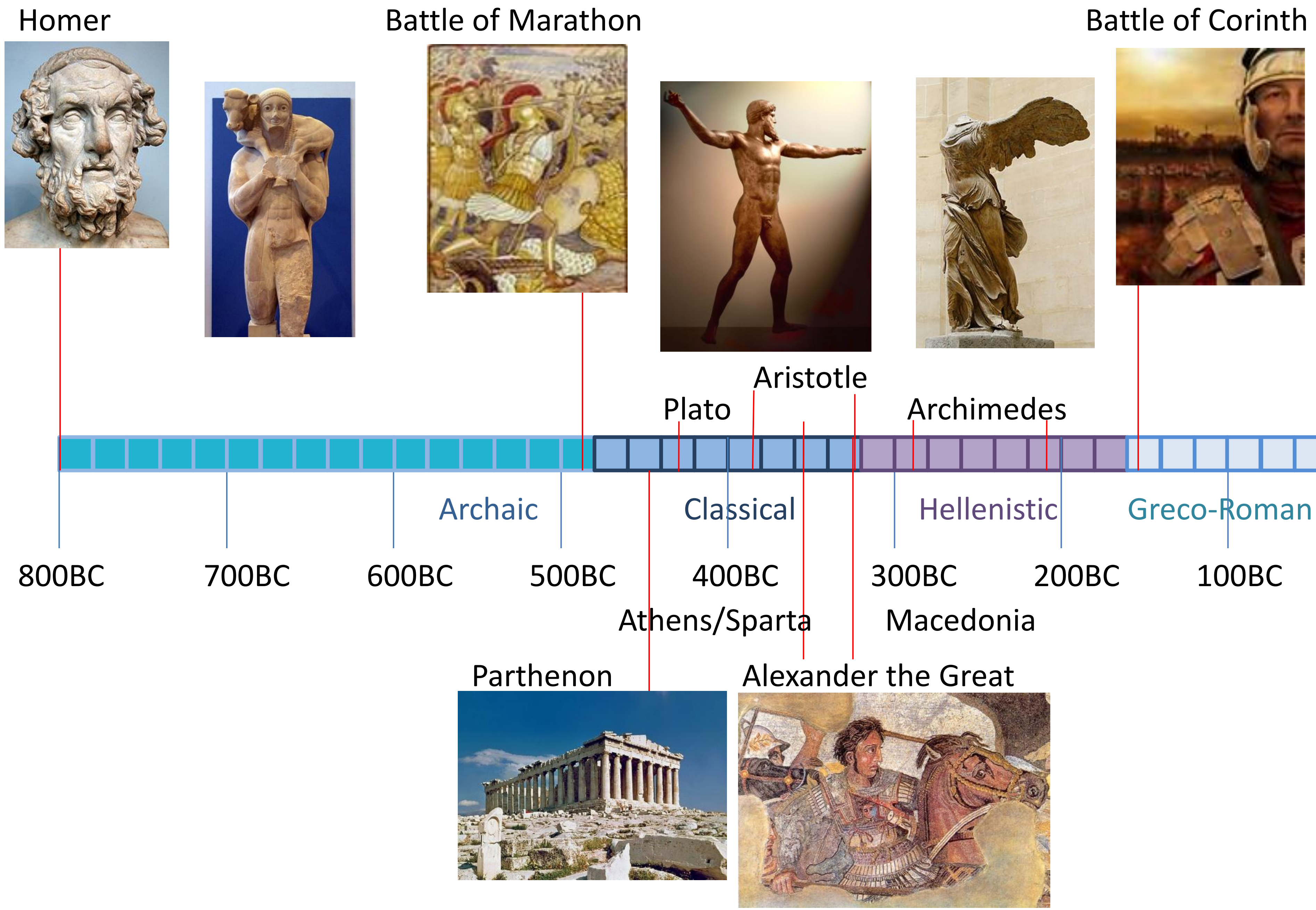
Second, you have to find an alternative….

Let’s go back in time:

What was the view on nature before the scientific revolution?

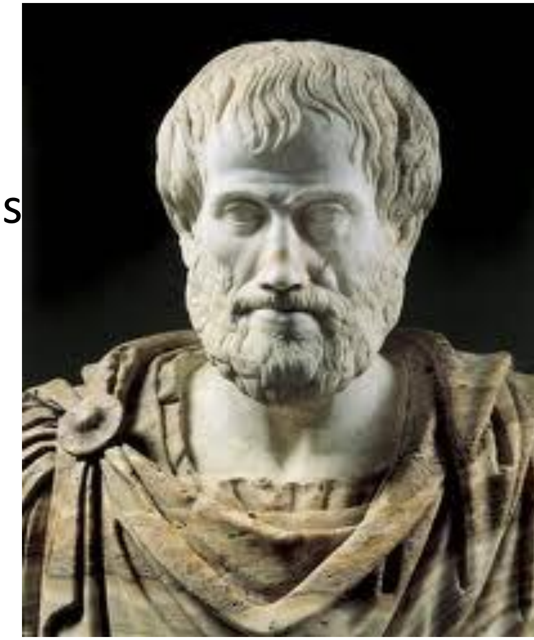
Aristotle’s writings were the dominating source of natural philosophy



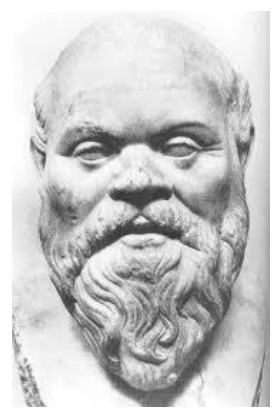

Homer
Battle of Marathon
Battle of Corinth
Aristotle
Plato
Archimedes
Archaic
Classical
Hellenistic
Greco-Roman
800BC
700BC
600BC
500BC
400BC
300BC
200BC
100BC
Athens/Sparta
Macedonia
Parthenon
Alexander the Great

- Aristotle was born in 384 BC in the small city of Stagira

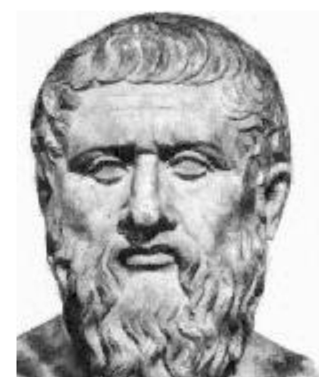
- At age 18, he became a student of Plato (who himself was a student of Socrates) in Athens



Aristotle (384 BC-322 BC)



Socrates



Plato

343 BC: Artistotle becomes the tutor of Alexander the Great (356 BC – 323 BC)

Other pupils of Aristotle

Cassander (350 BC – 297 BC), king of Macedonia

Ptolemy (367 BC – 283 BC), king of Egypt

Aristotle's metaphysics:

actual form

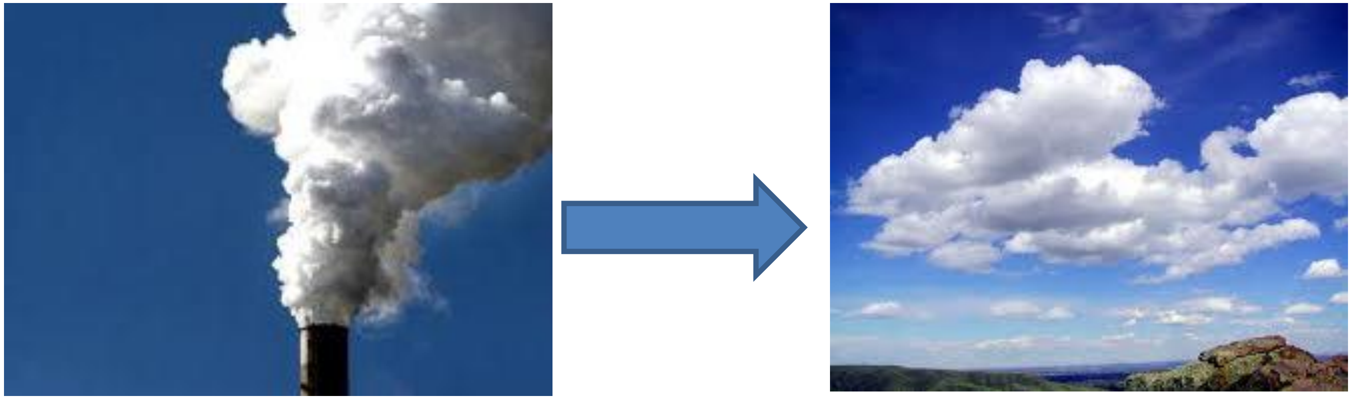
potential form

It is in the apple's *nature* to become an apple tree

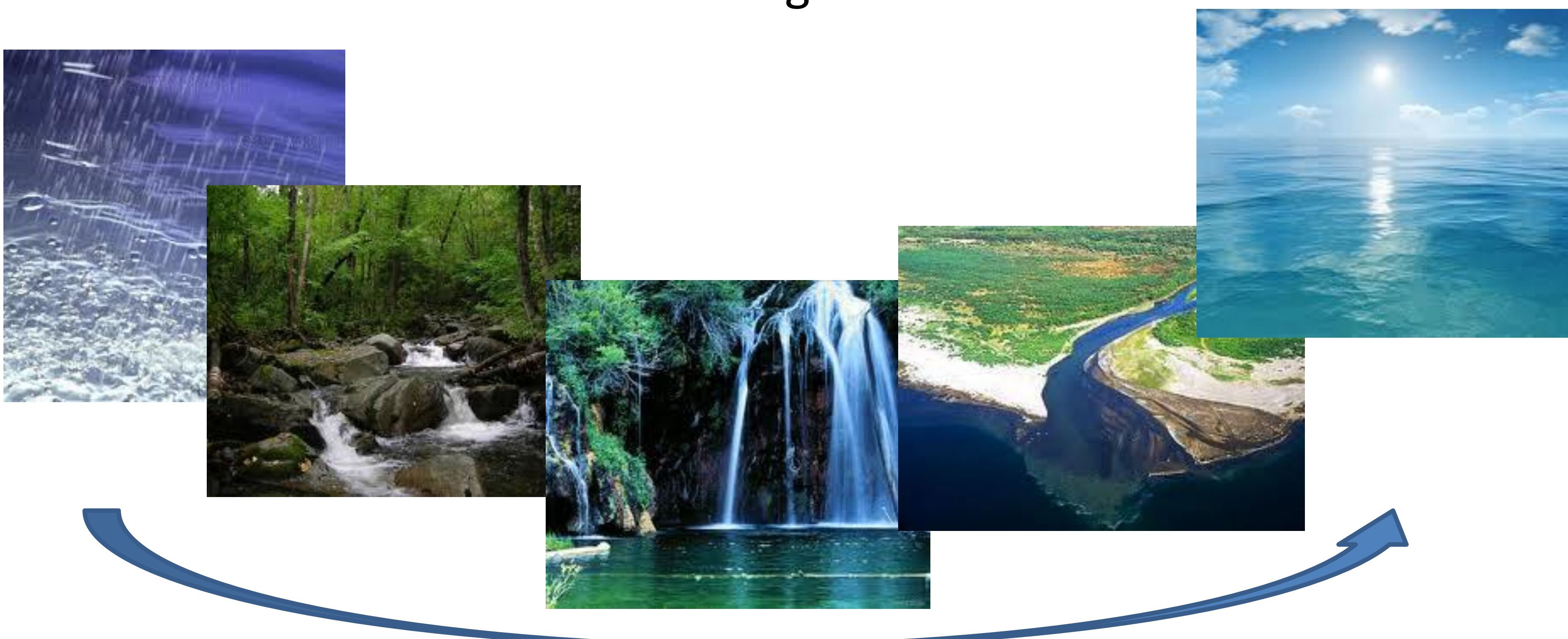
This very naturally leads to Aristotle's laws of motion

It is in the rock's *nature* to fall to the earth

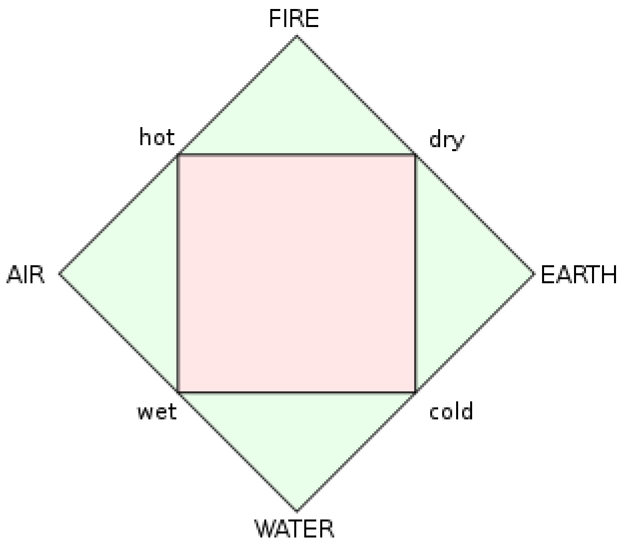
It is in the smoke's nature to go up to the clouds



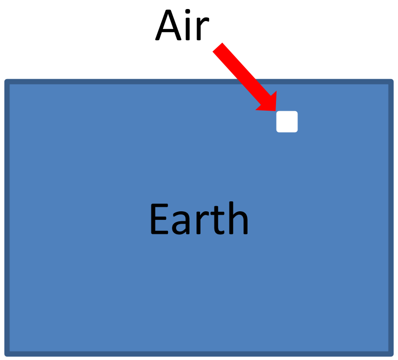
It is in the water's nature to go back to the ocean

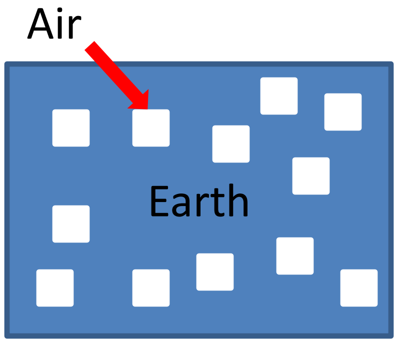


It also explains (qualitatively) why a rock falls faster then a feather



- a feather has more air in it than a rock, which mainly consists of earth
- A feather feels solid because it is mainly earth





This clearly does not explain every motion:

Natural motion

Forced motion

Since a object is at rest in its natural state, making it go forward will always require a force

However, what does such a force look like?

It must have something to do with the speed that you are moving

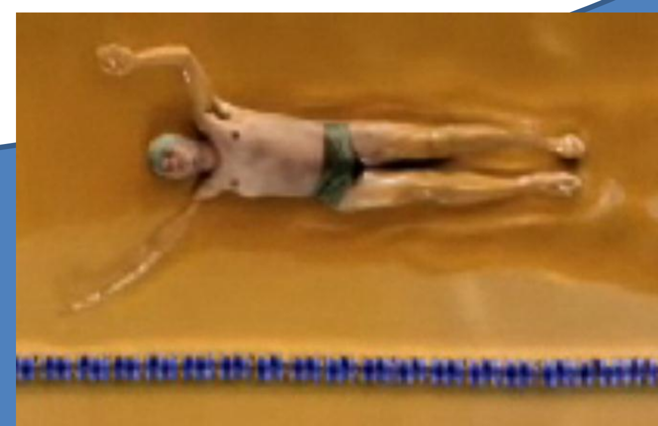
Low speed: less force

High speed: more force

It must have something to do with the difference in “nature”

more difference in nature

The less difference in nature, the more difficult to get out of your state of rest



less difference in nature

Equation:

$$F = R\,v$$

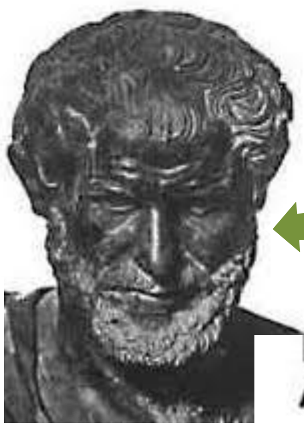
F is force

R is resistance due to difference in nature

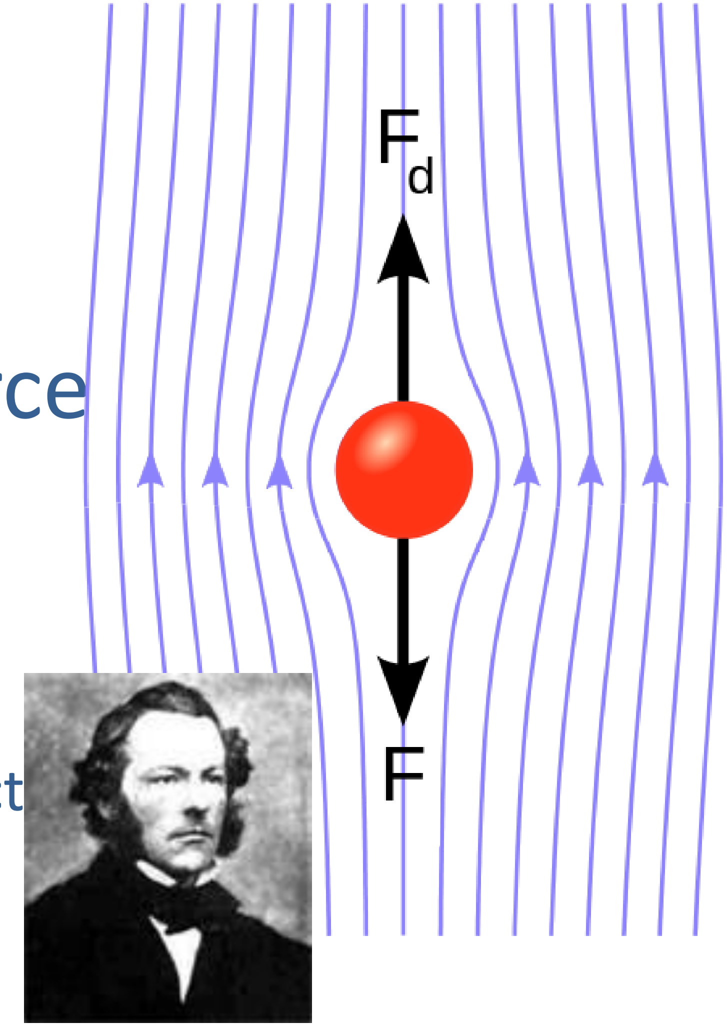
v is speed

1. To have something move at greater speed you need more force.
2. If the medium is close to the object, it is more likely to be at rest and the resistance R, and therefore the force F increases

Surely, this must be wrong, because Newton is right and Aristotle is wrong…



However, $F = R\,v$ is Stokes' equation



compare

$F = F_d$

with the drag or frictional force

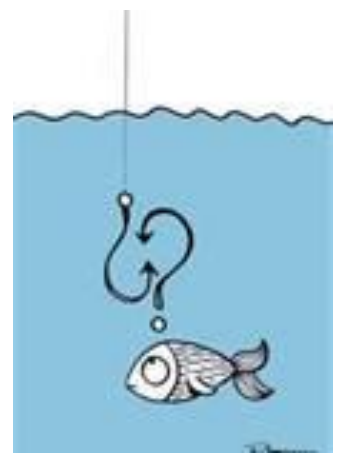
$F_d = R\,v$

Discovered by George Stokes in 1851 to describe the drag force on a spherical object in a viscous fluid

OK, this is complicated but the take-home message is that the equation itself is *not* wrong

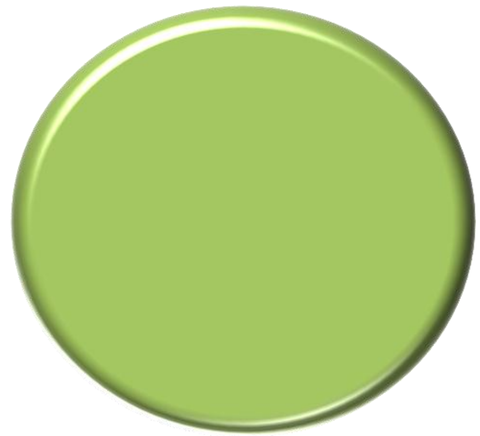
So if it is not wrong,

then what is the problem?

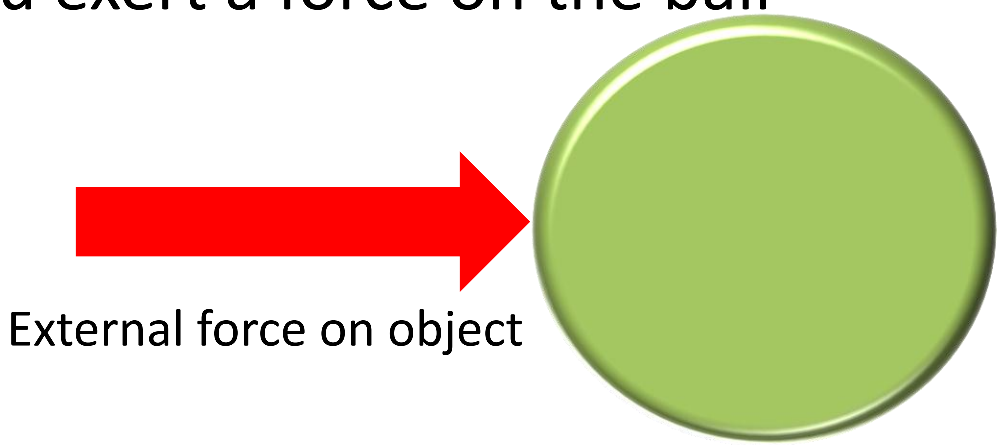


Newton, Galileo, and others

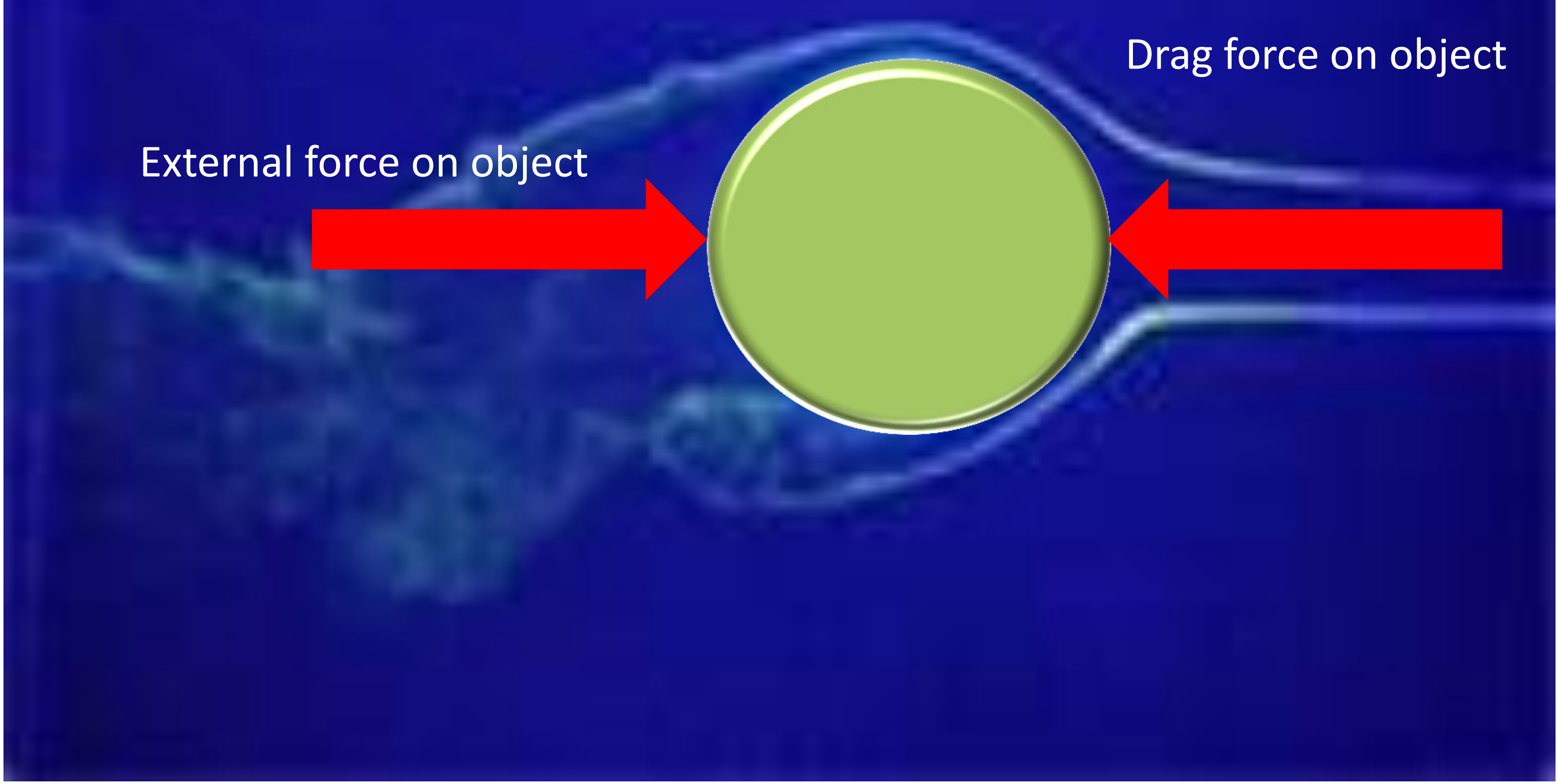
You want to move an object:



You exert a force on the ball



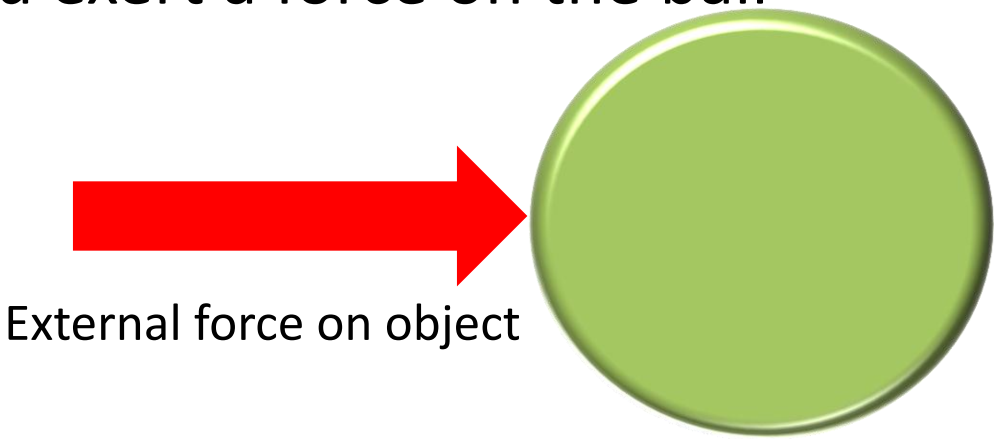
The medium then pushes back on the object:



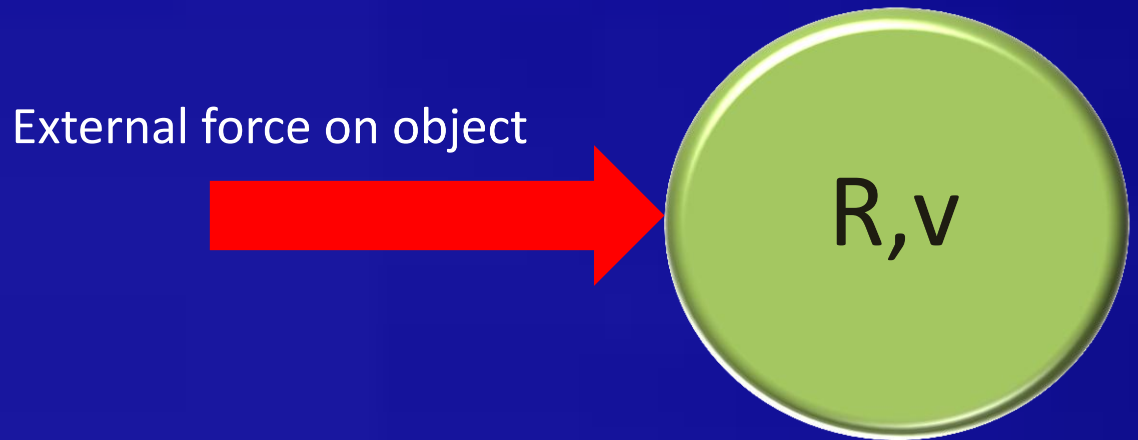
Aristotle:

You want to move an object:

You exert a force on the ball



The object then “decides” how much force needs to be applied to make it move from its natural state



Therefore:

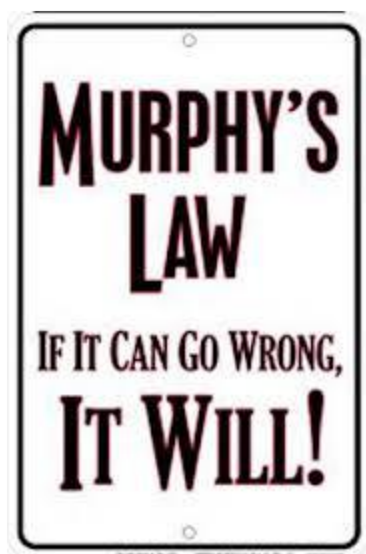
- Natural processes are internally goal directed (this does not mean that everything is alive)
- The reason a rock falls comes from the nature of the rock itself

But we arrive at the right equation, so what is the problem?

Just because you reach the right result,
does not make it correct

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

Interpreting a correct equation as a law
can lead to wrong results!



So let’s treat this as a law:

$$F = R\,v$$

F is force

R is resistance due to difference in nature

v is speed

We can also write this as:

$$F = R\,v \qquad \text{or} \qquad v = \frac{F}{R}$$

Given a force F, this equation tells you at what speed an object will move

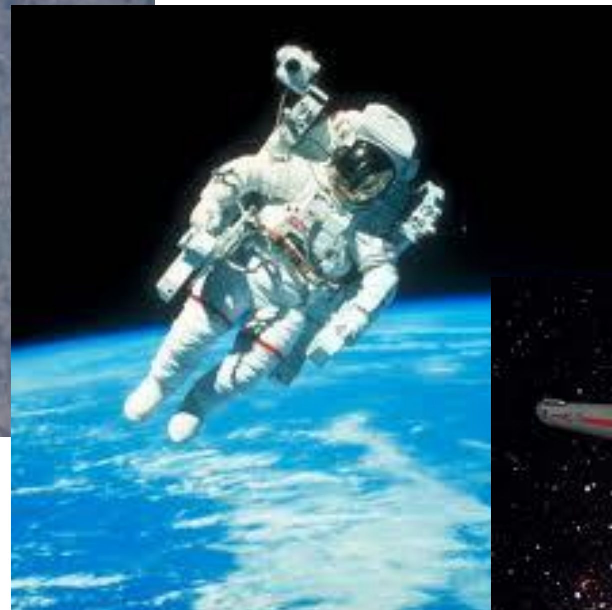
It must have something to do with the difference in “nature”

R big

The less difference in nature, the more difficult to get out of your state of rest, or the bigger R becomes

R small

Can we decrease R even more? Can we make it zero?



For the Greeks this is purely a thought experiment…

But they did think about it!

What if R=0?

$$v = \frac{F}{R} = \infty \qquad \text{or infinity}$$

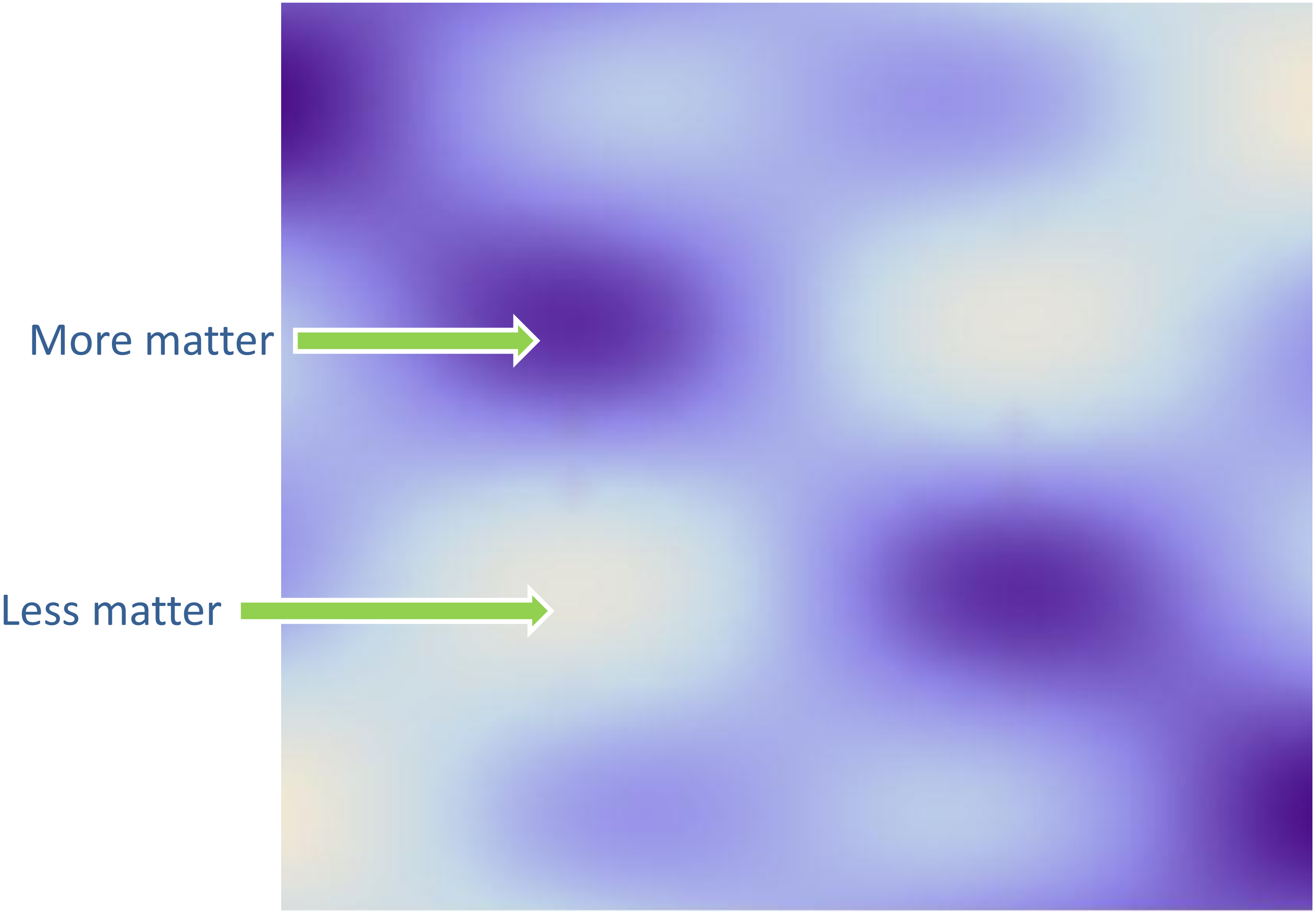
Even the Greeks realized the Impossibility of infinite speed!

However, instead of concluding that the equation is wrong, they concluded that a vacuum does not exist

Horror vacui
Nature abhors a vacuum

Hey, nobody had seen a vacuum in the first place…

Aristotle view on matter:
Matter is everywhere…

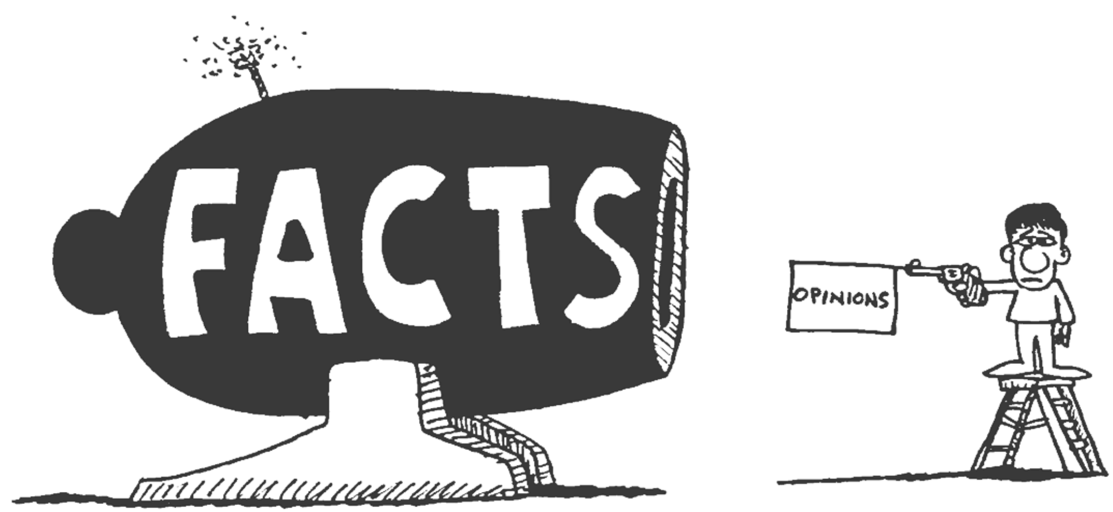


But never a vacuum! This issue remained problematic
Till the twentieth centure!

But how can you ignore measurements and facts?

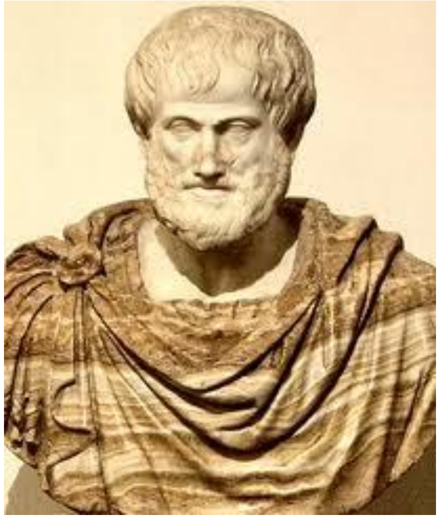
It's simple, everyone can do it.

- Measurements and producing facts are hard
- Understanding them is sometimes even harder
- We ignore measurements/facts when they conflict with our philosophy, opinion, religious beliefs, etc.



When it came to deciding the ‘truth’, a coherent philosophy was more important than actual measurements

- No problem understanding why a rock falls to the earth
- Heavier objects fall faster than lighter ones
- No problem understanding why an apple becomes an apple tree: it’s in their nature (very tough for modern science)
- Earth in the center of the universe



Aristotle: All objects have an internal goal

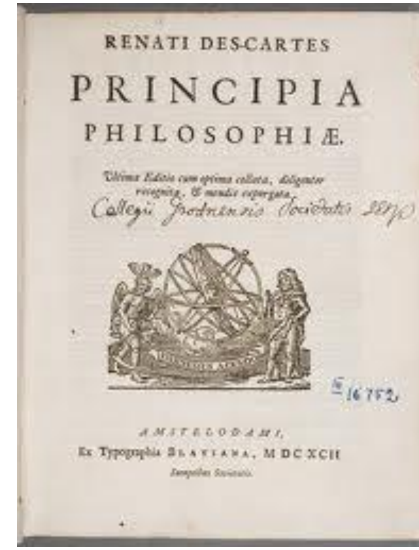
it's in my nature to go down and come to rest

This had to go too….


I'm going down!!
What's pulling me down??

Philosophy and facts

René Descartes (1596 -1650) introduced the mechanical philosophy



separated the purpose/goal, thinking, and spiritual world from the laws of physics

minds = thinking substance

Cogito ergo sum
I think, therefore I am

bodies = extended substance

Laws of motion apply only to bodies

God = infinite substance

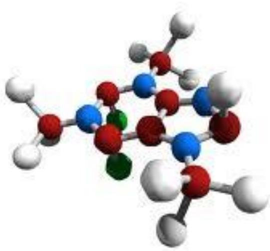
Spiritual world

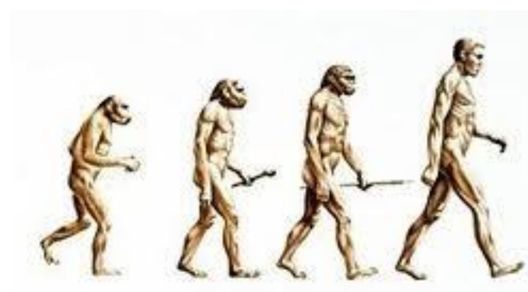
Ok, makes sense a rock is just a rock without a internal goal

=

However, where is the purpose/goal?

a mountain earth the universe……?

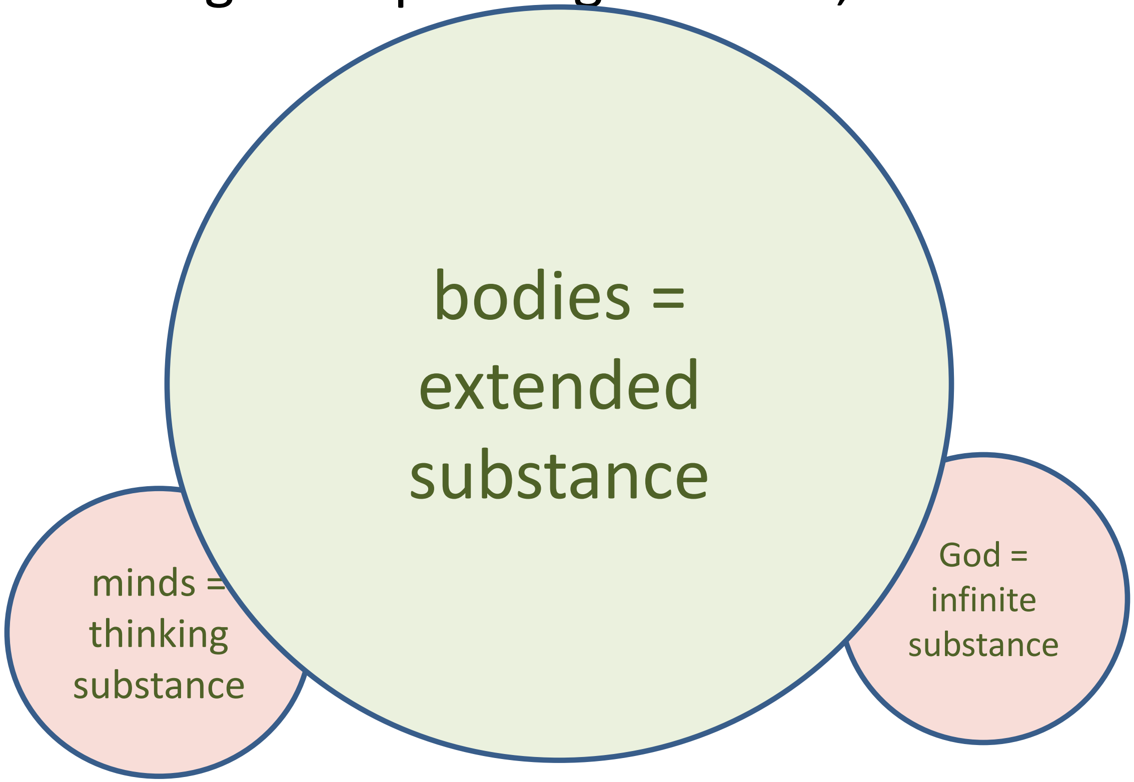




A molecule DNA cells humans….?

Where does purpose start? Or is there no purpose….?

Scientific knowledge is expanding so much,



The thinking and spiritual can feel threatened.

Is thinking just a bunch of chemical reactions…?

Is there only place for the spiritual at the beginning and the end?

ABOUT FACTS

Factual correctness is so "post-scientific revolution"

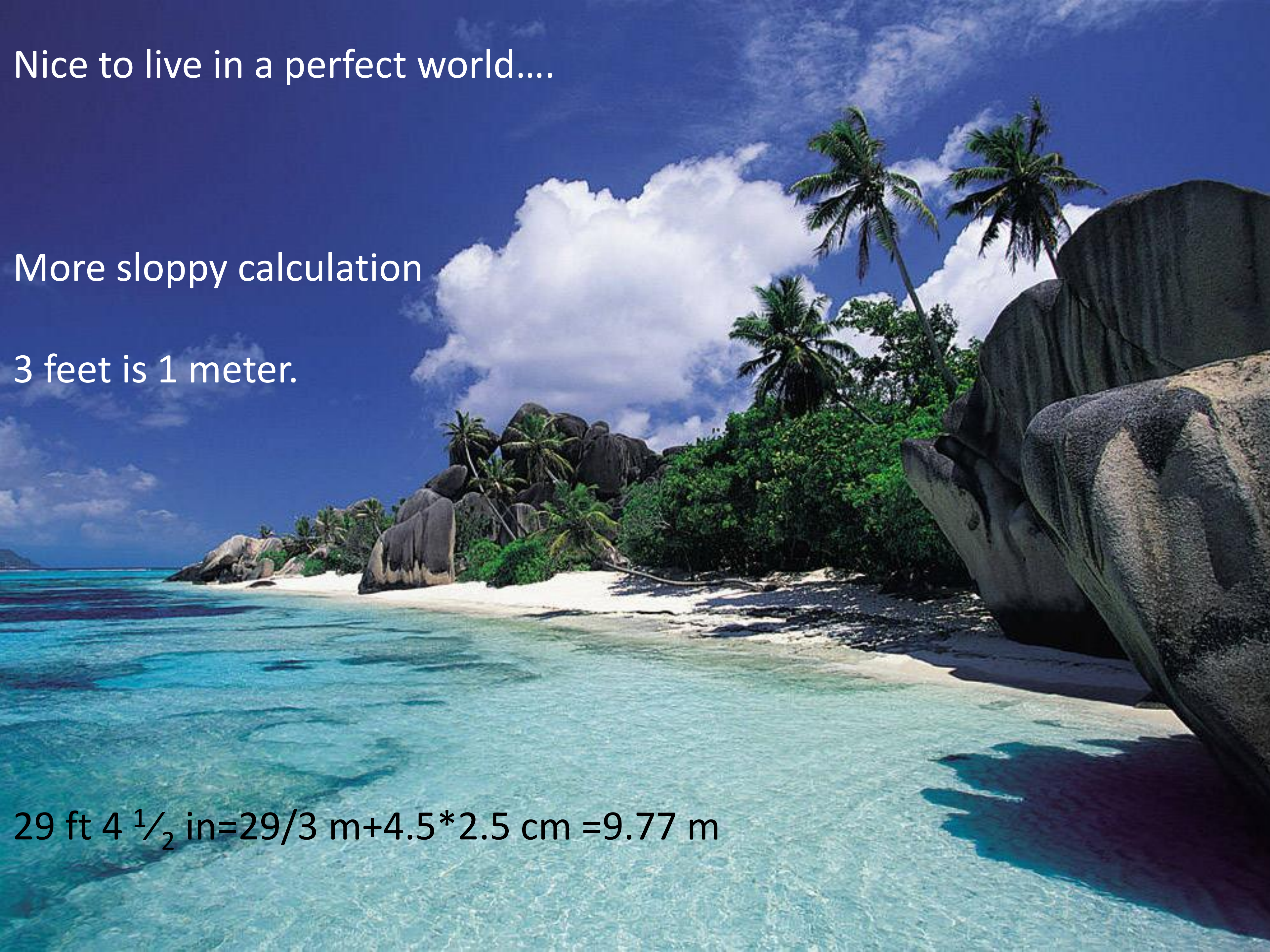
Example:

World record long jump

American: the record is 29 ft 4 $^1/_2$ in

Dutch guy: How much is that in units that everyone else uses?

American: 8.95 m



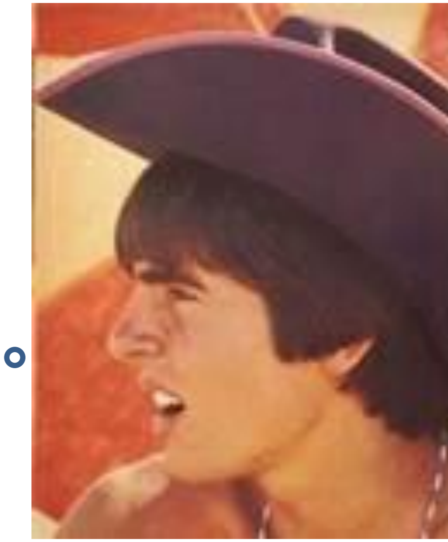
Nice to live in a perfect world….

More sloppy calculation

3 feet is 1 meter.

29 ft 4 $\frac{1}{2}$ in=29/3 m+4.5*2.5 cm =9.77 m

I have no idea
how much a
meter is……

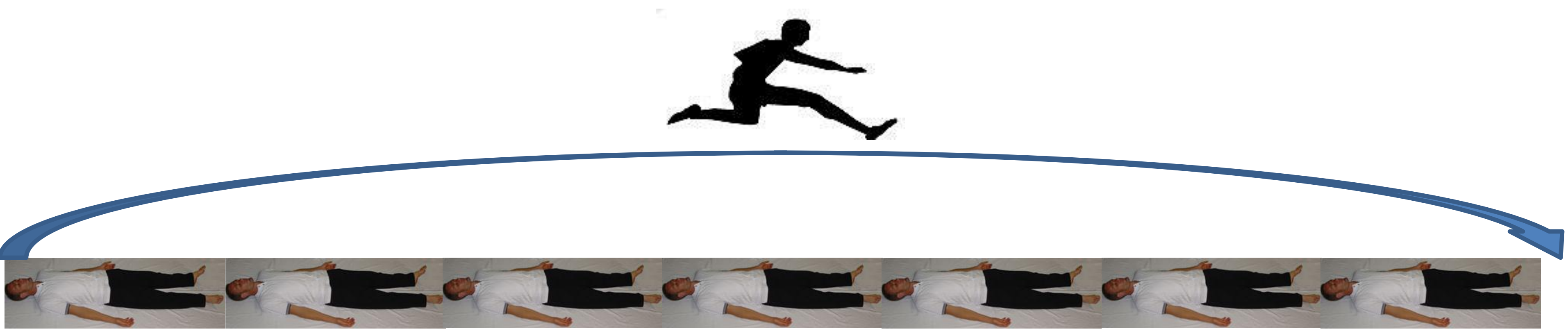


It’s about this many human beings in a row:

Ok, that just looks wrong

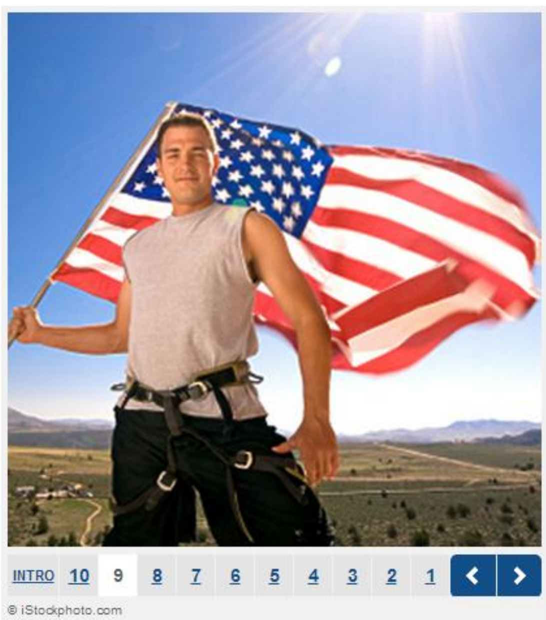
That’s better!

Wait as sec…That just doesn’t sound right. This long jumper dude is really great. I don’t want this Dutch guy to get a bad impression. This record has stood since 1991, which is awesome! Also he’s American!! So let’s add a person for good measure.



Obviously, the Dutch guy might have some opinions about what nationality is really the greatest…..

Top 10: Tallest Countries



No.9 USA

No.1 Netherlands

But the record in the mean time has crept up to 12.65 m or 41 feet and 6¼ inch

This sounds like a fish tale!

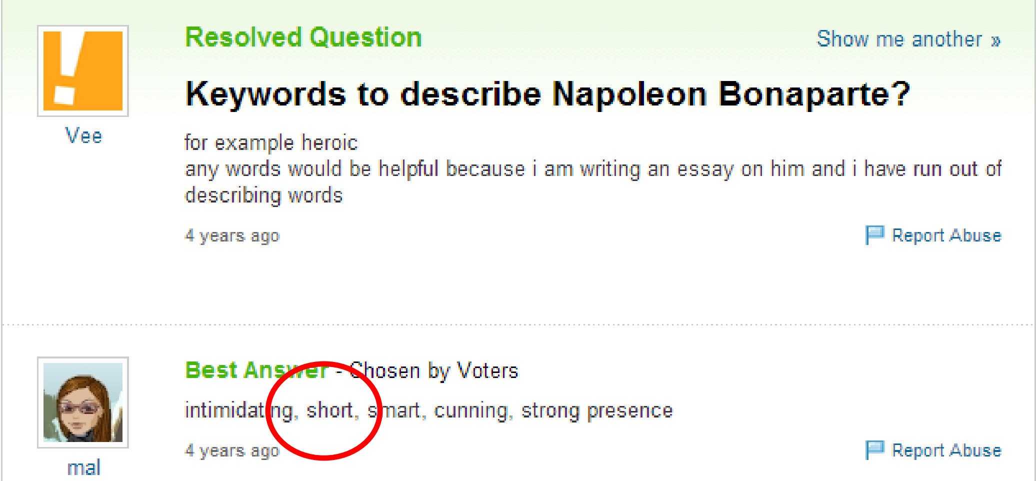
This is no longer a scientific fact,
but does that mean it is not true?

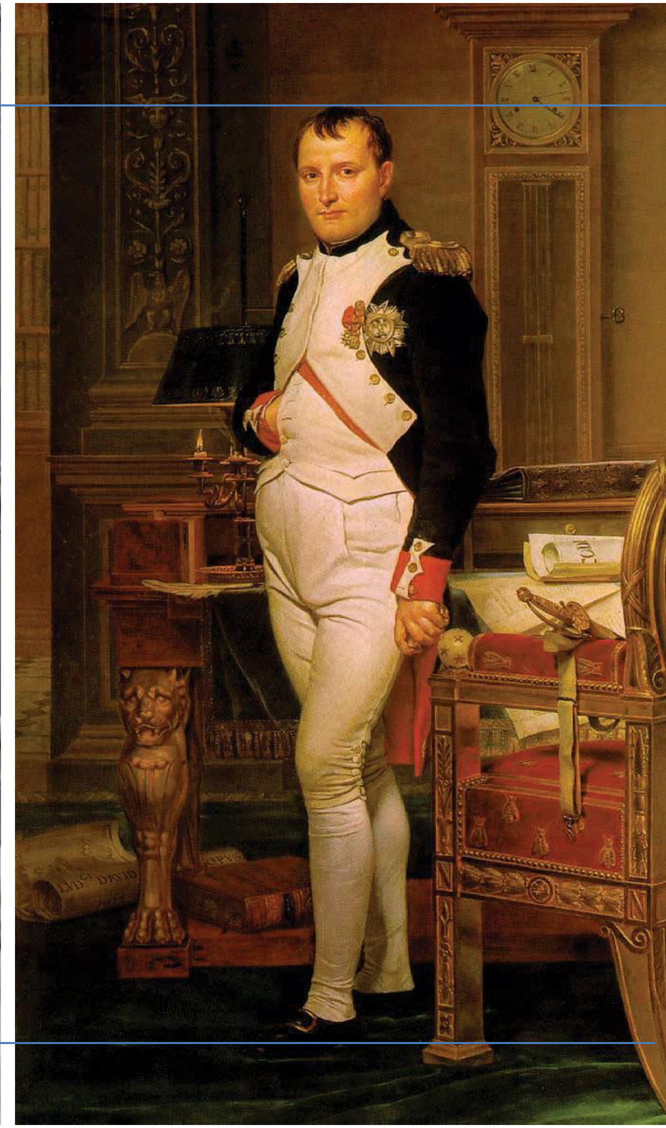
People familiar with the conventions of fish tales understand it. It actually becomes better/“truer” than real life.

Look, nowadays we just stick to the facts…

Quick:
Think of a word to describe Napoleon!

SHORT, Anyone?



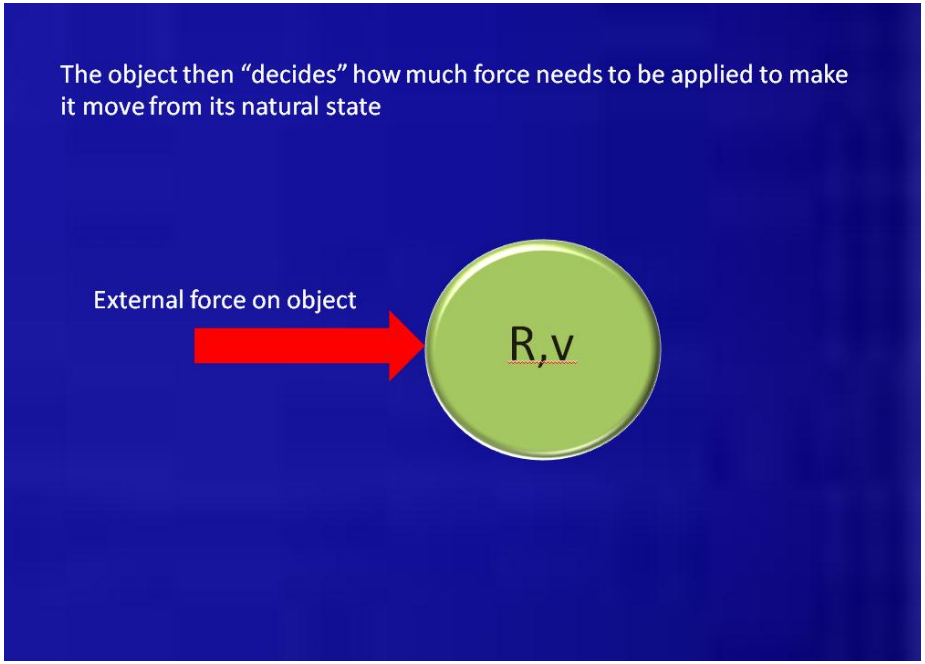


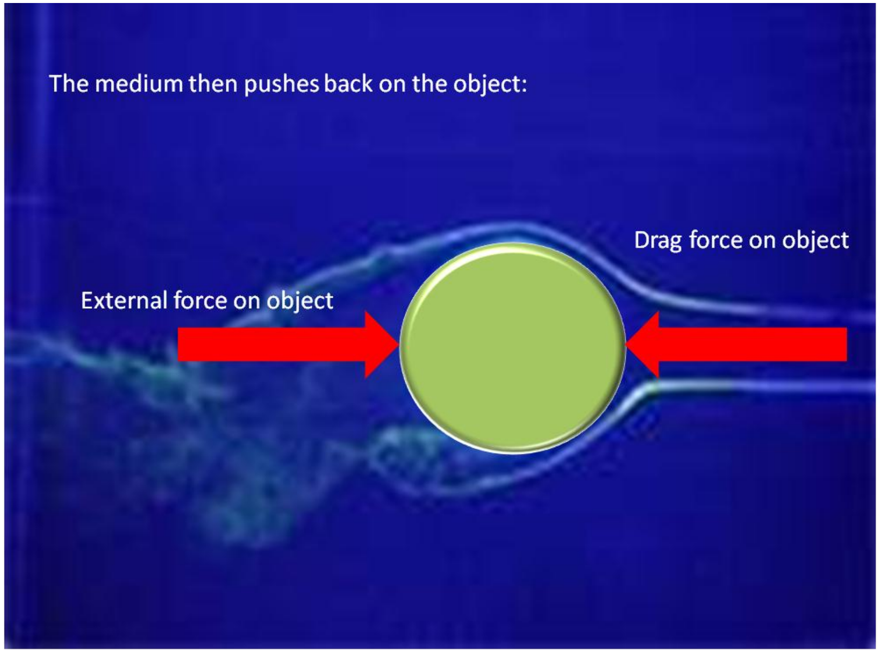
Typical Frenchman: 5’3’’

Napoleon: 5’6’’ (maybe 5’7’’)

However, the French units differed from the British and military records showed Napoleon as 5’2’’.

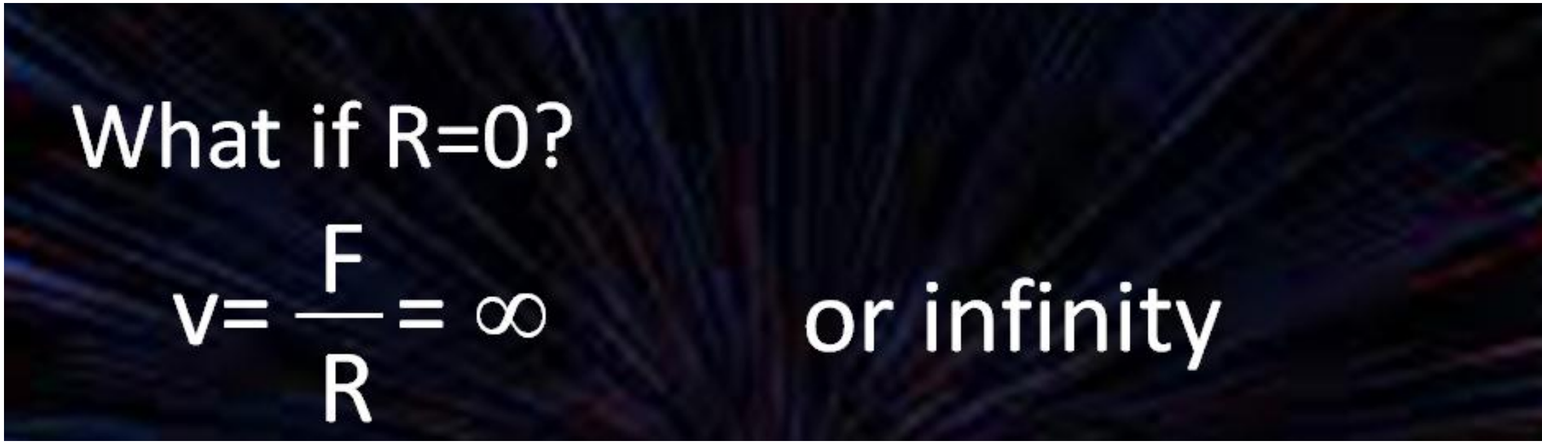
British propaganda obviously had a field day with that!

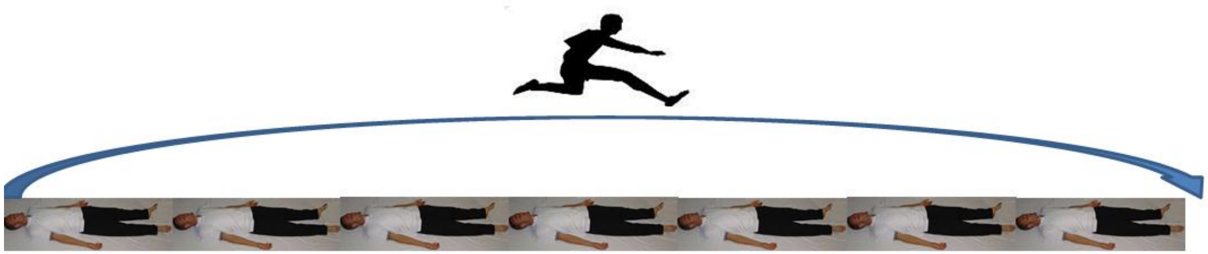




$$F = R\, v$$

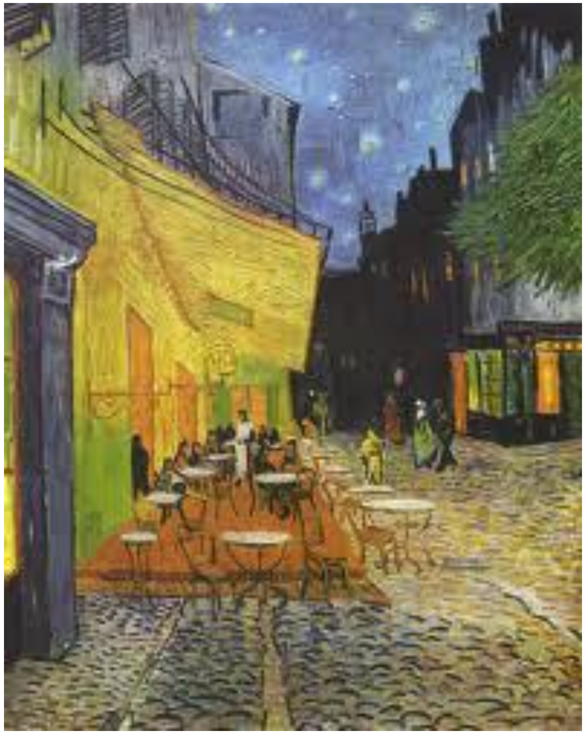
In vacuum





- Bad units
- No proper measurements
- Different agenda (philosophical, religious, political)

Jeanne Louise Calment
21 February 1875 – 4 August 1997
Lived 122 years, 164 days



Lived in Arles,
met van Gogh in 1888

Calment at age 20 in 1895

First emperor of Japan:
711 BC-585 BC

126 years!

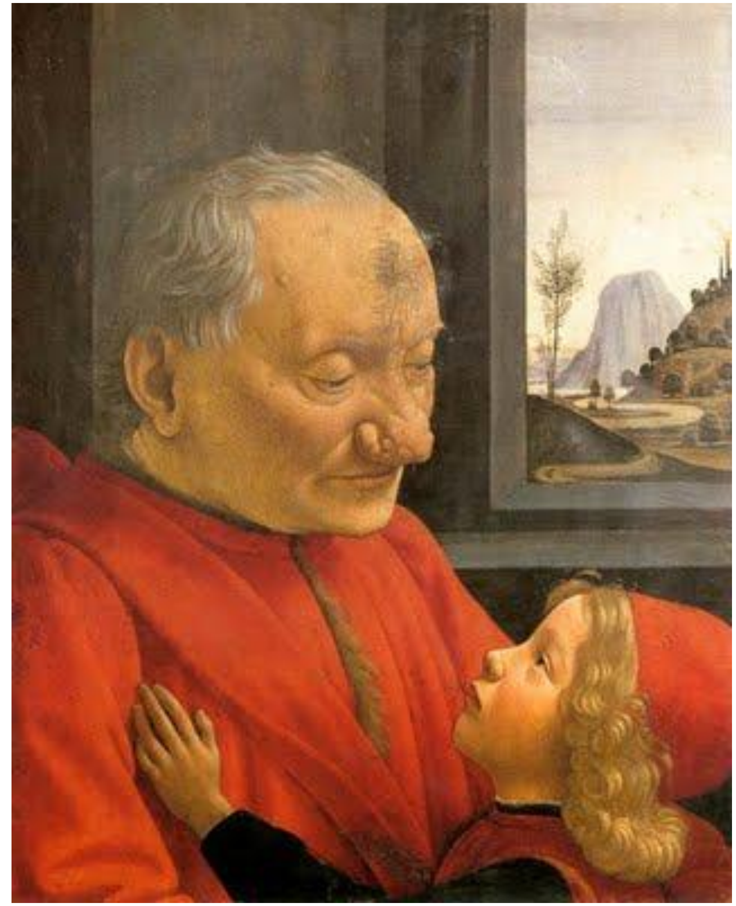
Kay Kavus, shah of Persia.

500 years!!!

Ok, scientific fact or legend…..

Methuselah Genesis 5:1-32

969 years!

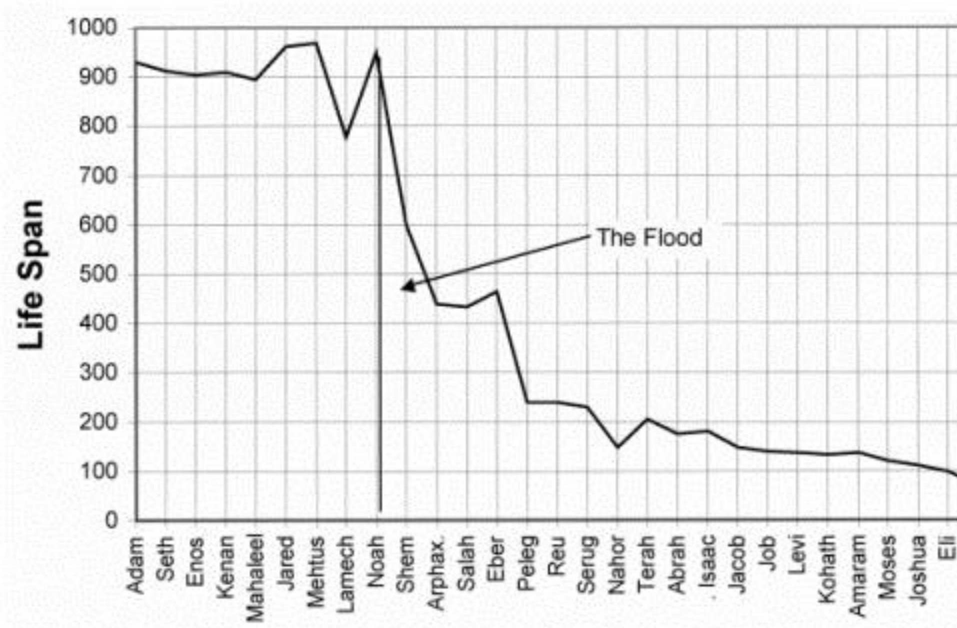


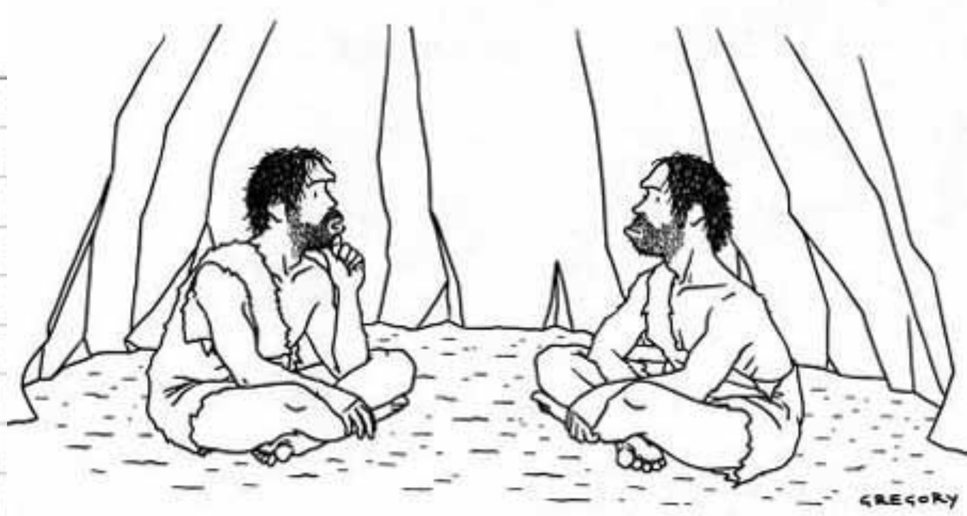
Oldest tree 4600 years. Methuselah tree…Great bristlepone pine tree

We can interpret these ages in two ways:

Scientific fact

- Condition were ideal
- After the flood people started eating meat
- In the flood where many people died , the longevity genes were lost (then again, Noah died when he was 950 years old…)





"Something's just not right—our air is clean, our water is pure, we all get plenty of exercise, everything we eat is organic and free-range, and yet nobody lives past thirty."

OR….

Literary device to indicate that older times were superior

- The bible was never meant to be written as a scientific text. That is post-scientific revolution.
- It was not uncommon to quote very long life spans in ancient literature. Sumerian kings lived more than 10,000 years

The bible is scientific fact, because it is inspired by God

Take the flood story:

- God plans to destroy humans because they have gone astray
- God warns Noah to build a boat and use pitch
- Every species of animal […] are to be saved […]
- The flood comes and destroys all life on the Earth
- The waters subside slowly and Noah sends out a dove, […] and raven
- The boat comes to rest on the top of Mt. Ararat

Clearly this comes from divine inspiration!

The bible is scientific fact, because it is inspired by God

Take the flood story:

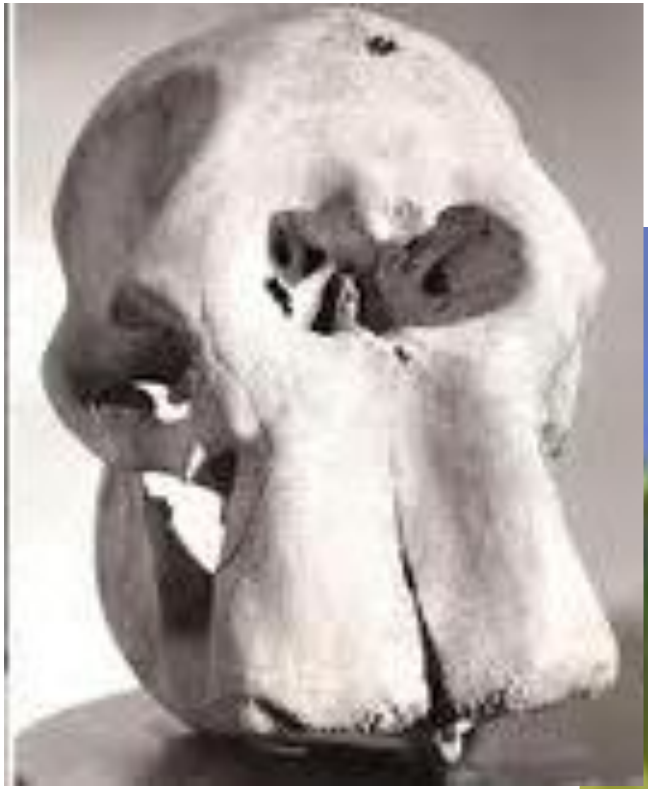
- The gods' plan_ to destroy humans because they have gone astray
- The god Ea warns Utnapishtim to build a boat and use pitch
- Every species of animal and of craftsmen are to be saved, as well as their family
- The flood comes and destroys all life on the Earth
- The waters subside slowly and Utnapishtim sends out a dove, swallow and raven
- The boat comes to rest on the top of Mt. Nisir

- However, this was a modified version of the flood epic in the Gilgamesh epic
- It is significantly older (~2000 BC) than the Bible (Exodus ~1290-1235 BC during the reign of Rameses II).
- Still this does not make it wrong, but if it is divine inspiration then why does it mention gods (plural!)

Why flood myths?

- There could have been a really big flood (tsunami)!
- Fossils of shells and fish are found on mountain tops

People have made up interesting explanations for things they did not understand:



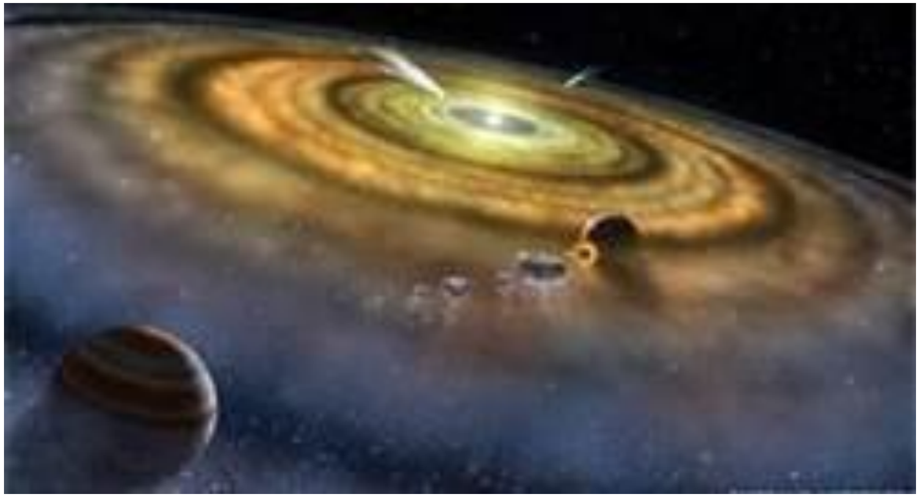
Cyclops skull??!!

No, the skull of a dwarf elephant.

How old is the earth:

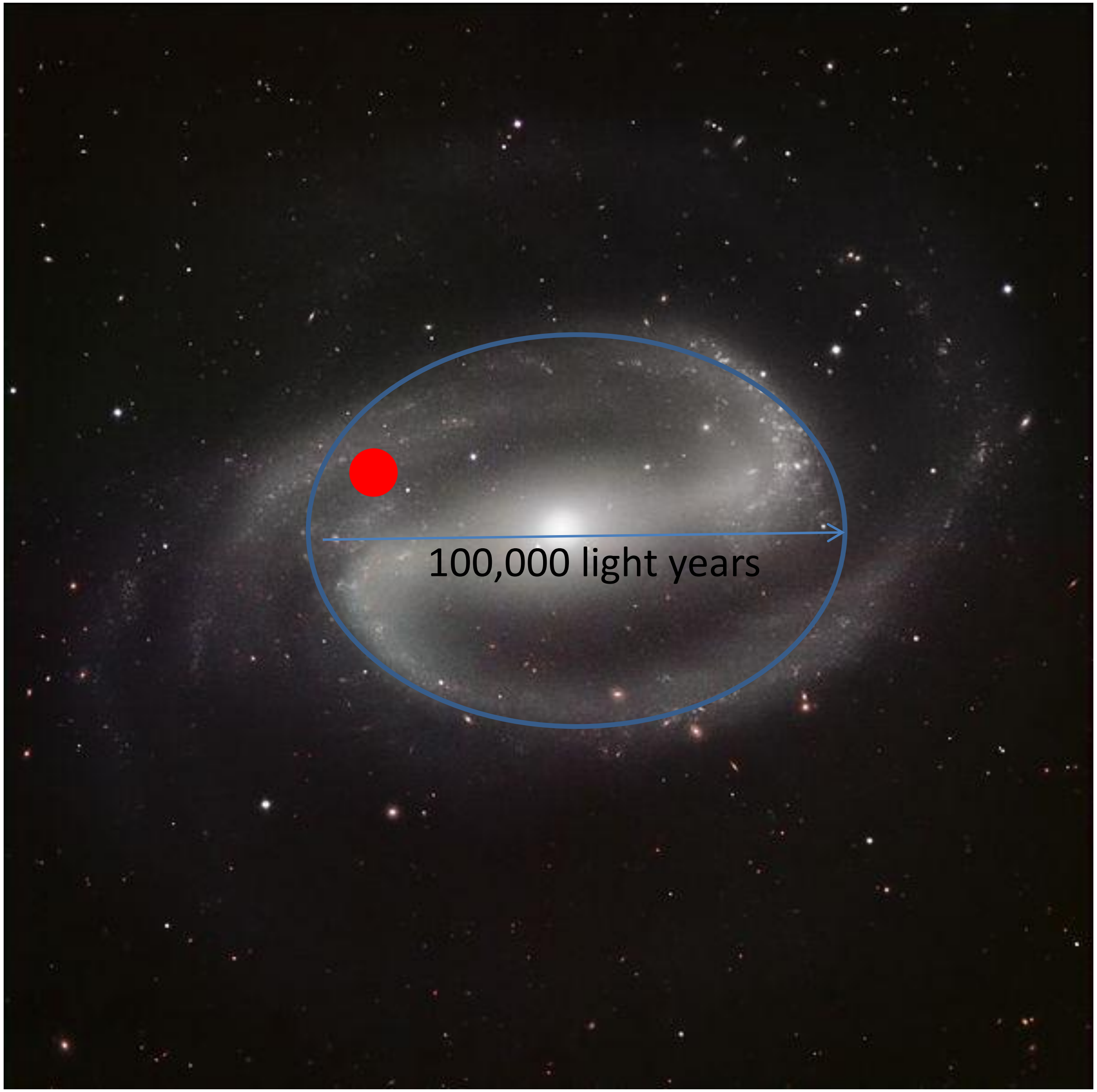
Counting the genealogies, about 6,000 years:

Treat Bible as scientific book and see 6,000 years as the geological age.



Contradicts current scientific thought of 4.5 billion years.

We would only be able to see the stars coming from less than 6,000 light years away



1 light year: the distance light travels in one year

But how can we see the Andromeda constellation?

2.5 million light years

What about these guys: up to 12 billion light years…..

Other option: Do not read the Bible as a scientific document:
It was written before the scientific revolution and the geological development of the earth was not really relevant in ancient times

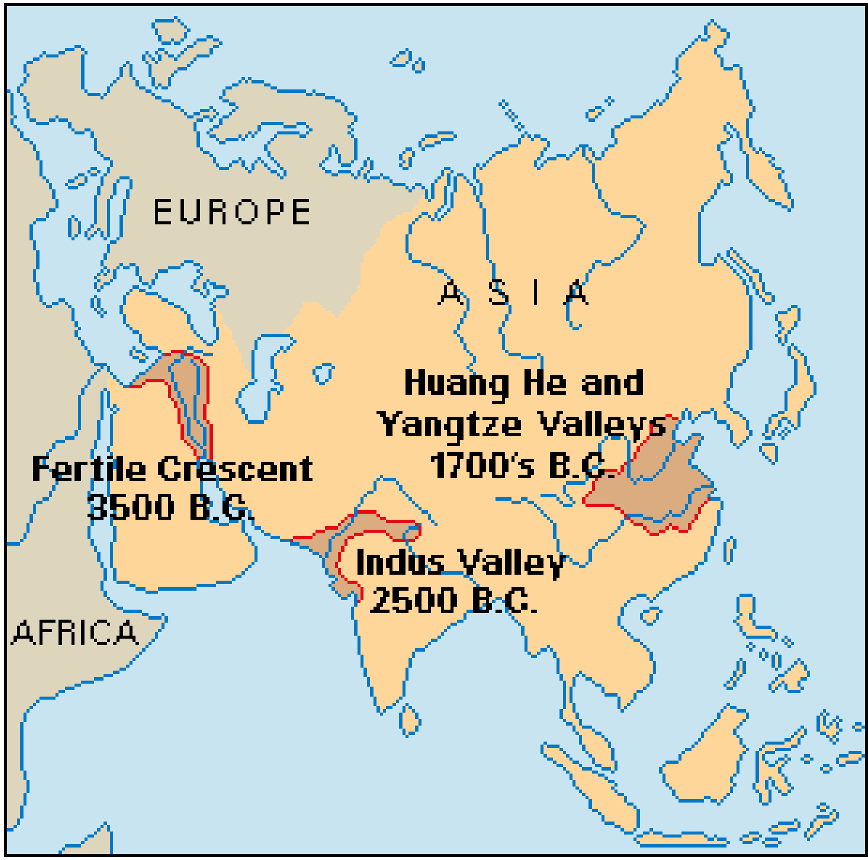
Just because something is not 100% correct, does not mean it is not true

Let us pose a different question, of more relevance in the ancient world

When did civilization start?

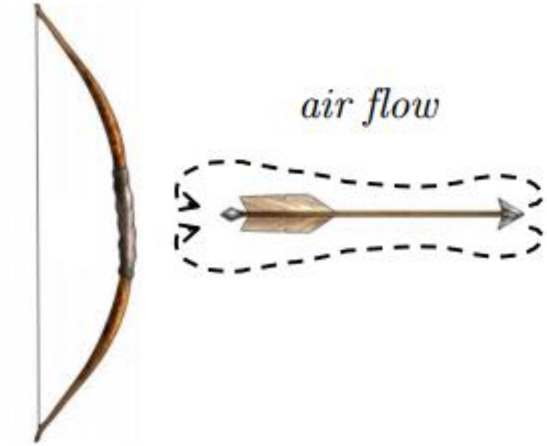
Eufrates and Tigris river civilizations started 5,500 years ago.

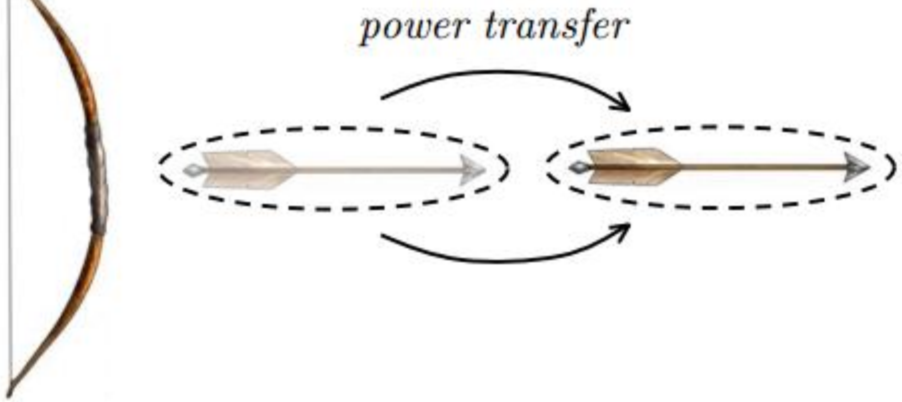
The "world" began 6,000 years ago, not a bad estimate at all!



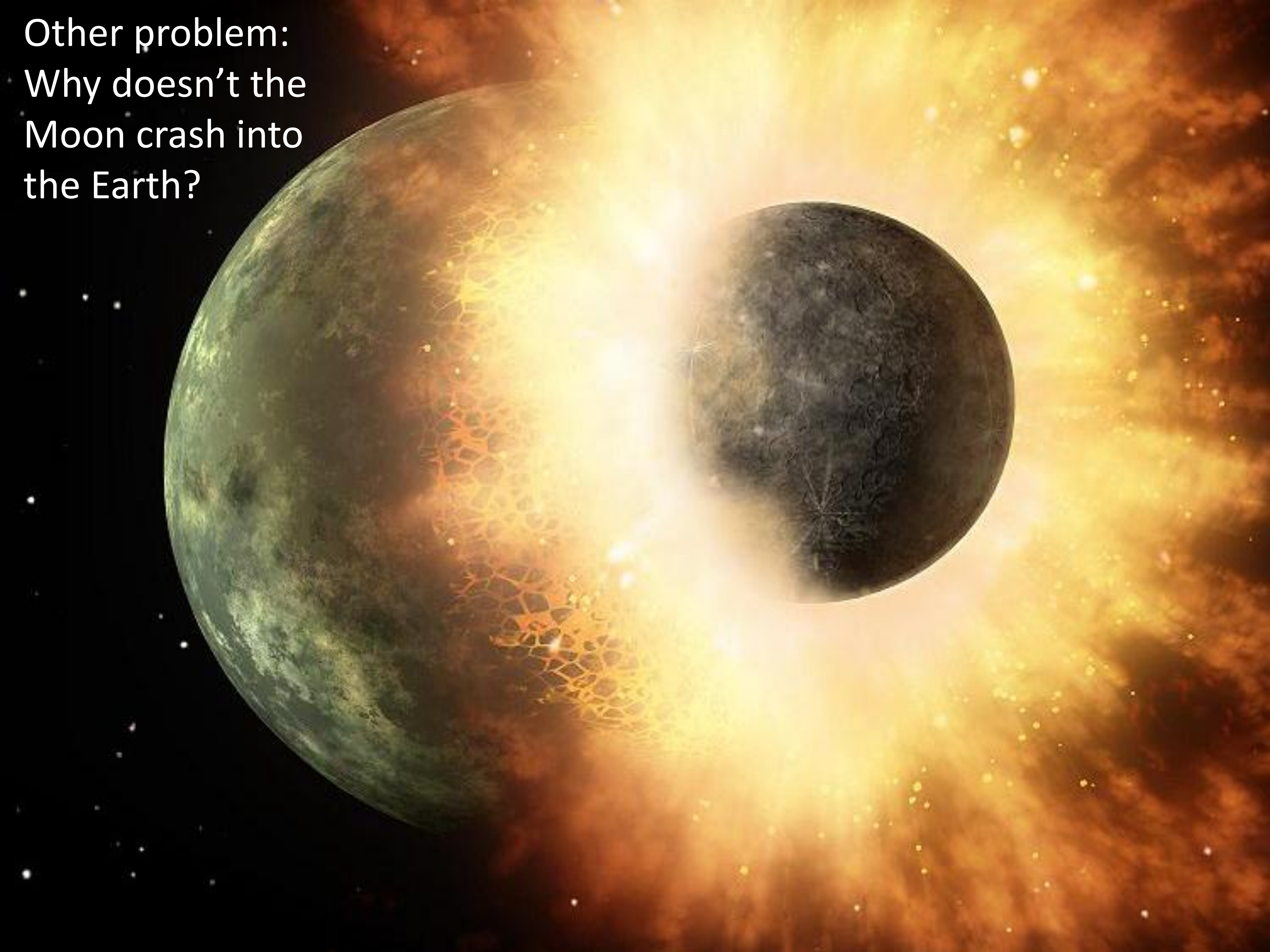
Problems with Aristotle’s natural philosophy

Aristotle's problem with projectiles:
If an object needs a force to keep moving, how does an arrow fly?





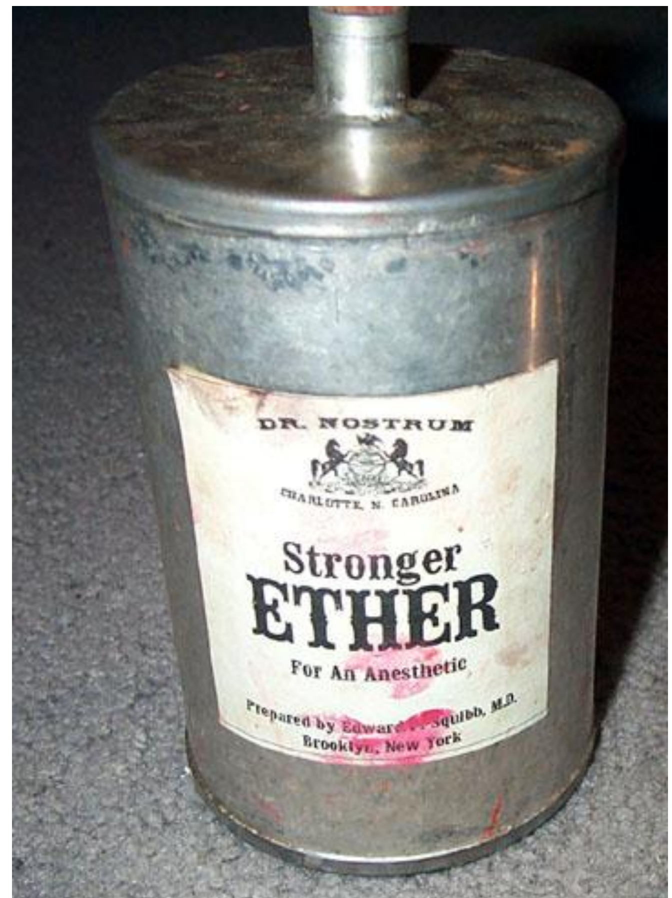
All kinds of convoluted explanations, but none really satisfactory


Other problem:
Why doesn't the Moon crash into the Earth?

If it is in the rock's *nature* to fall to the earth,
Then why doesn't the Moon drop to Earth.

Couple of options:

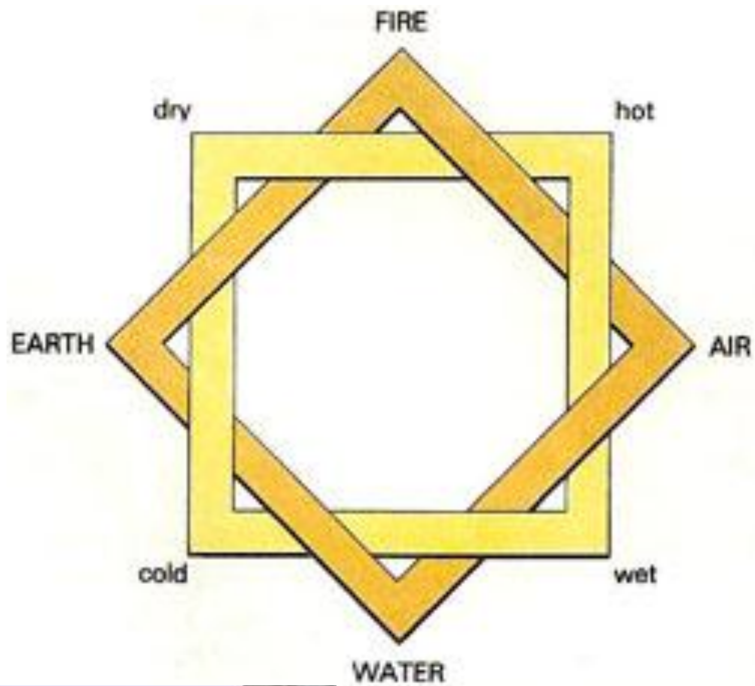
- The Earth, planets, Sun, and stars are stuck to a solid sphere (the firmament) beyond which the Gods live
- The celestial elements are in a different element aether, where the natural laws do not apply



The usual four classical elements:

combustion/chemical reactions

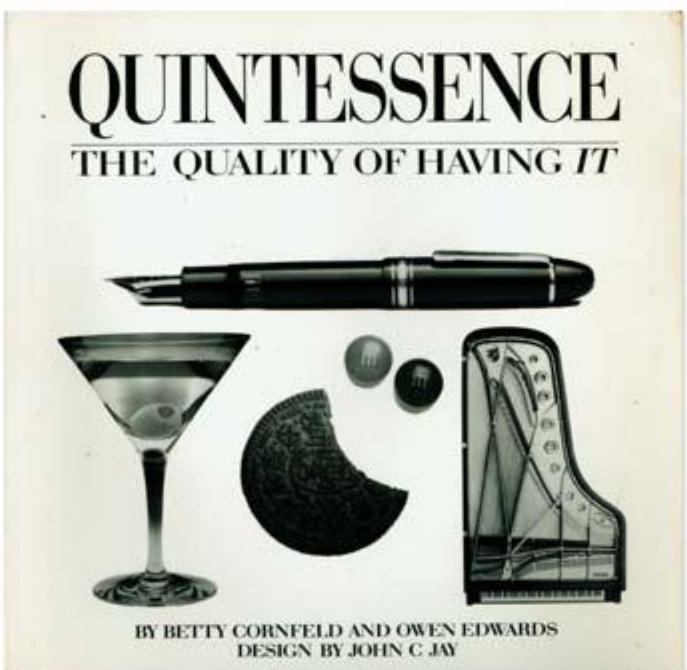
solids

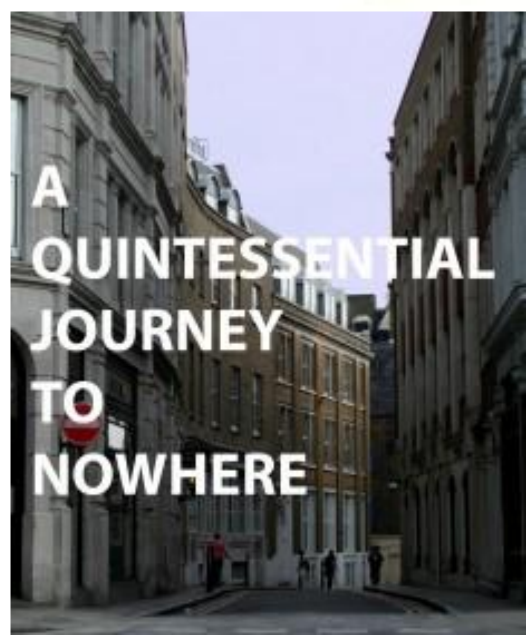


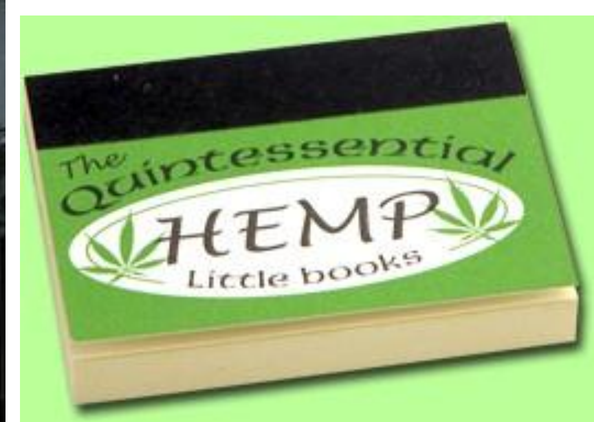
gas

liquid

Aether=fifth element
=quintessential







Surprisingly common:

Classical Elements V · T · E

Babylonian

Earth
Sea Fire **Sky**
Wind

Greek

Air
Water **Aether** Fire
Earth

Hinduism (Tattva)
Buddhism (Mahābhūta)
Jainism (Tattva)

Vayu
Ap Akasha Agni
Prithvi

Chinese (Wuxing)

Wood (木)
Water (水) Fire (火)
Metal (金) Earth (土)

Japanese (Godai)

Air (風)
Water (水) Void (空) Fire (火)
Earth (地)

Tibetan (Bön)

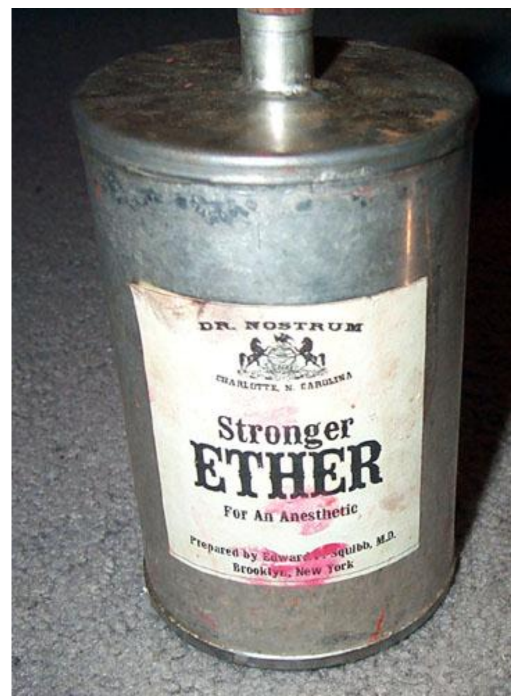
Air
Water **Aether** Fire
Earth

Medieval Alchemy

Air (□)
Water (□) **Aether** (□) Fire (□)
Earth (□)
Sulphur (□) Mercury Salt (□)

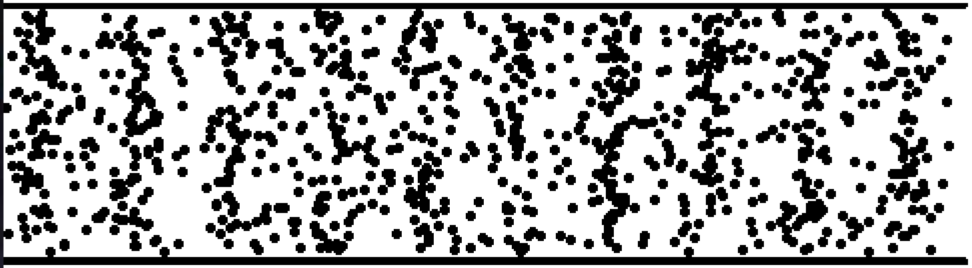
And

Aether: surprisingly hard to get rid off....

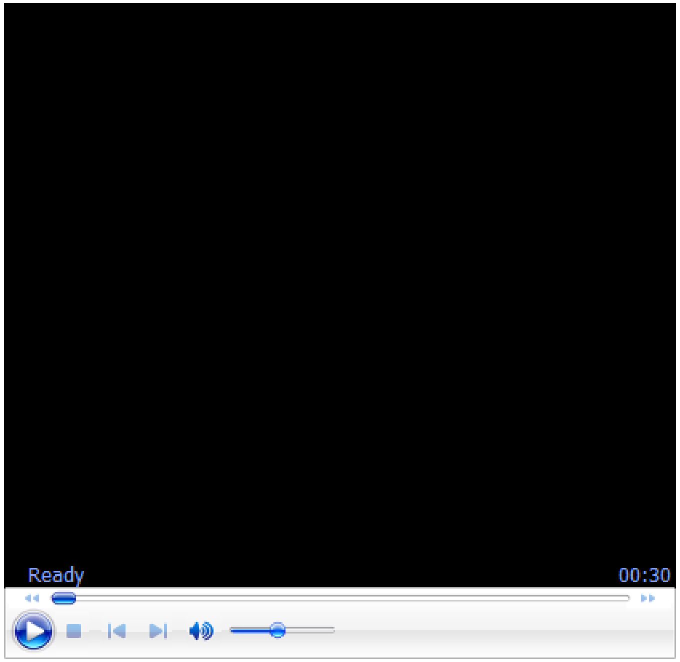



Fundamental question: How does light propagate?

Common thought: just like sound, lights needs some medium



Sounds waves are vibrations of the molecules in the air



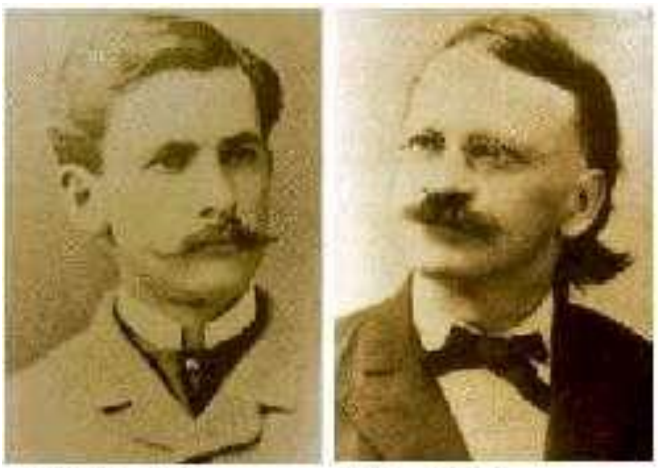
Change the medium,
Change the sound

We know that light travels in space

But we cannot talk in outer space

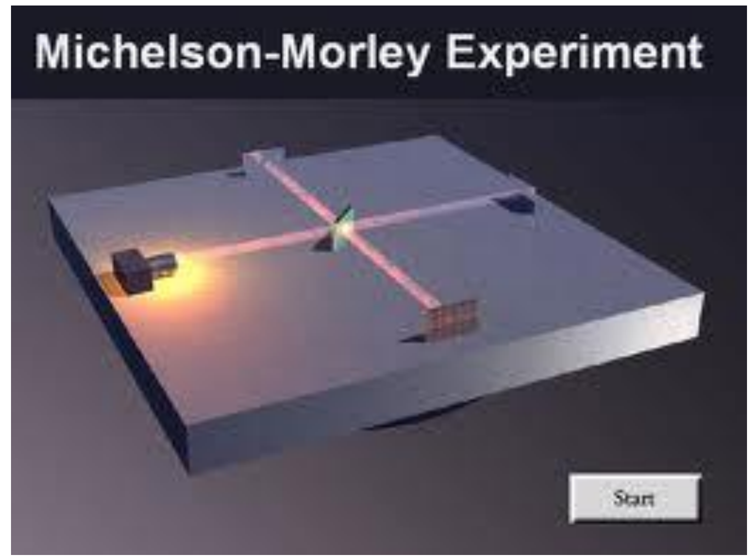
Light travels in aether…

Aristotle could live with that since he did not believe a vacuum existed in the first place



A.A. Michelson
1852 - 1931

E.W. Morley
1838 - 1923



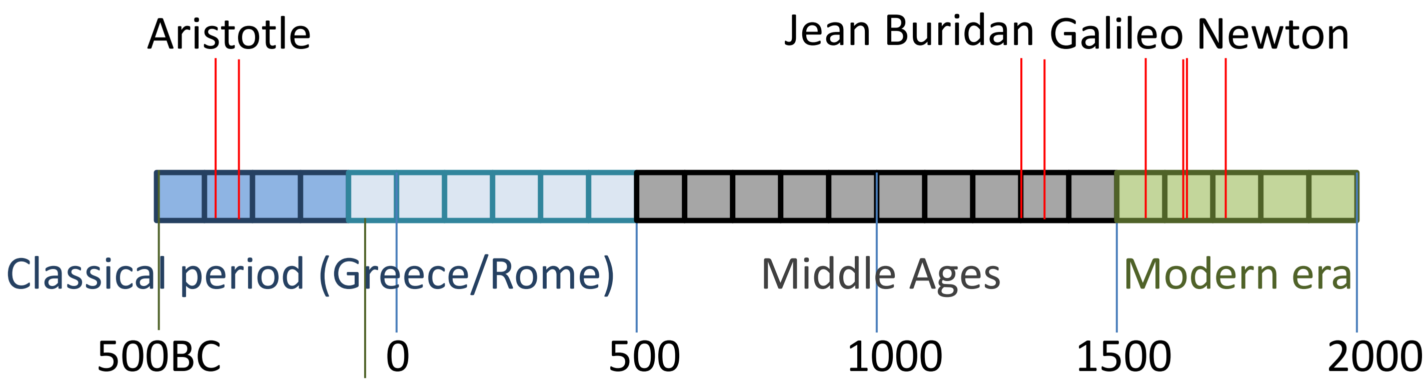
1887 Albert Michelson and Edward Morley show that light does not need a medium to propagate!!

The aether does not exist!!

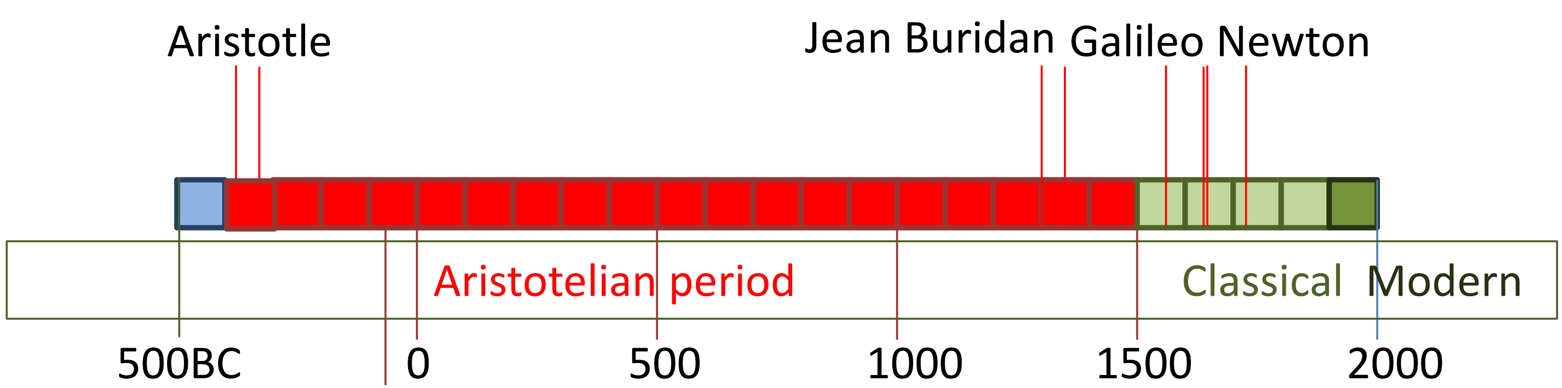
(just a couple of centuries more than two millennia to disprove this nonsense

Constant velocity

Aristotle's ideas lasted for about two millenea!



However, from scientific/natural philosophy point of view, a lot less happened

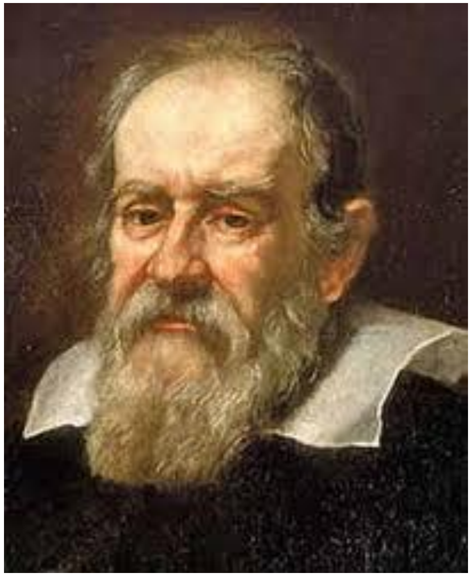


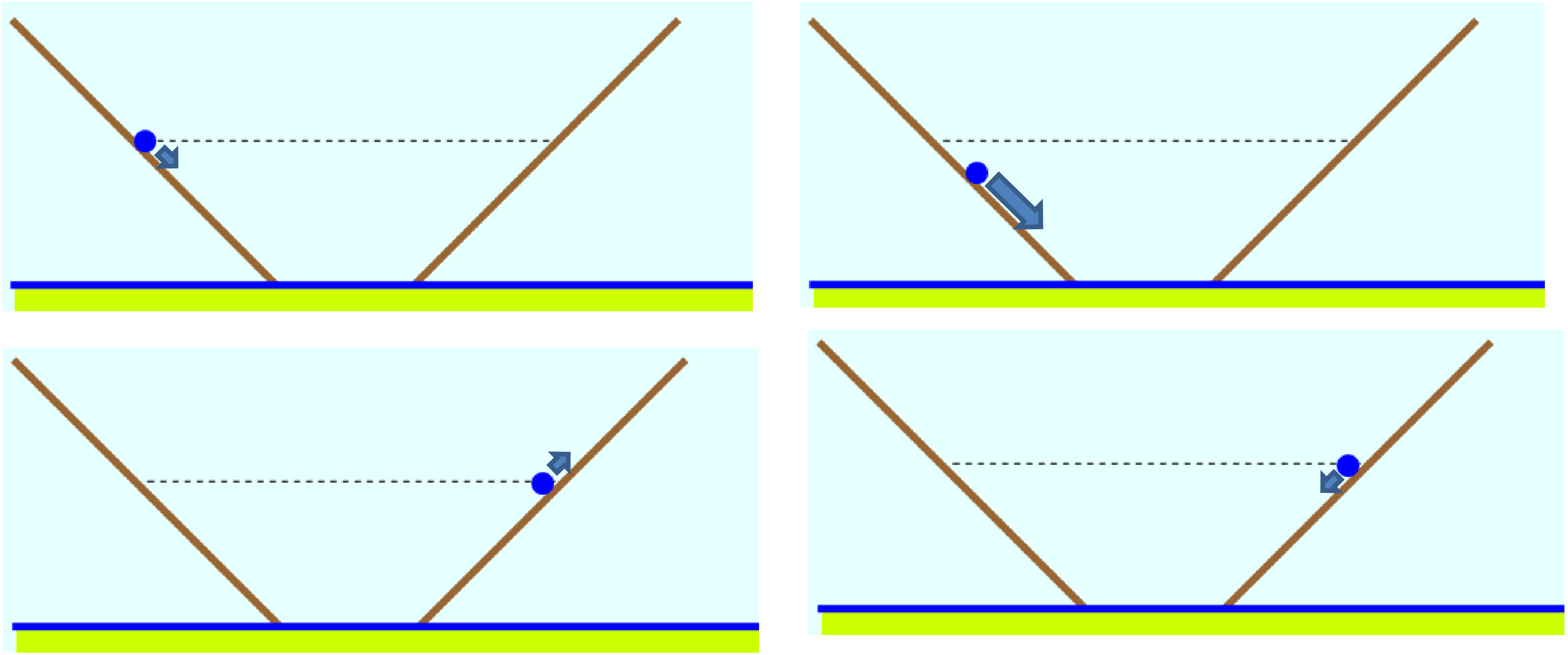
Jean Buridan (c1300-1358)

...after leaving the arm of the thrower, the projectile would be moved by an impetus given to it by the thrower and would continue to be moved as long as the impetus remained stronger than the resistance, and would be of infinite duration were it not diminished and corrupted by a contrary force resisting it or by something inclining it to a contrary motion

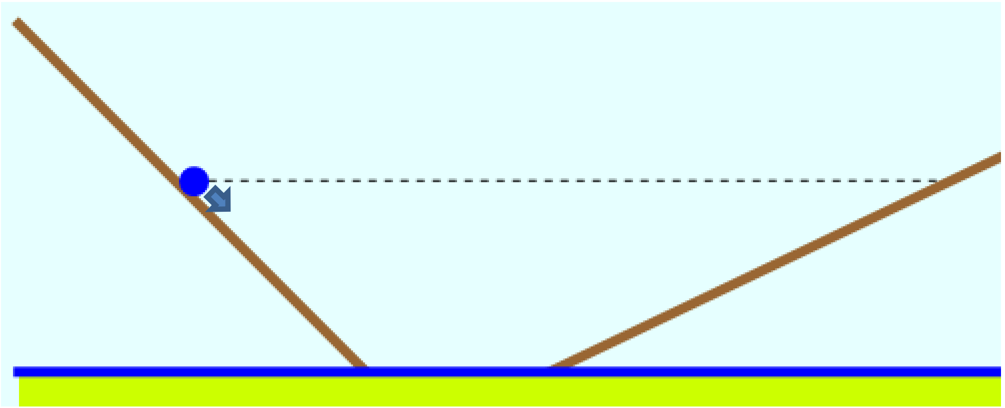
Galileo Galilei (1564-1642)

Without resistance the object will return to its original height

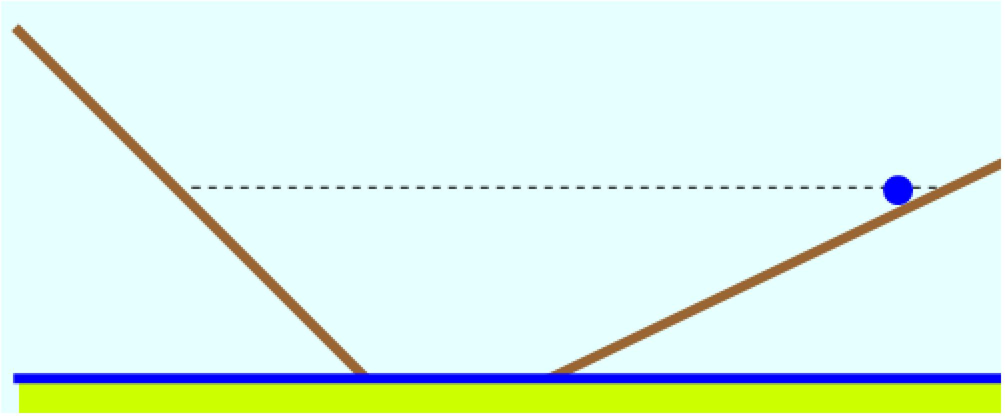




Now let's make the right arm flatter

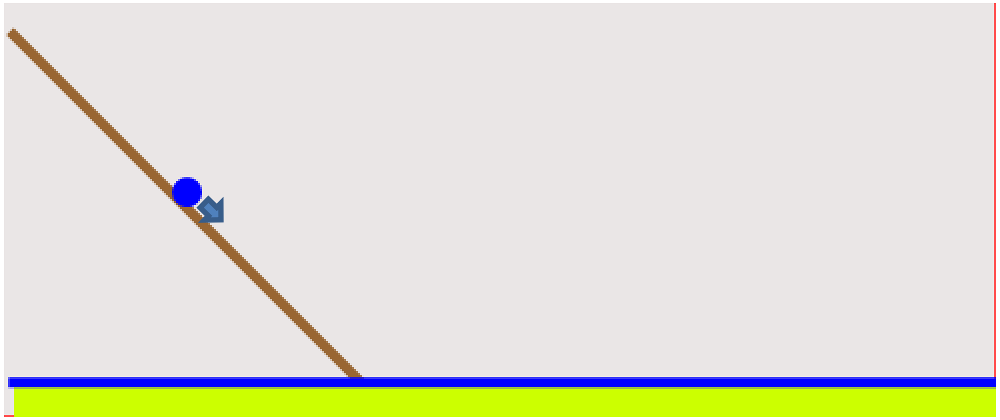


Again it will reach the same height

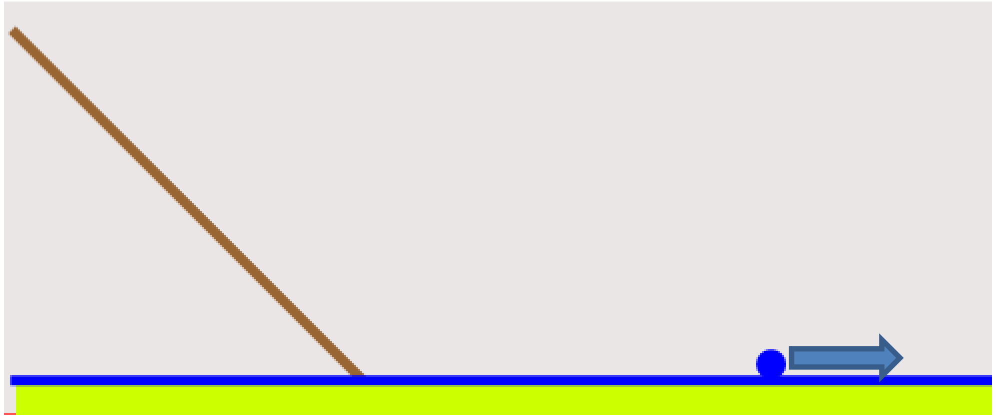


But it will travel further

Now let’s remove the right arm



Again, the object will try to reach the same height



However, it will never reach it and continue moving forever!

How do we obtain the distance travelled?

$$x = vt$$

With

x : position

v : velocity

t : time

Example:

$$v = 2 \text{ m/s} \quad \text{and} \quad t = 10 \text{ s} \quad \Rightarrow \quad x = vt = 2 \times 10 = 20 \text{ m}$$

However, we do not always want the object at x=0 at t=0. A more general formula is

$$x = vt + x_0$$

Example:

Suppose an object is at $x = 10$ km at $t = 0$. Where will the object be at $t = 4$ h later given an average velocity of 40 km/h.

Answer:

$$x = vt + x_0 = 40t + 10 \ \ \text{km} \quad \Rightarrow$$

$$\text{at} \quad t = 4 \ \ \text{h}, \ \ \text{we have} \ \ x = 40 \times 4 + 10 \ \ \text{km} = 170 \ \ \text{km}$$

Example:

Two cars start driving towards each other. One car starts in a city 100 km away and drives 40 km/h. The other car drives at a speed of 60 km/h. If the cars drive towards each other when and where do they meet?

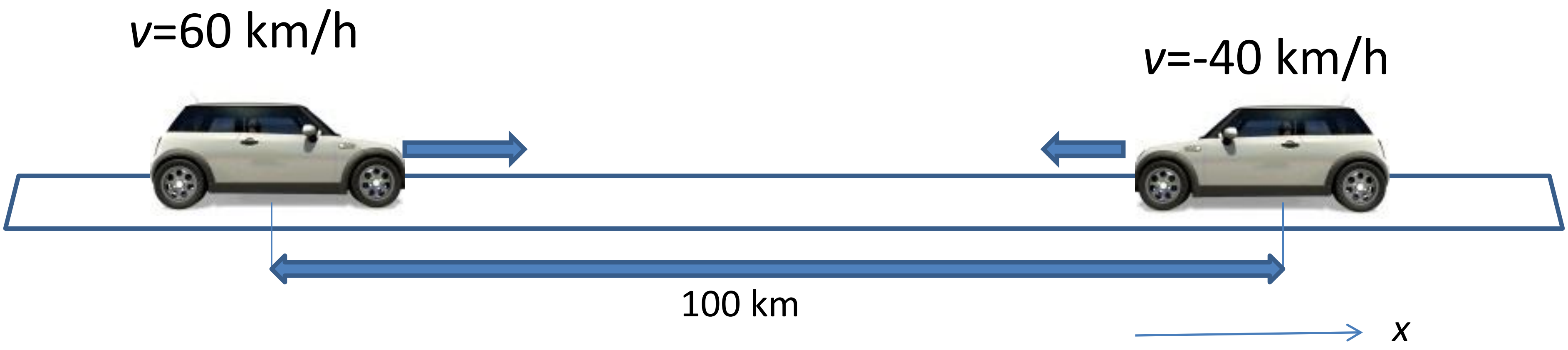
First, we have to be careful about the difference between speed and velocity.

The planes can all fly with the same speed, their velocities are still different, since they fly in different directions

Velocity in one dimension

speed=60 km/h
velocity=60 km/h

speed=40 km/h
velocity=-40 km/h

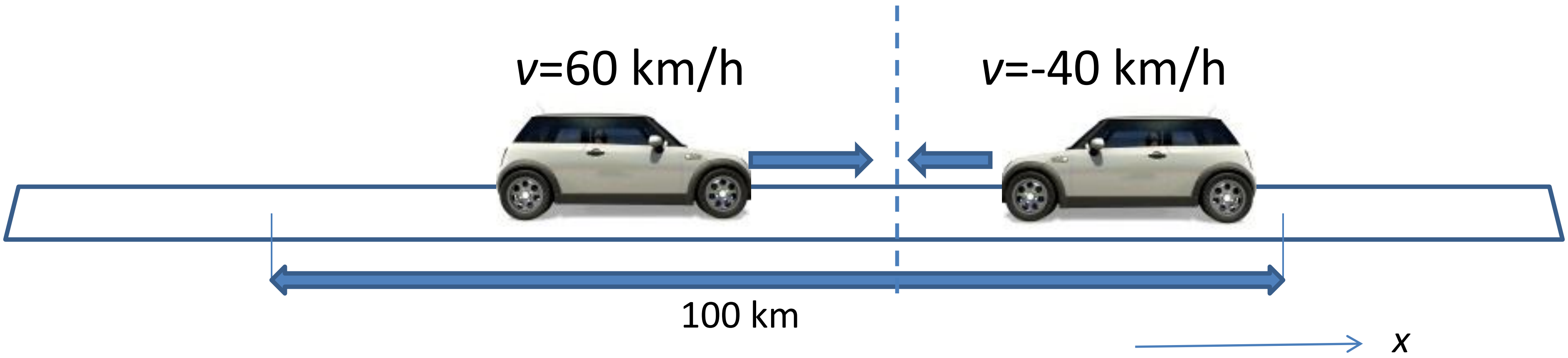


Equations of motion

$$x_1 = v_1 t + x_{10} = 60t \ \ \mathrm{km}$$

$$x_2 = v_2 t + x_{20} = -40t + 100 \ \ \mathrm{km}$$

When do they meet?



$$x_1 = x_2 \quad \Rightarrow \quad 60 t_{\text{meet}} = -40 t_{\text{meet}} + 100 \ \ \text{km}.$$

We can rewrite this as

$$100 t_{\text{meet}} = 100 \quad \Rightarrow \quad t_{\text{meet}} = 1 \ \ \text{h}.$$

Where do they meet?

Where are the cars at the time of the meeting? We can find this by inserting t_{meet} in one of the equations of motion

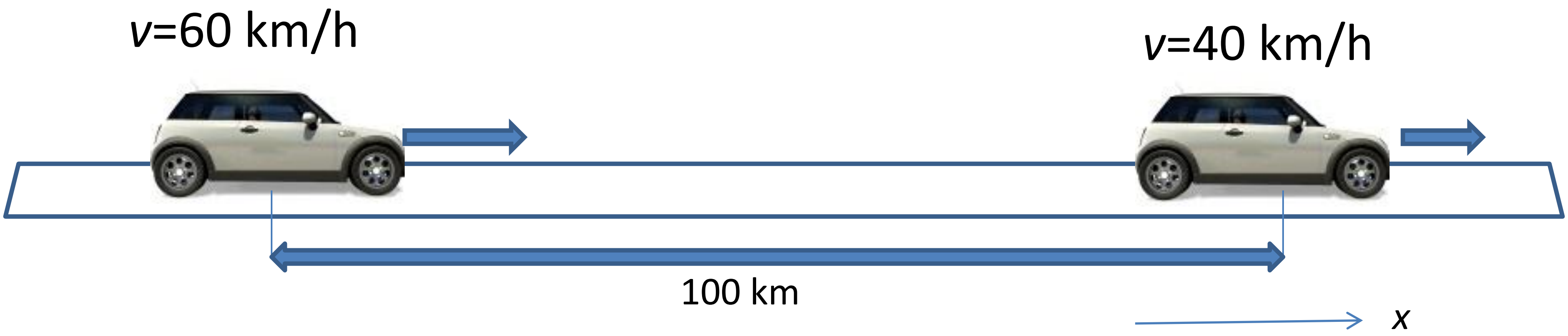
$$x_1 = 60t_{\text{meet}} = 60 \times 1 = 60 \text{ km}. \tag{1.12}$$

Obviously, we find the same value if we insert it in the second equation of motion

$$x_2 = -40t_{\text{meet}} + 100 = -40 \times 1 + 100 = 60 \text{ km}, \tag{1.13}$$

Note that there is nothing in the equations that prevents the cars from being in the same place at the same time!!!

What if we had messed up the sign: then the cars are driving in the same

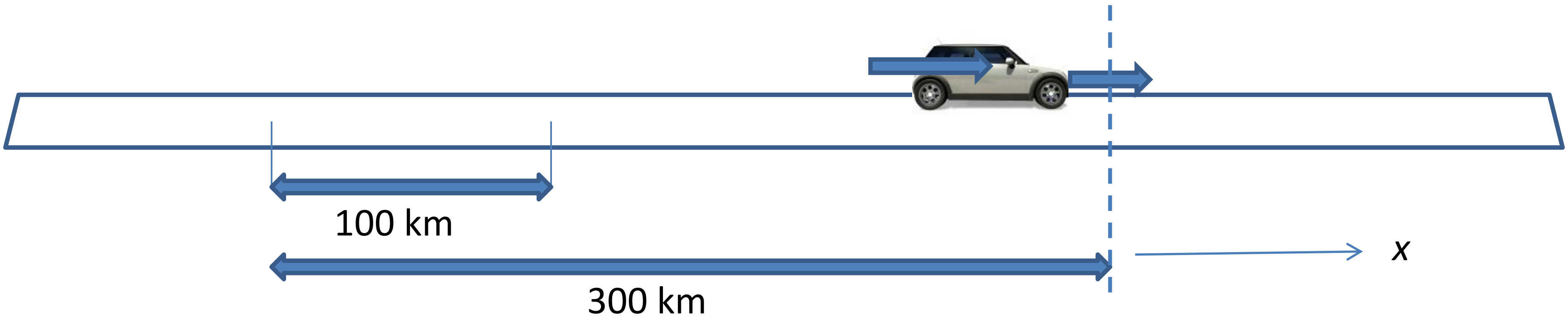


When do they meet?

$$60t_{\text{meet}} = 40t_{\text{meet}} + 100 \quad \text{km} \quad \Rightarrow \quad 20t_{\text{meet}} = 100$$
$$\Rightarrow \quad t_{\text{meet}} = 5 \quad \text{h.}$$

They now meet at

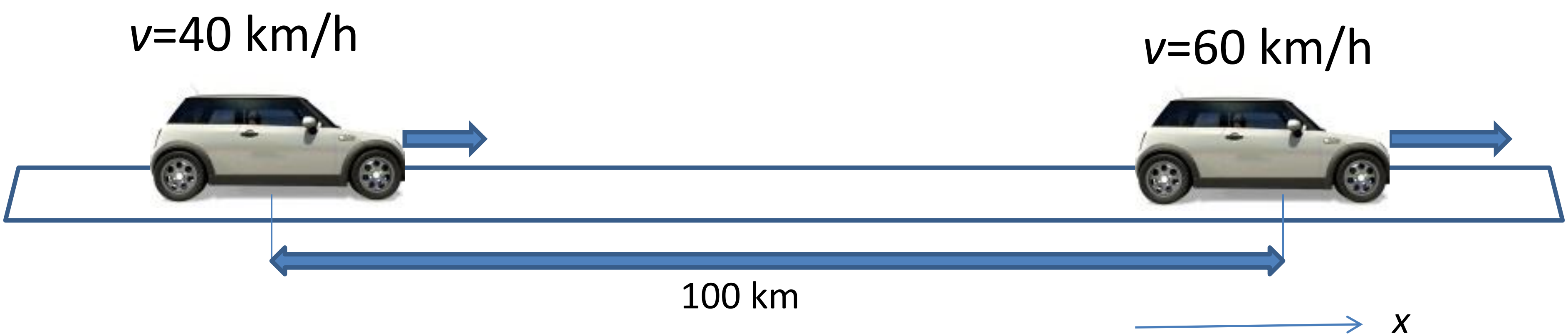
$$x = 60t_{\text{meet}} = 300 \quad \text{km.}$$



What if we messed up the velocities

$$x_1 = v_1 t + x_{10} = 40t \quad \text{km}$$

$$x_2 = v_2 t + x_{20} = 60t + 100 \quad \text{km},$$

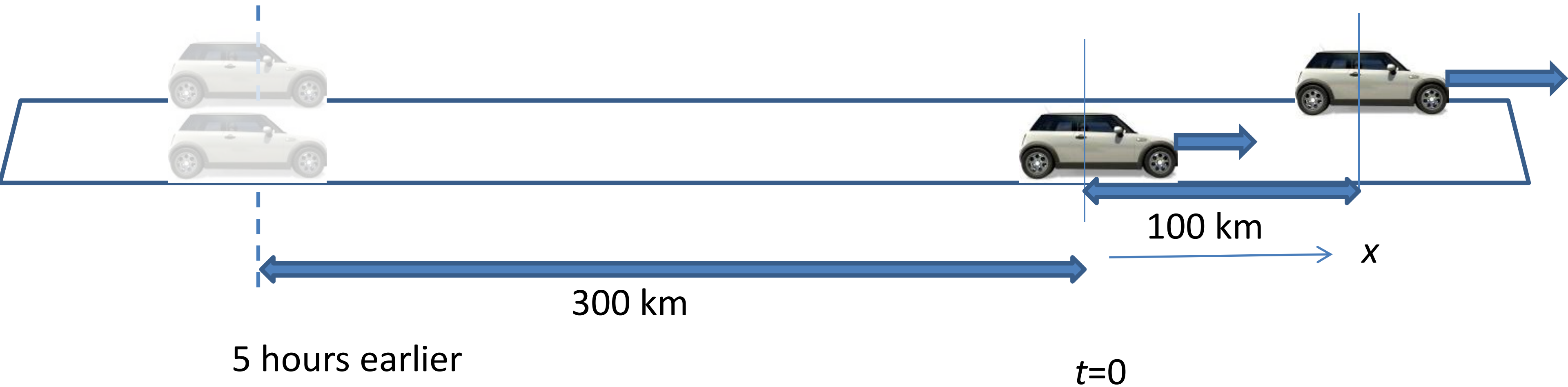


When do the meet now

$$x_1 = x_2 \quad \Rightarrow \quad 40t_{\mathrm{meet}} = 60t_{\mathrm{meet}} + 100 \quad \mathrm{km}$$

The solution is now

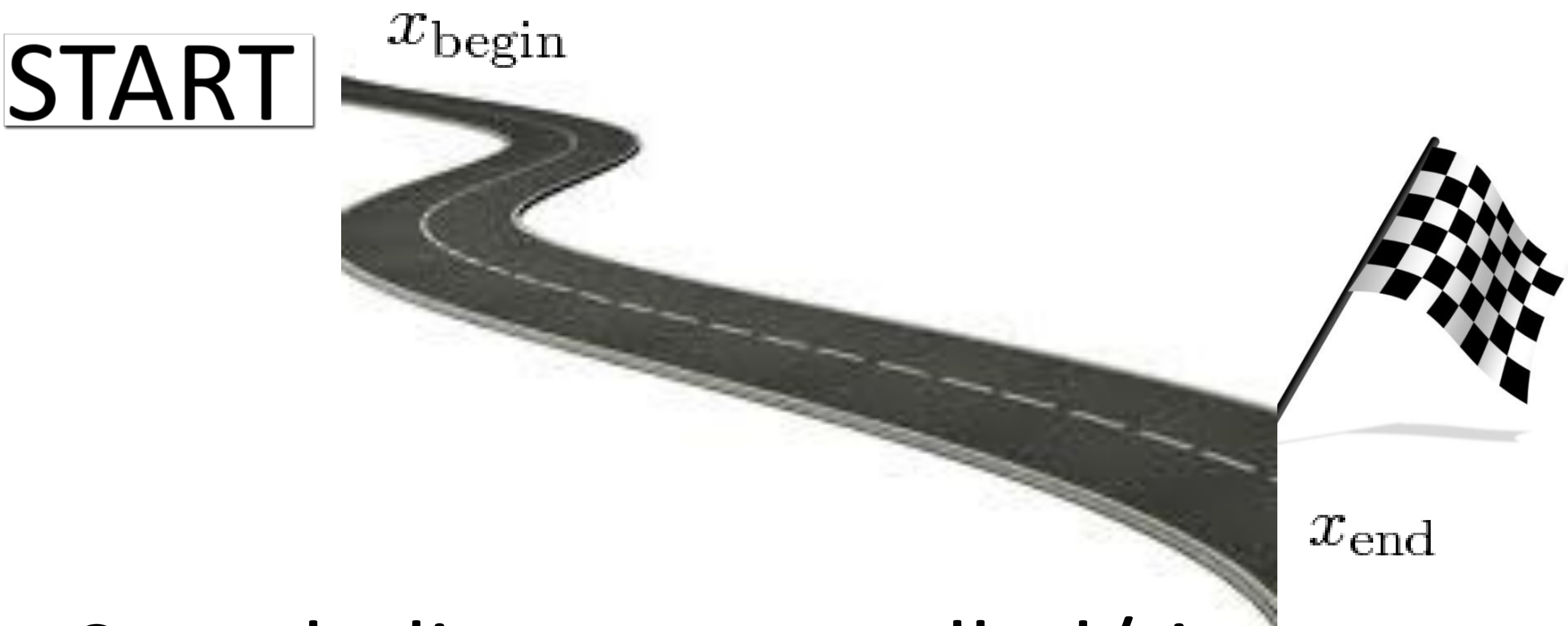
$$-20t_{\mathrm{meet}} = 100 \quad \Rightarrow \quad t_{\mathrm{meet}} = -5 \quad \mathrm{h}.$$



If at time $t = 0$ two cars are 100 km apart. The car driving 60 km/h is ahead of the car driving 40 km/h. If the cars started at the same point, when did they start driving?
The answer is then 5 hours earlier.

Constant acceleration

But first, what is speed and velocity?



Speed=distance travelled/time

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = x_{\text{end}} - x_{\text{begin}}$$

$$\Delta t = t_{\text{end}} - t_{\text{begin}}$$

$$v = \frac{x_{\text{end}} - x_{\text{begin}}}{t_{\text{end}} - t_{\text{begin}}}$$

EXAMPLES

#1

50 miles in 1 hour   speed=50/1=50 miles/hour

#2

1 km in 10 minutes

speed= 1 / (1/6)= 6 km/h

#3

100 m in 10 seconds

speed=100/10 = 10 m/s

m/s (meters per second) is a very common unit in physics

We can shuffle this equation around to obtain other quantities

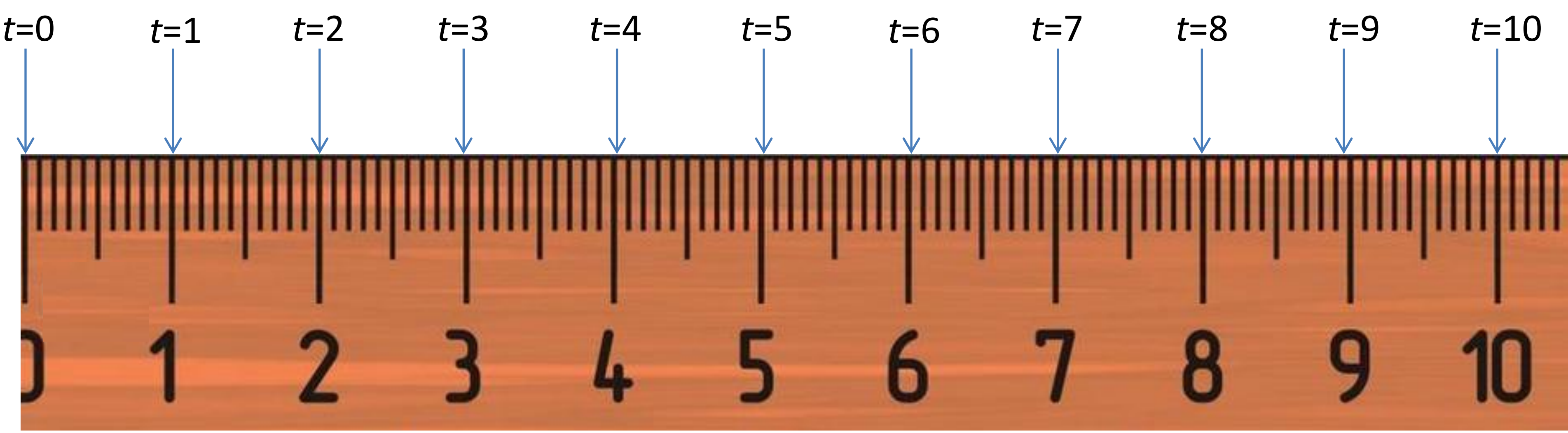
$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v \Delta t \qquad \Delta t = \frac{\Delta x}{v}$$

For a constant velocity, the position is given by

$$x = vt,$$

For v=1



If we know the positions as a function of time, we should be able to determine the velocity.
So, given

$$x = vt.$$

We can obtain the velocity from

$$v = \frac{x_{\text{end}} - x_{\text{begin}}}{t_{\text{end}} - t_{\text{begin}}} = \frac{v(t + \Delta t) - vt}{t + \Delta t - t} = \frac{v\Delta t}{\Delta t} = v.$$

Which works (as it should)

Suppose you are in an accelerating car

starting time

t=0 s

v=70 km/h

20 seconds later

t=20 s

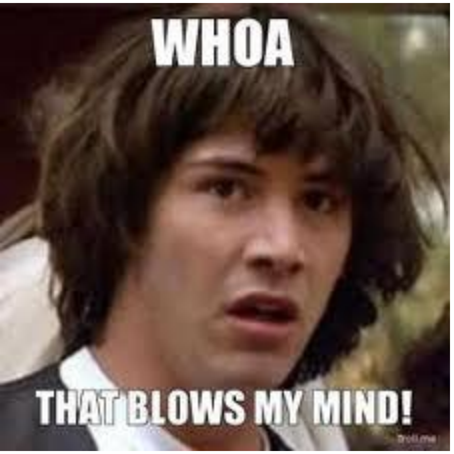
v=90 km/h

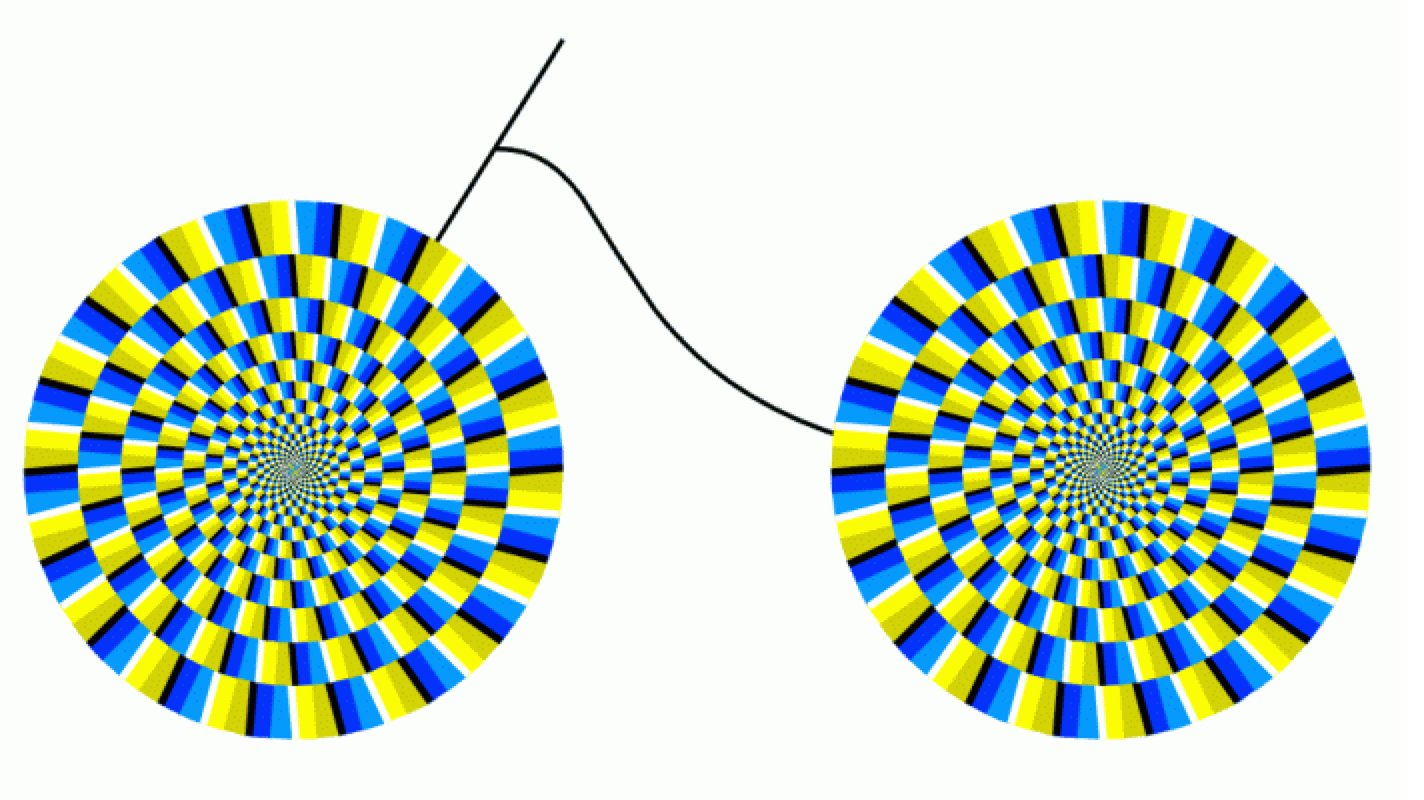
The speed is changing all the time!

However, if you could freeze time,
The speed would be constant!

- When we look at the speedometer, it is fixed at a constant speed
- When we look at the object, it is standing still

So how can something be moving at a constant speed and standing still at the same time?

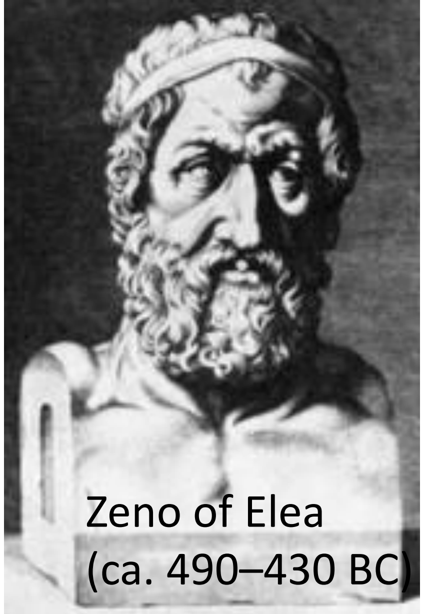


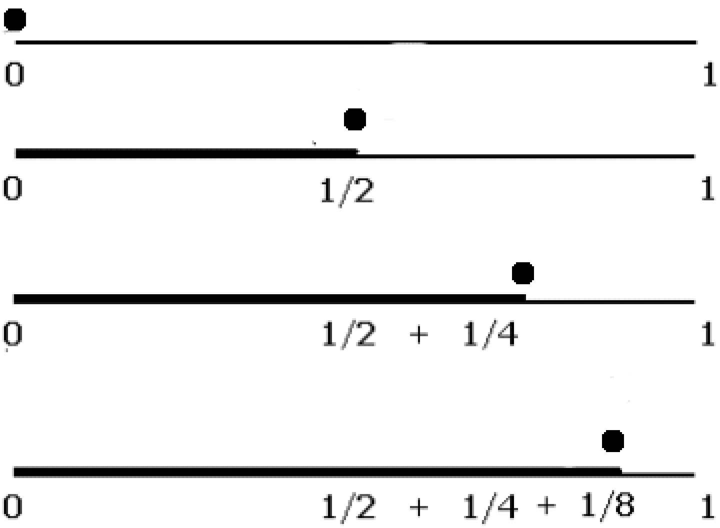


Paradoxes with infinite and infinitesimal

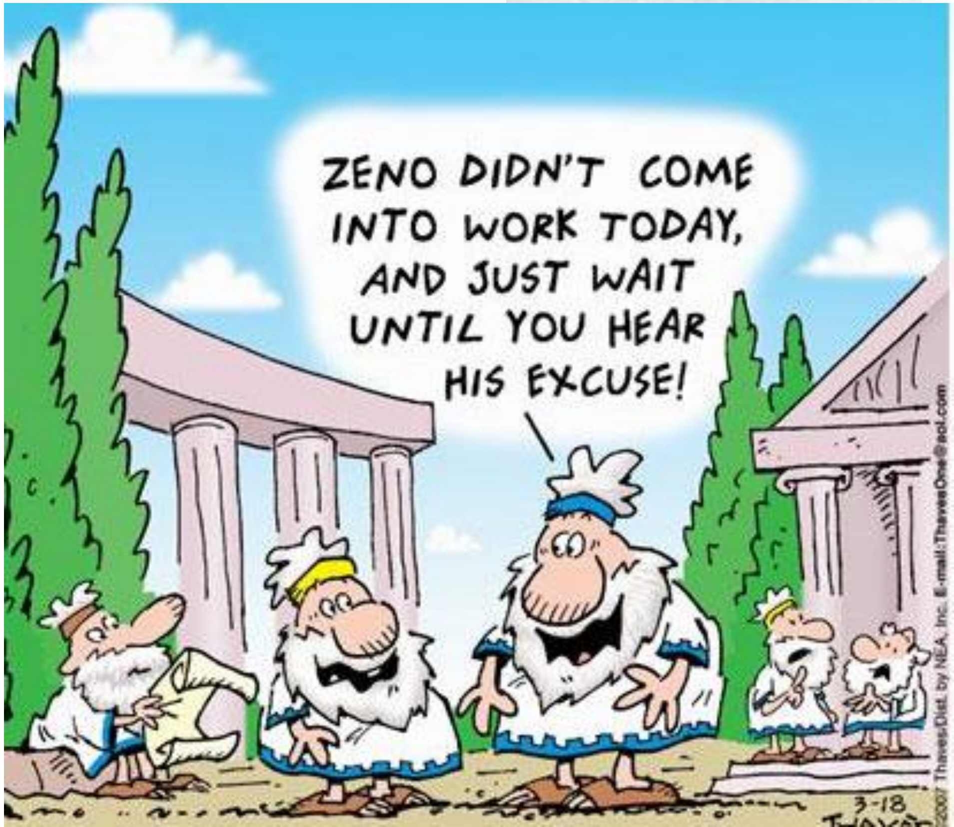
That which is in locomotion must arrive at the half-way stage before it arrives at the goal.

Therefore, it has to perform an infinite number of tasks and therefore will never get there!


Zeno of Elea (ca. 490–430 BC)



$$\Delta t = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

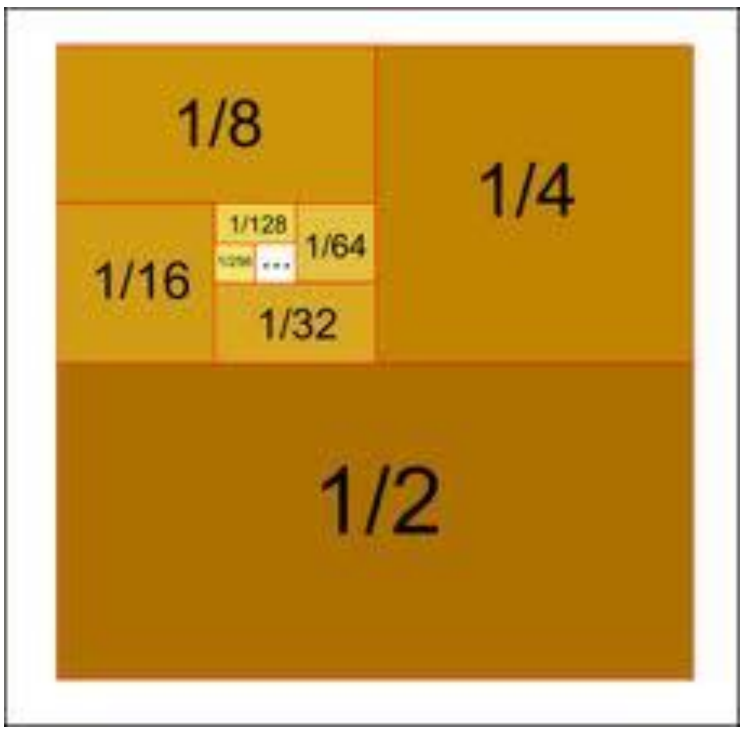


However, this paradox is not really a paradox at all. Let us multiply Δt by $1/2$

$$\frac{1}{2}\Delta t = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots . \tag{1.7}$$

However, the right-hand side is equal to $\Delta t - \frac{1}{2}$. Therefore,

$$\frac{1}{2}\Delta t = \Delta t - \frac{1}{2} \quad \Rightarrow \quad \frac{1}{2}\Delta t = \frac{1}{2} \quad \Rightarrow \quad \Delta t = 1 \ \text{ s}. \tag{1.8}$$



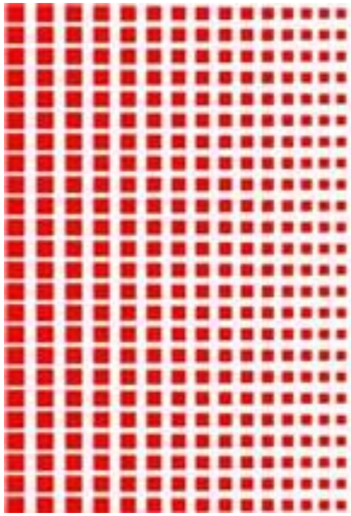
We can also see this by looking at the sum in terms of areas

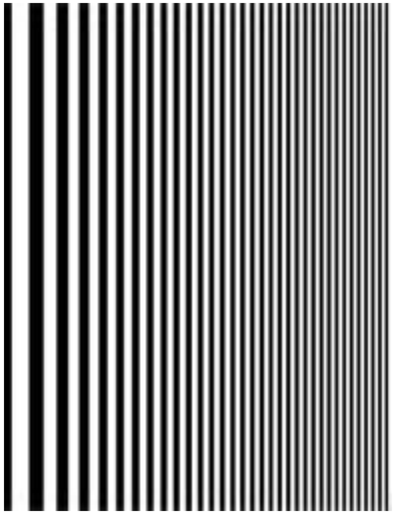
Total area is 1!

Of course it takes 1 s to cross 1 m at 1 m/s.

Apparently, freezing time: not a good idea

But, how about taking a very small time difference?

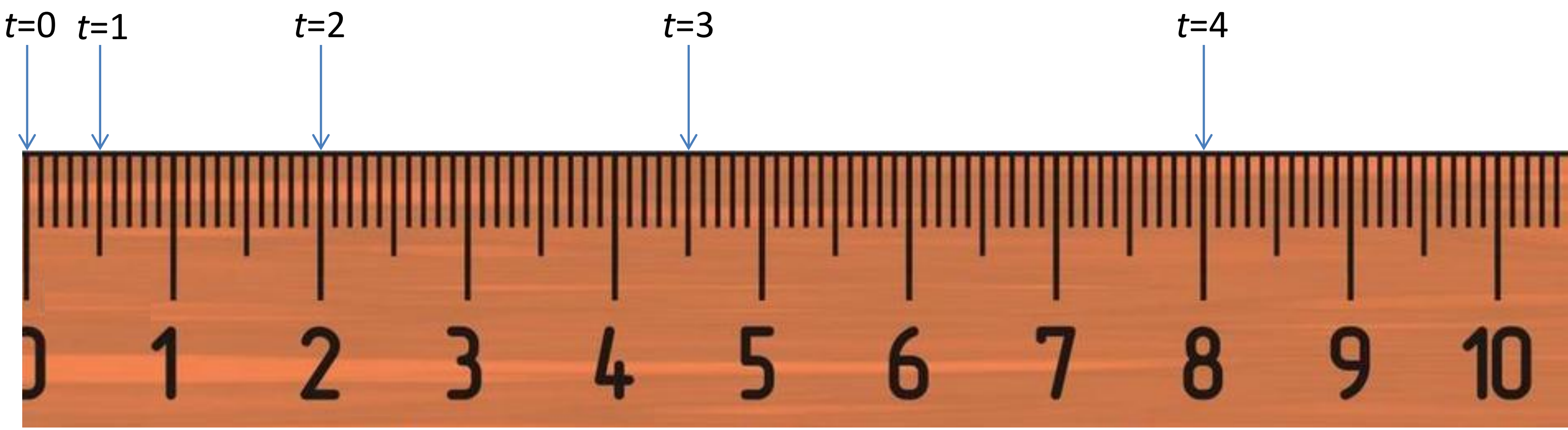




Let us take an example where *x* is not linear in *t*

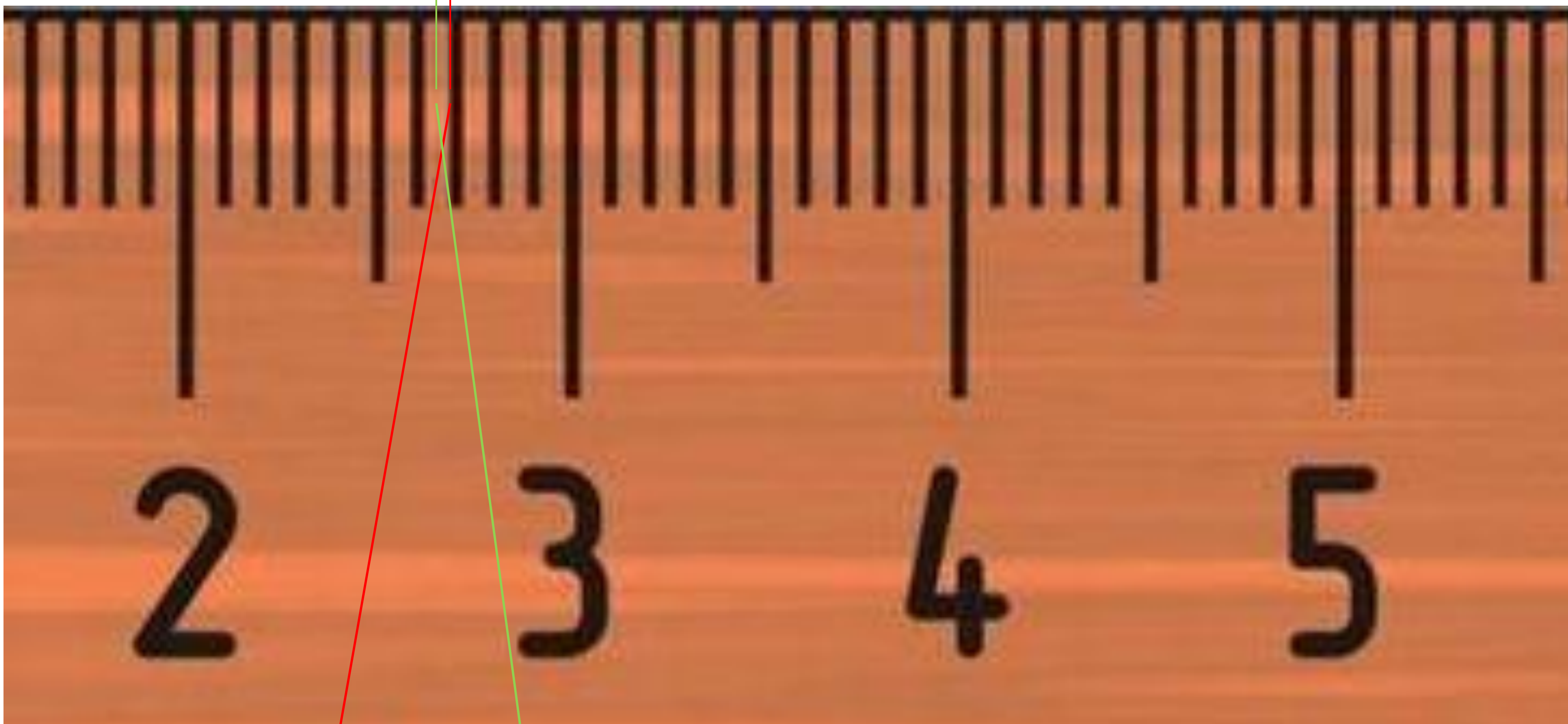
Let us take: $x = \frac{1}{2}at^2$

For a=1:



$$x = \frac{1}{2}at^2$$

$$t \quad t + \Delta t$$



$$v = \frac{x_{\text{end}} - x_{\text{begin}}}{t_{\text{end}} - t_{\text{begin}}} = \frac{\frac{1}{2}a(t+\Delta t)^2 - \frac{1}{2}at^2}{(t+\Delta t) - t}$$

Now we can write:

$$(t + \Delta t)^2 = (t + \Delta t)(t + \Delta t) = t^2 + 2t\Delta t + (\Delta t)^2$$

If Δt is small then $(\Delta t)^2$ is even smaller. For example

$$\Delta t = 0.001, \text{ then } (\Delta t)^2 = 0.000001$$

So, approximately:

$$(t + \Delta t)^2 \cong t^2 + 2t\Delta t$$

and therefore:

$$v \cong \frac{\frac{1}{2}a(t^2 + 2t\Delta t) - \frac{1}{2}at^2}{\Delta t} = \frac{at\Delta t}{\Delta t} = at$$

Therefore, we can directly relate the position and the speed:

$$x = \frac{1}{2}at^2 \qquad \Leftrightarrow \qquad v = at$$

The velocity is no longer constant. The change in velocity is given by

$$\frac{\Delta v}{\Delta t} = \frac{a\Delta t}{\Delta t} = a$$

Where *a* is the acceleration.

Example I

Example.– Given an acceleration of 5 $\mathrm{m/s^2}$, what are the velocity and position at $t = 10$ s?

$$x = \frac{1}{2}at^2 = \frac{1}{2}5 \times 10^2 = 250 \text{ m}, \qquad v = at = 5 \times 10 = 50 \text{ m/s}.$$

Example II

Example.– If the acceleration is 4 m/s^2, what is the velocity at $x = 50$ m?

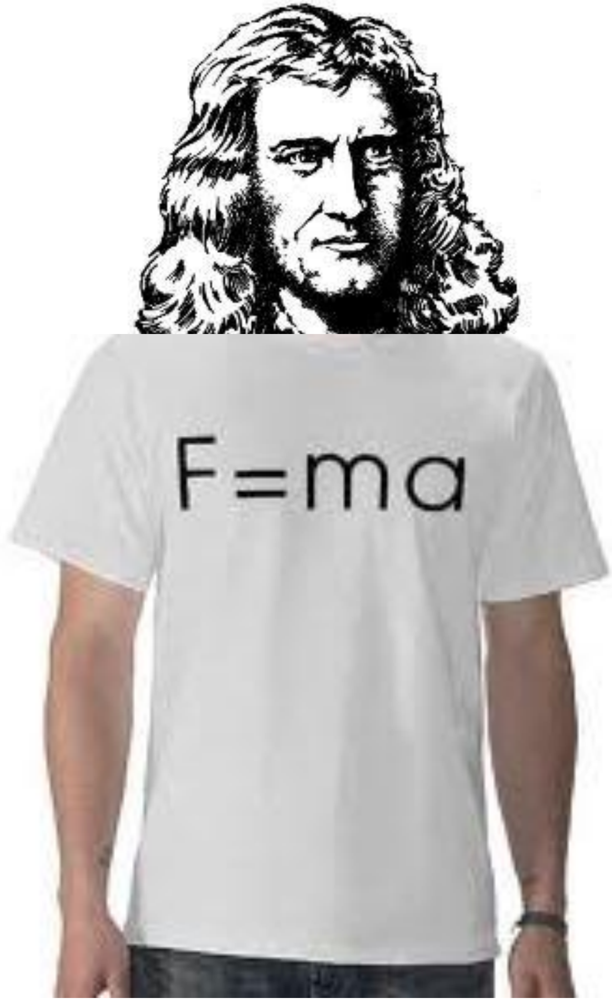
This problem splits into two parts. First, we need to find the time that the object is a $x = 50$ m. This can be done via

$$x = \frac{1}{2}at^2 = \frac{1}{2}4 \times t^2 = 50 \quad \Rightarrow \quad t^2 = 25 \quad t = 5 \ \ \text{s}. \tag{1.37}$$

Secondly, we need to find the velocity at that time:

$$v = at = 4 \times 5 = 20 \ \ \text{m/s}. \tag{1.38}$$

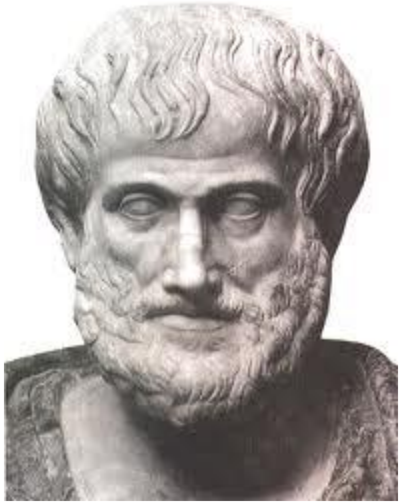
Newton’s second law



We know that we can change how an object moves by applying a force

But on what part of the motion does the force act

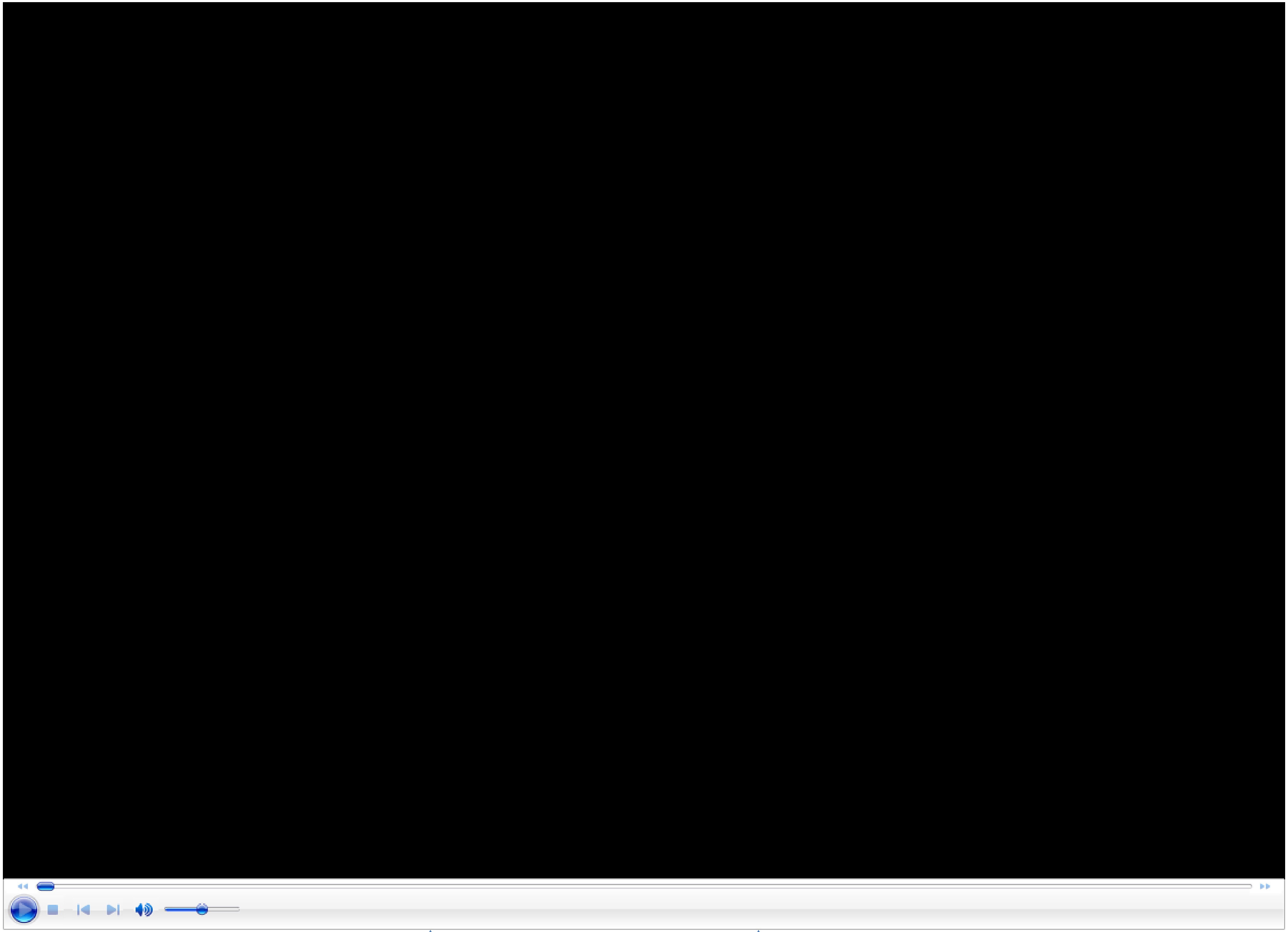
Position x? Velocity v? Acceleration?



Aristotle (remember: wrong!)

$F = R\,v$ or $v = F/R$

- Constant speed requires continuous force
- A force is need to change the position
- The bigger R (rock vs. feather), the smaller the speed



2:00

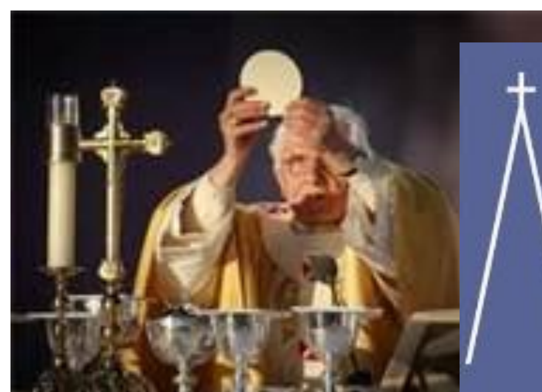
Different balls, same force (approximately): Balls do not move the same!

Note: no force, but balls are still moving!

The quantity that makes objects behave differently under influence of a force is called

MASS

Multi Ammunition Softkill System (MASS)

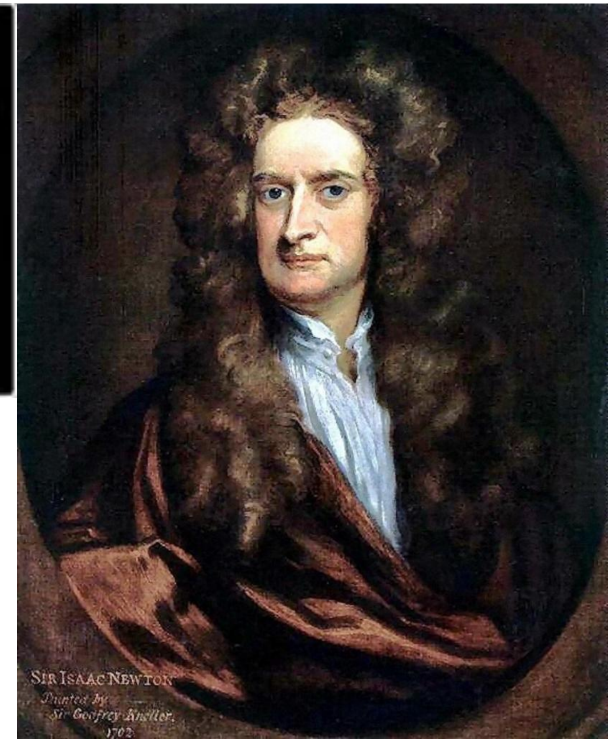


MASS

No, not those!

MASS!

Yes, that's the one!



Newton (RIGHT!)

F = m a   or   a=F/m

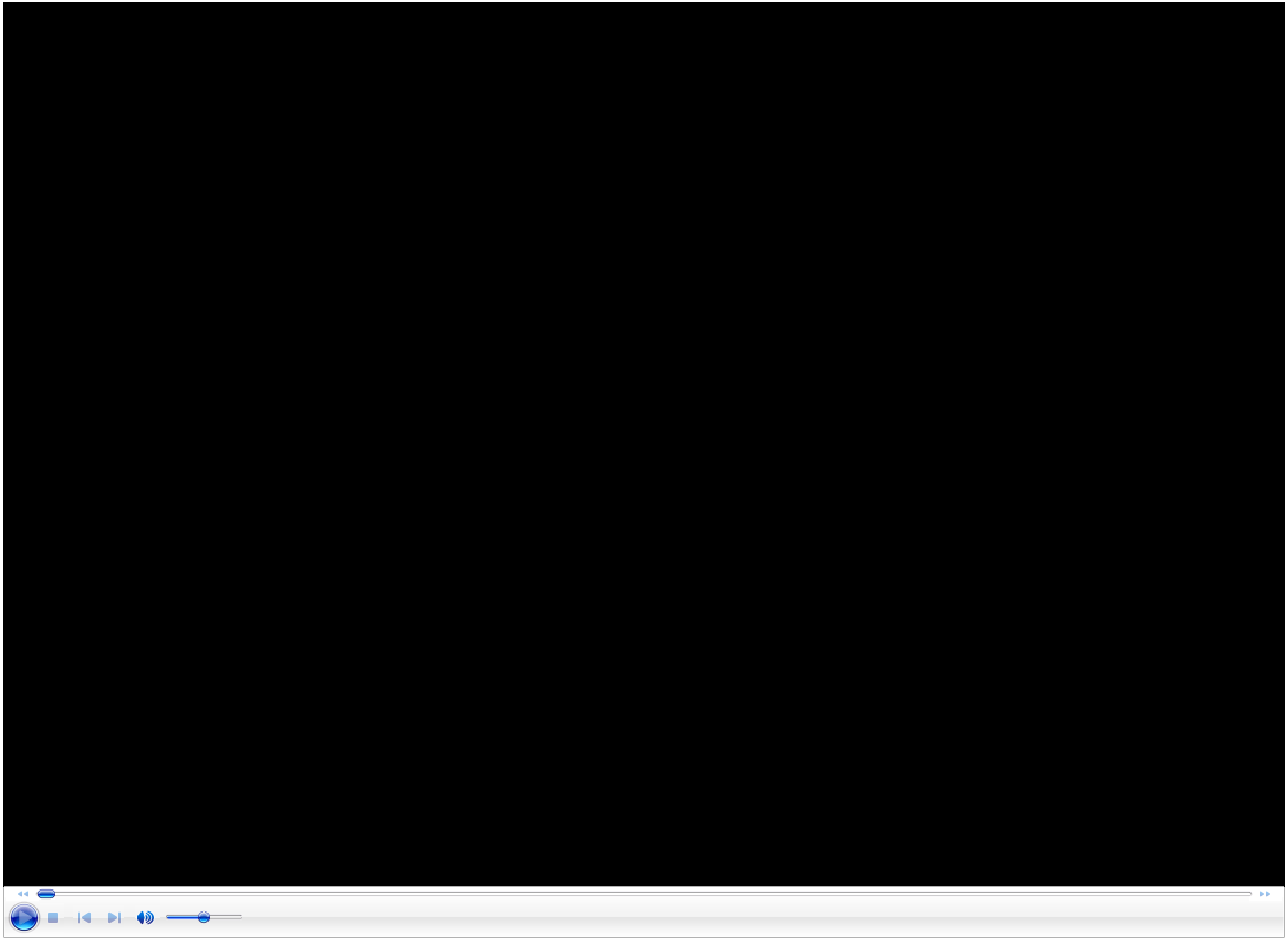
- Constant speed requires no force
- A force is needed to change the velocity
- Applying a force makes the object go faster!
- The greater the mass, the smaller the acceleration

This also means, if not enough force is applied objects will maintain the same velocity

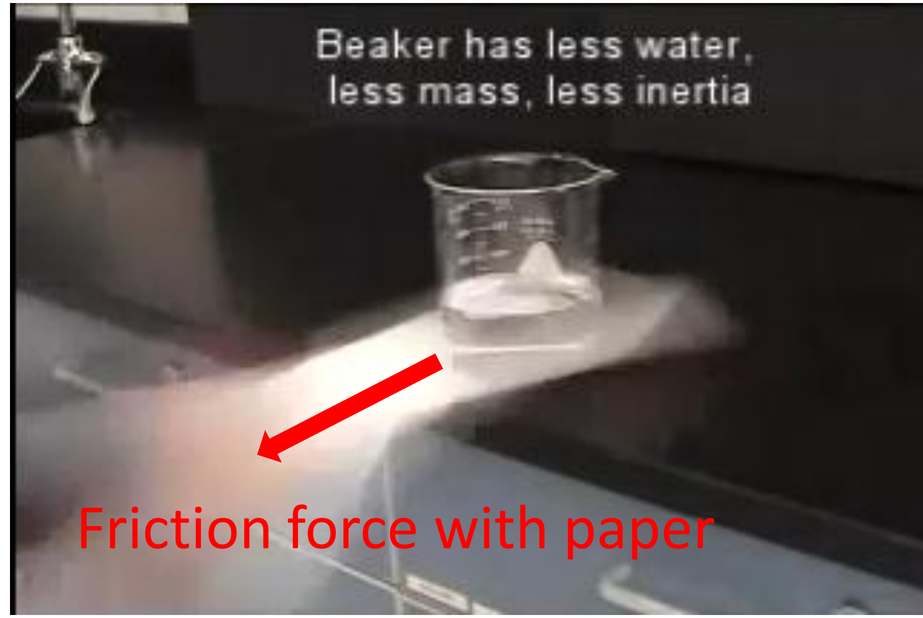
This is called: INERTIA

- Objects at rest tend to stay at rest

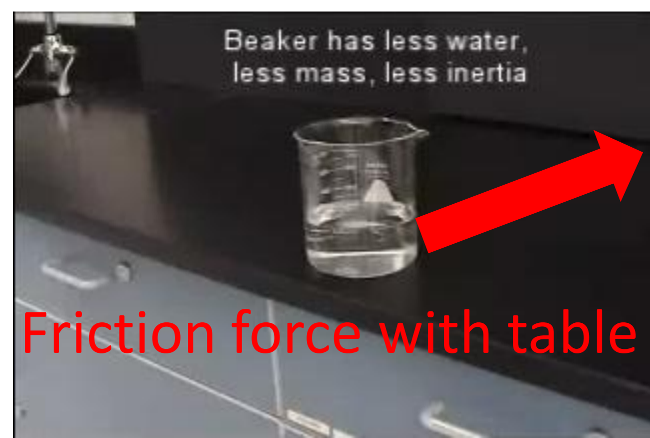
- Objects in motion tend to stay in motion

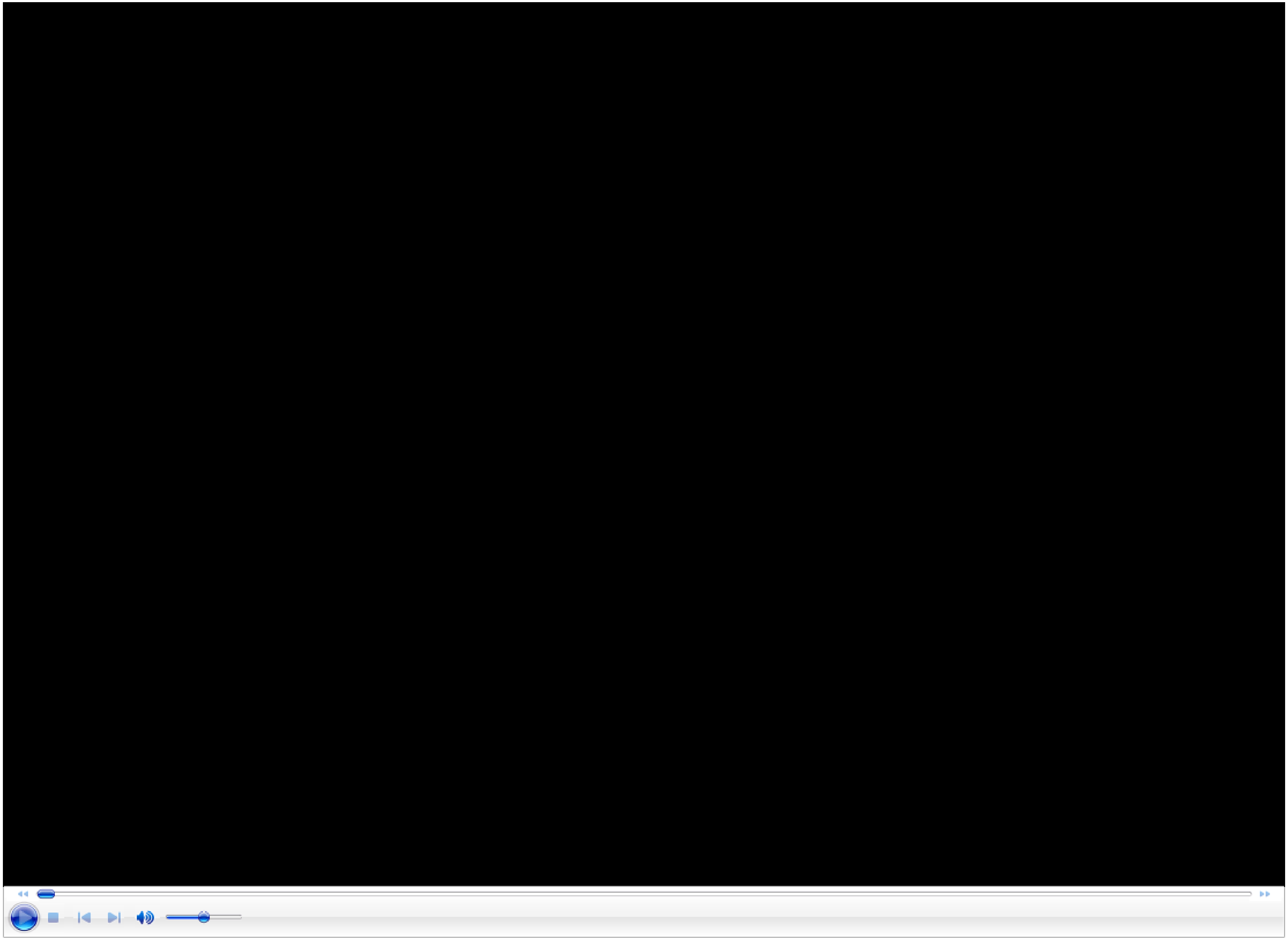


Object at rest:



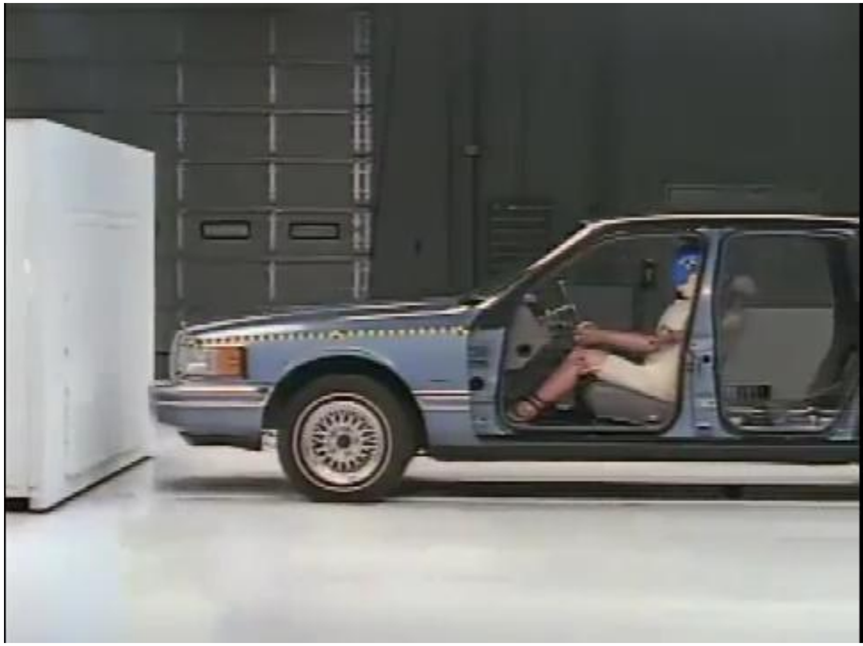
- There is a force working on the beaker: the friction between the beaker and the paper
- Friction force is not enough to give beaker large acceleration
- Beaker has not moved enough before paper is gone
- Friction force with table stops the beaker



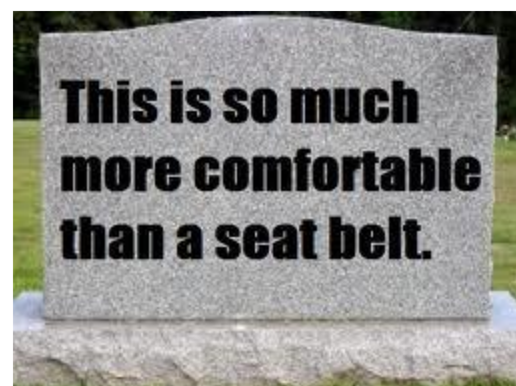


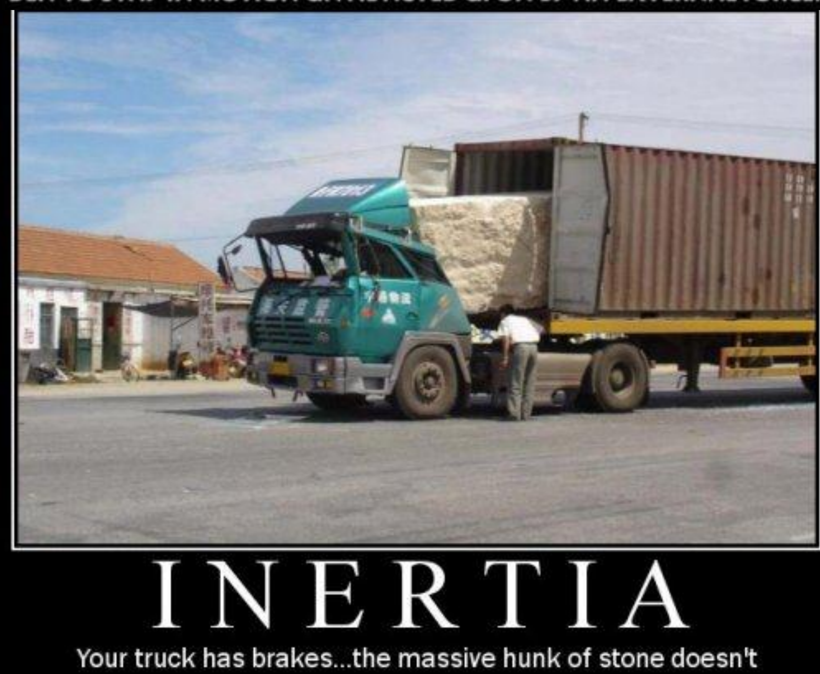
Vehicle in motion

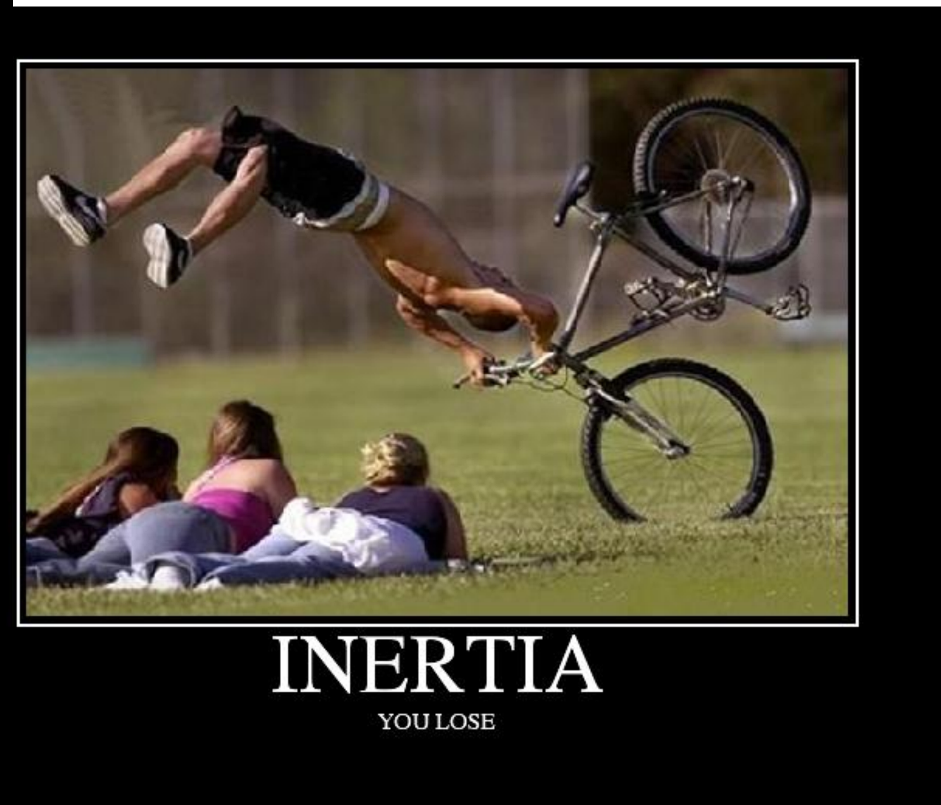
- Car and driver move at a certain speed
- Car's motion is suddenly decellerated
- Friction with seat not sufficient to stop motion of driver



Unfortunately, impact with wind screen and dash board cause a significant deceleration





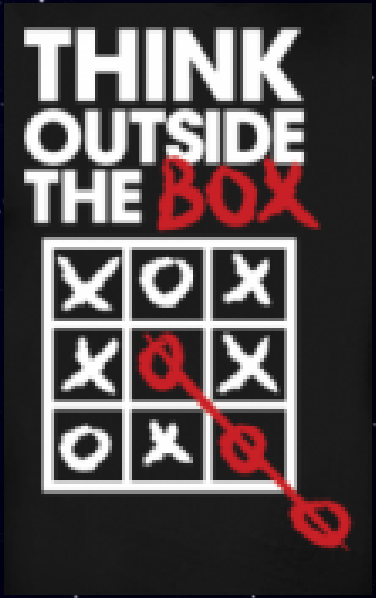


the greater the mass,
the greater the inertia!

Newton, Galileo and others managed to find fundamental laws by imagining ideal conditions (no friction).

$$F=ma$$

Great outside-the-box thinking



$$F = ma.$$

The unit of force is

$$\mathrm{kg} \times \frac{\mathrm{m}}{\mathrm{s}^2} \equiv \mathrm{N},$$

where N stands for Newton, since kilogram meter per second squared is such a mouth full.

EXAMPLE I

Example.— If the mass if 10 kg and the force applied on it is 2 N, then what is the acceration.

$$F = ma \quad \Rightarrow \quad 2 = 10a \quad \Rightarrow \quad a = \frac{2}{10} = \frac{1}{5} = 0.2 \ \frac{\mathrm{m}}{\mathrm{s}^2}. \tag{1.43}$$

EXAMPLE II

Example.– If the velocity after 40 seconds is 80 m/s and the mass is 2 kg, what was the applied force?

Step one is to calculate the acceleration

$$v = at \quad \Rightarrow \quad 80 = a40 \quad \Rightarrow \quad a = 2 \ \ \frac{\mathrm{m}}{\mathrm{s}^2}. \tag{1.44}$$

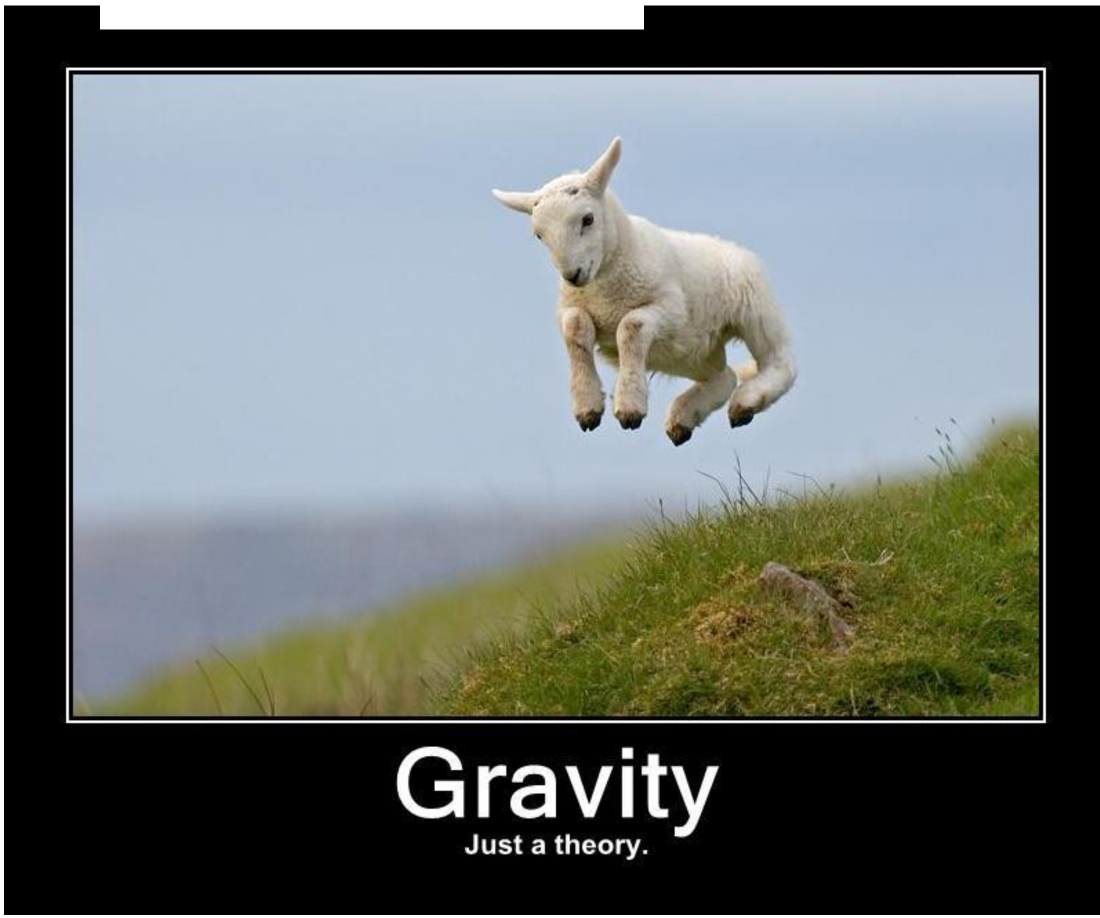
From the acceleration, we can obtain the applied force

$$F = ma = 2 \times 2 = 4 \ \ \mathrm{N}. \tag{1.45}$$

Gravitational acceleration

Let us look at a prototypical way of accelerating

GRAVITY



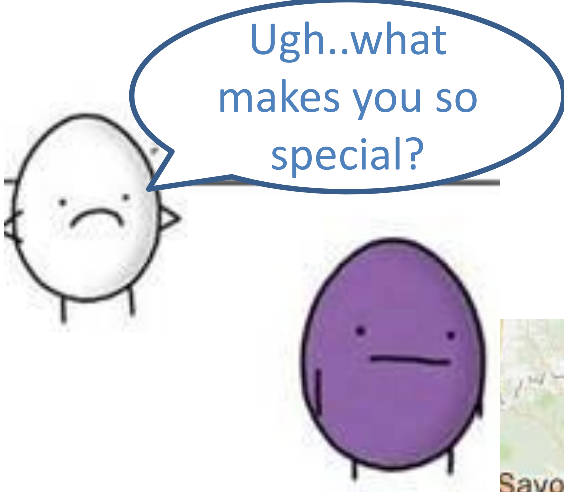
Galileo did his famous experiments on gravitation by dropping balls of different mass from the leaning tower in Pisa

Inspired by the beautiful renaissance architecture

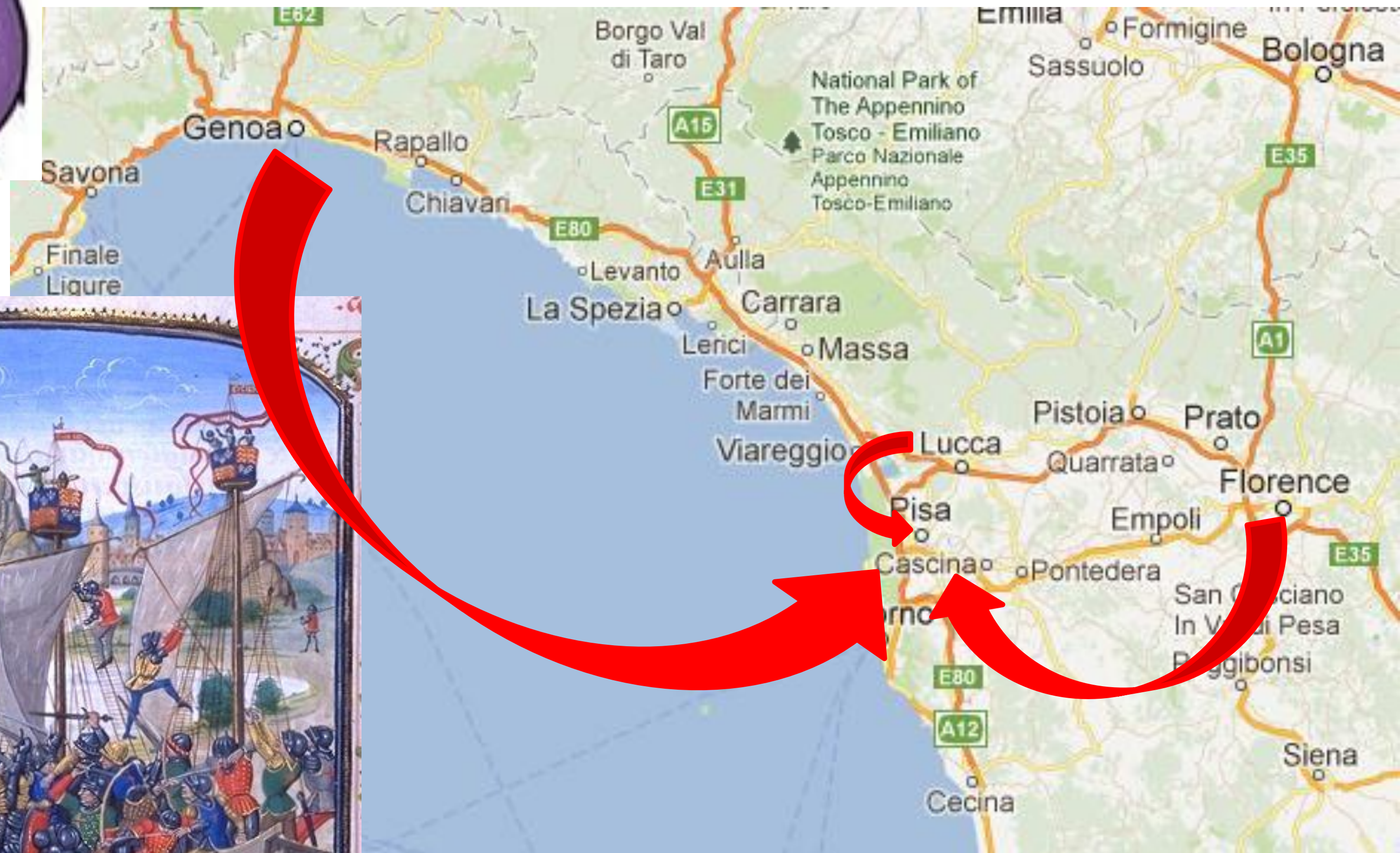
The Piazza dei Miracoli was contructed from 1064-1372

Pisa benefited a lot in the 11th and 12th centuries from trade with the eart, in particular the Byzantine Empire

Maybe not dark.
Some renaissance propaganda?



This created some friction with other city state neighbors…



1284: Crushing defeat against Genoa, who destroyed Pisa's harbor in 1290s.

Also: Did Galileo really throw his balls of the leaning tower?

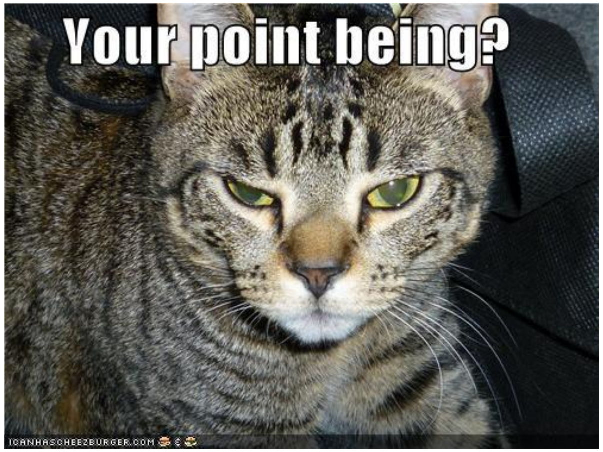
that sounds so wrong….

Or was it Girolamo Borro in 1575?

We also know that Simon Stevin (1548-1620) Threw objects from the Nieuwe Kerk in Delft (the Netherlands)

Nieuwe kerk (1396-1496)

Oude kerk (1250-1350)
He should have done it here because this tower is at least leaning….

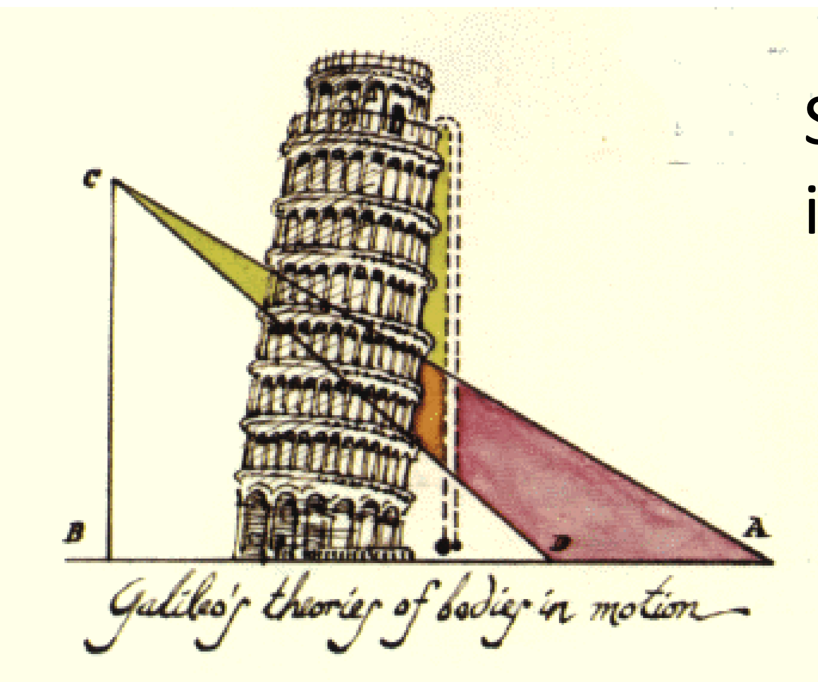


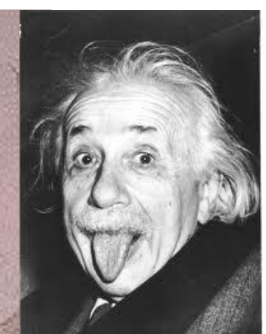
The point being:

Regardless of where or what was thrown and by whom:

The balls of different weight hit the ground at the same time

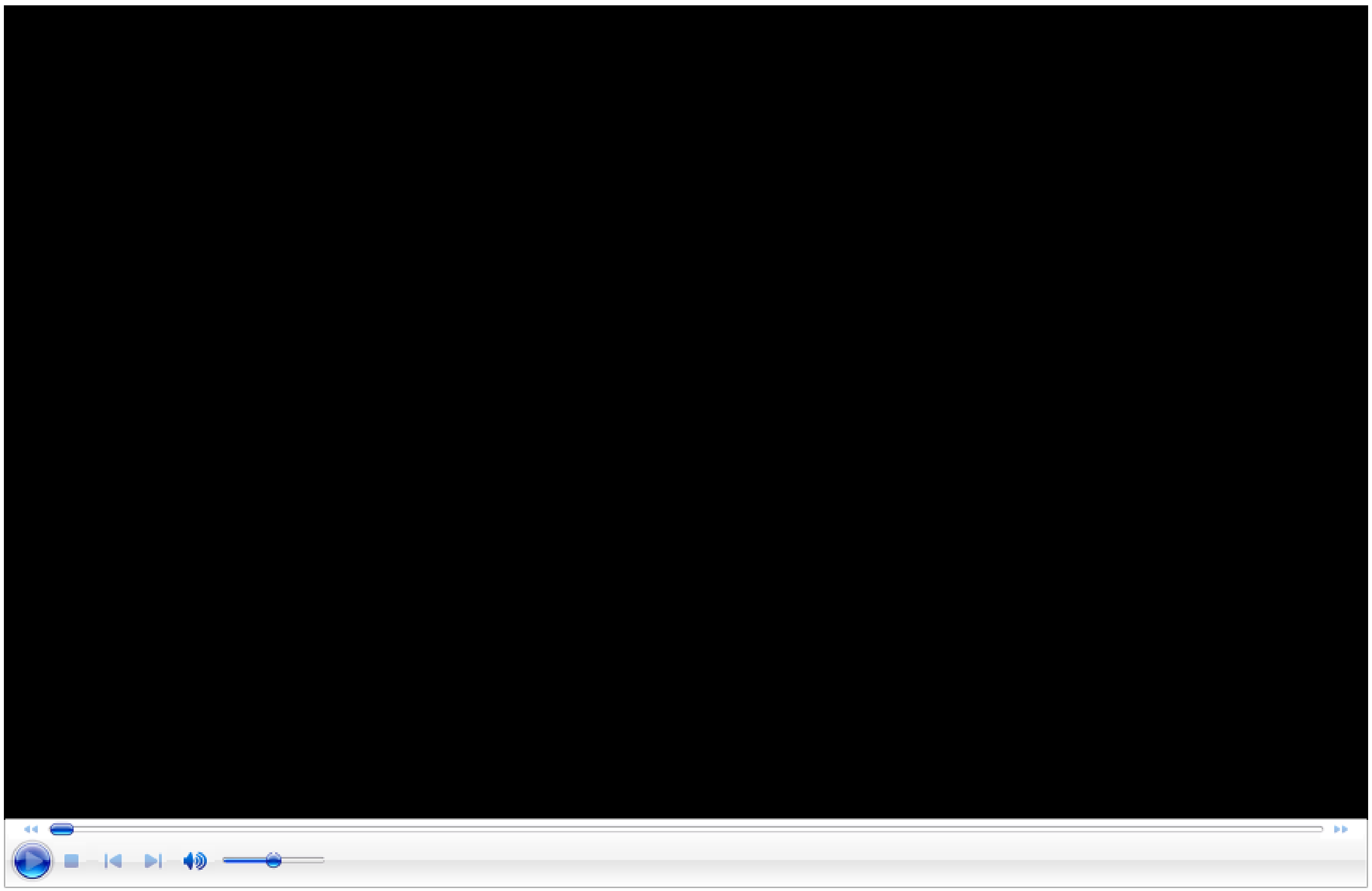
Showing that Aristotle was incorrect!!





Aristotle could still say: a wooden and metal ball have more or less the same nature

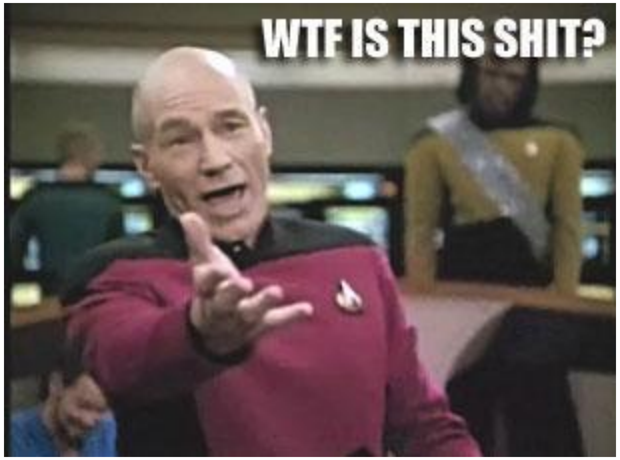
But now let us throw a ball and a feather at the same time



Feather and ball drop in exactly the same way!

However:

- Intuition tells me that the heavier ball should fall faster
- Gravity is working on the feather and the ball
- F=m a
- Well, that means that the ball should actually fall more slowly



Galileo's inclined plane experiment
More accurate that just dropping stuff!

Mathematica demonstration

GalileosInclinedPlaneExperiment-source

Galileo found that the ration between the intervals were independent of the inclination:

1, 3, 5, 7, 9,....

Or the positions were:

0, 1, 4, 9, 16, 25,.... Or t^2

That is the same as we treated before:

$$x = \frac{1}{2}at^2 \quad \Leftrightarrow \quad v = at$$

NO: gravity is not just any force!

Before we had a constant force,

But gravity depends on the mass *m* of the object, so if the force is mass times the gravitational acceleration, or *F=m g*, then

$$F = ma \quad \Rightarrow \quad mg = ma \quad \Rightarrow \quad a = g$$

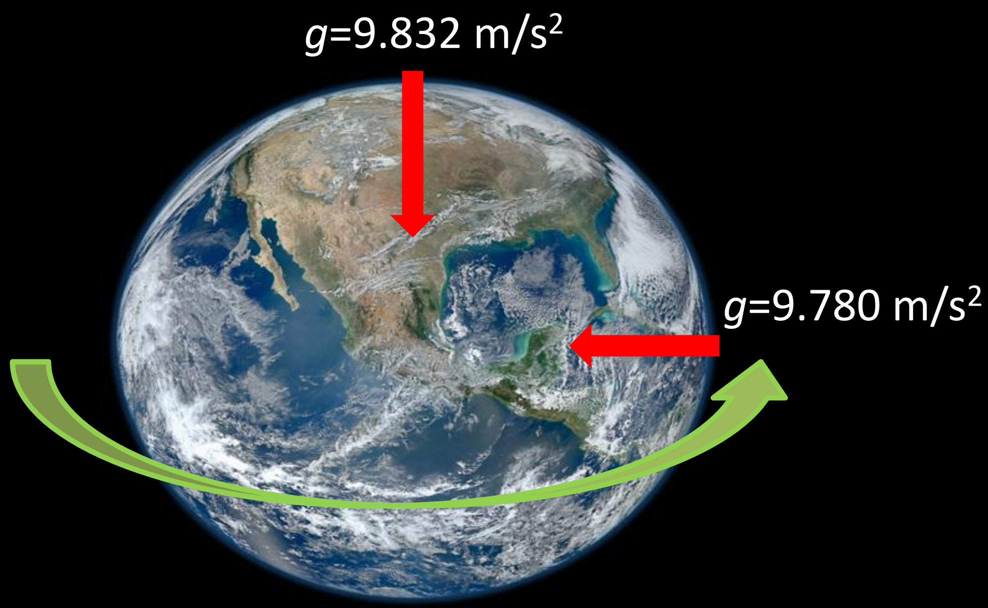
How much is g?

On average g=9.806 m/s^2

The earth is not a sphere!

????????

No more like this

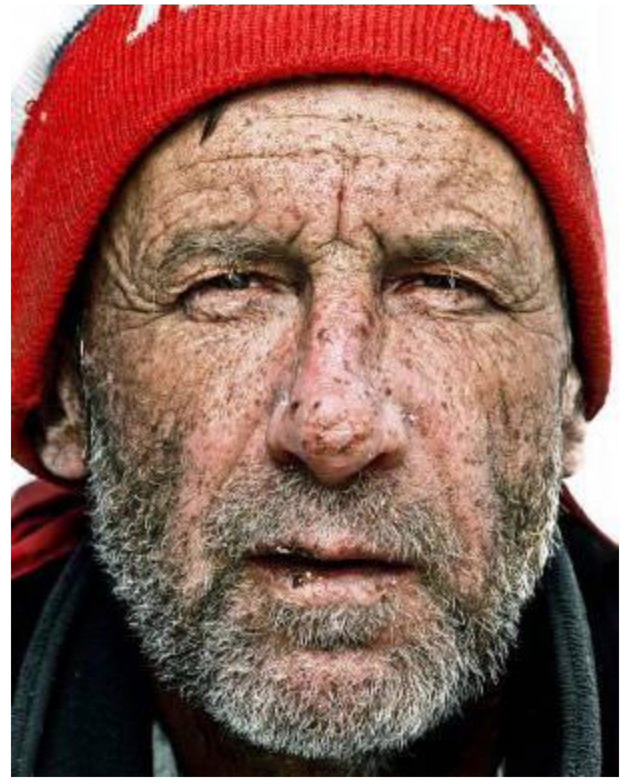


- Different distance to the center of the Earth
- Plus the spinning of the Earth

Different gravitational accelerations in different places


Gravity 0.28% smaller
8,848 meter

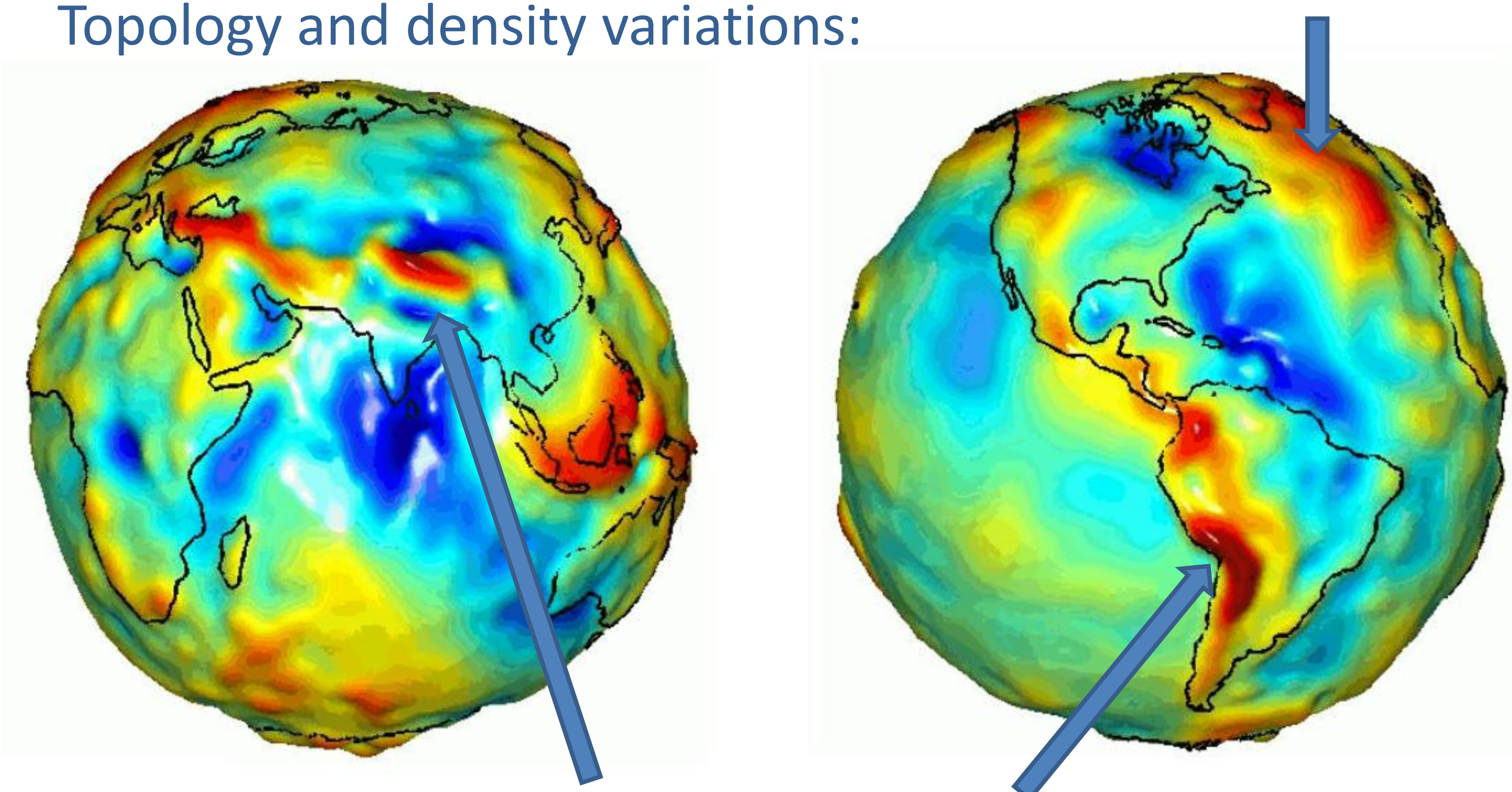
Ok, maybe climbing Mount Everest is not the best way to lose half a pound....



But even after taking all of this into account, there are still variations

High density

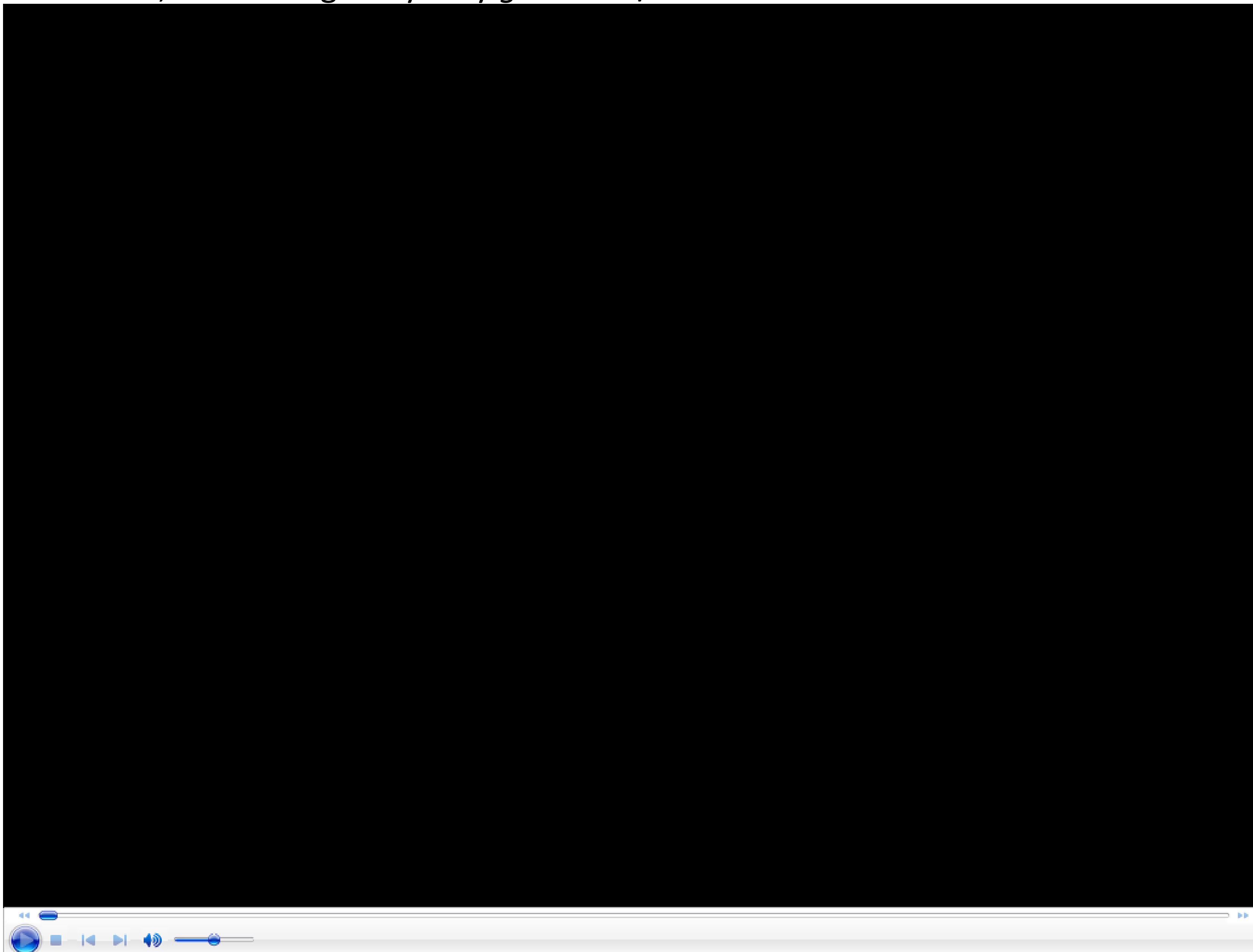
Topology and density variations:



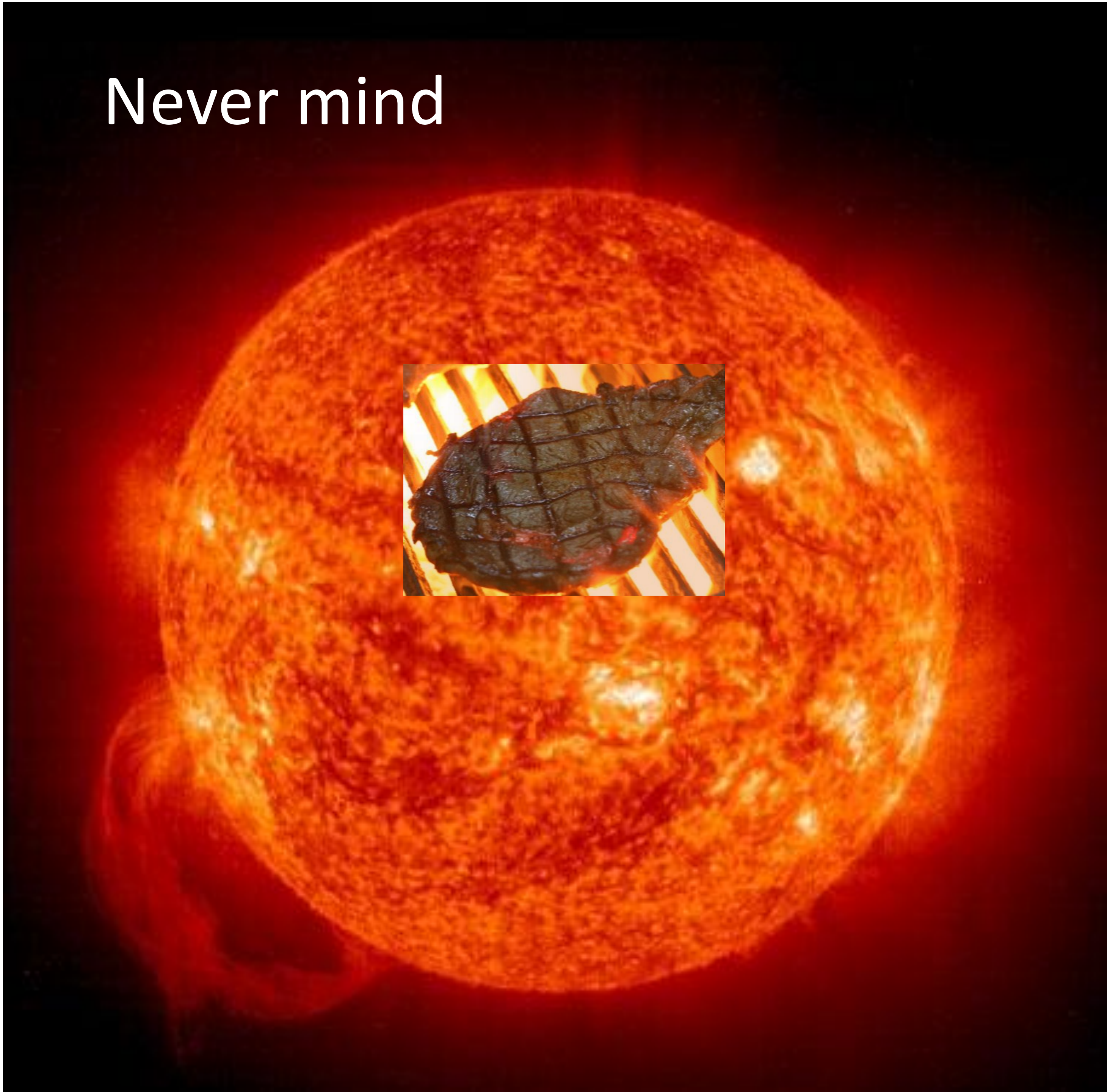
Himalayas

Andes

However, on Moon gravity only g=1.62 m/s^2 or 6 times smaller



But how about jumping on the Sun?


Never mind

Example.– If an object hits the ground with a velocity of 20 m/s, from what height was it dropped?

The time can be expressed in terms of the gravitational acceleration and the velocity as

$$v = gt \quad \Rightarrow \quad t = \frac{v}{g}. \tag{1.47}$$

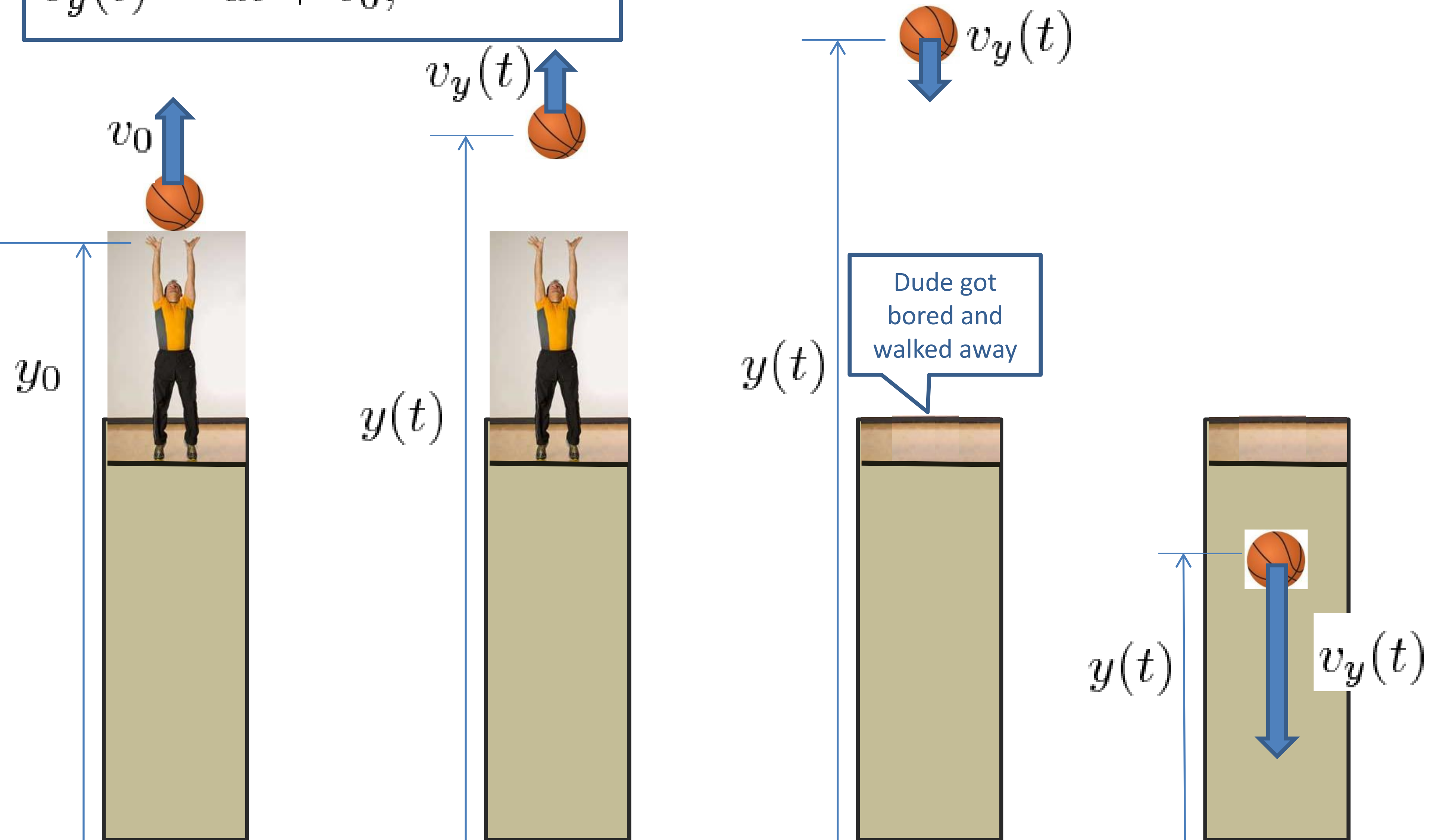
The height is then given by

$$h = \frac{1}{2} g t^2 = \frac{1}{2} g \left(\frac{v}{g} \right)^2 = \frac{1}{2} \frac{v^2}{g} = \frac{1}{2} \frac{20^2}{9.8} \cong 20.4 \ \ \text{m}. \tag{1.48}$$

More general formulas for arbitrary initial position and velocity

$$y(t) = \frac{1}{2}at^2 + v_0 t + y_0$$

$$v_y(t) = at + v_0,$$

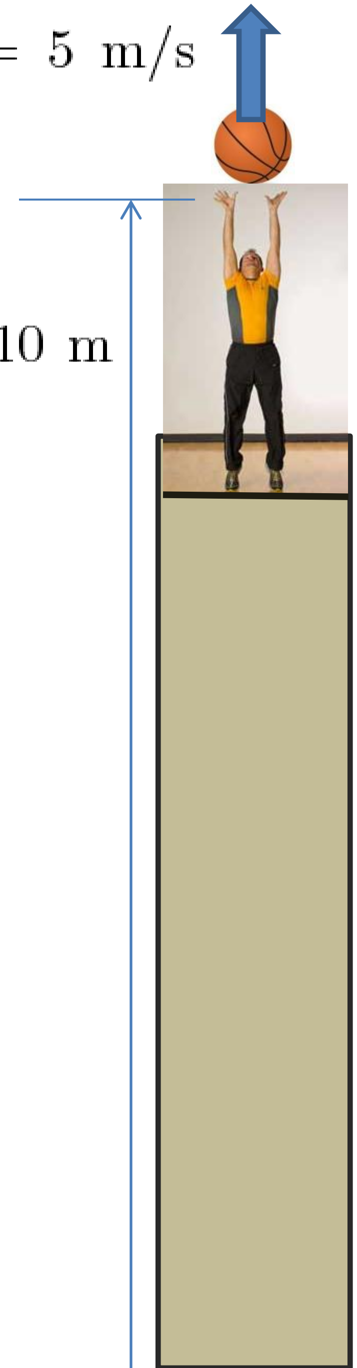


Example:

$$y_0 = 10 \text{ m}$$

$$v_0 = 5 \text{ m/s}$$

$v_0 = 5 \text{ m/s}$



$y_0 = 10 \text{ m}$

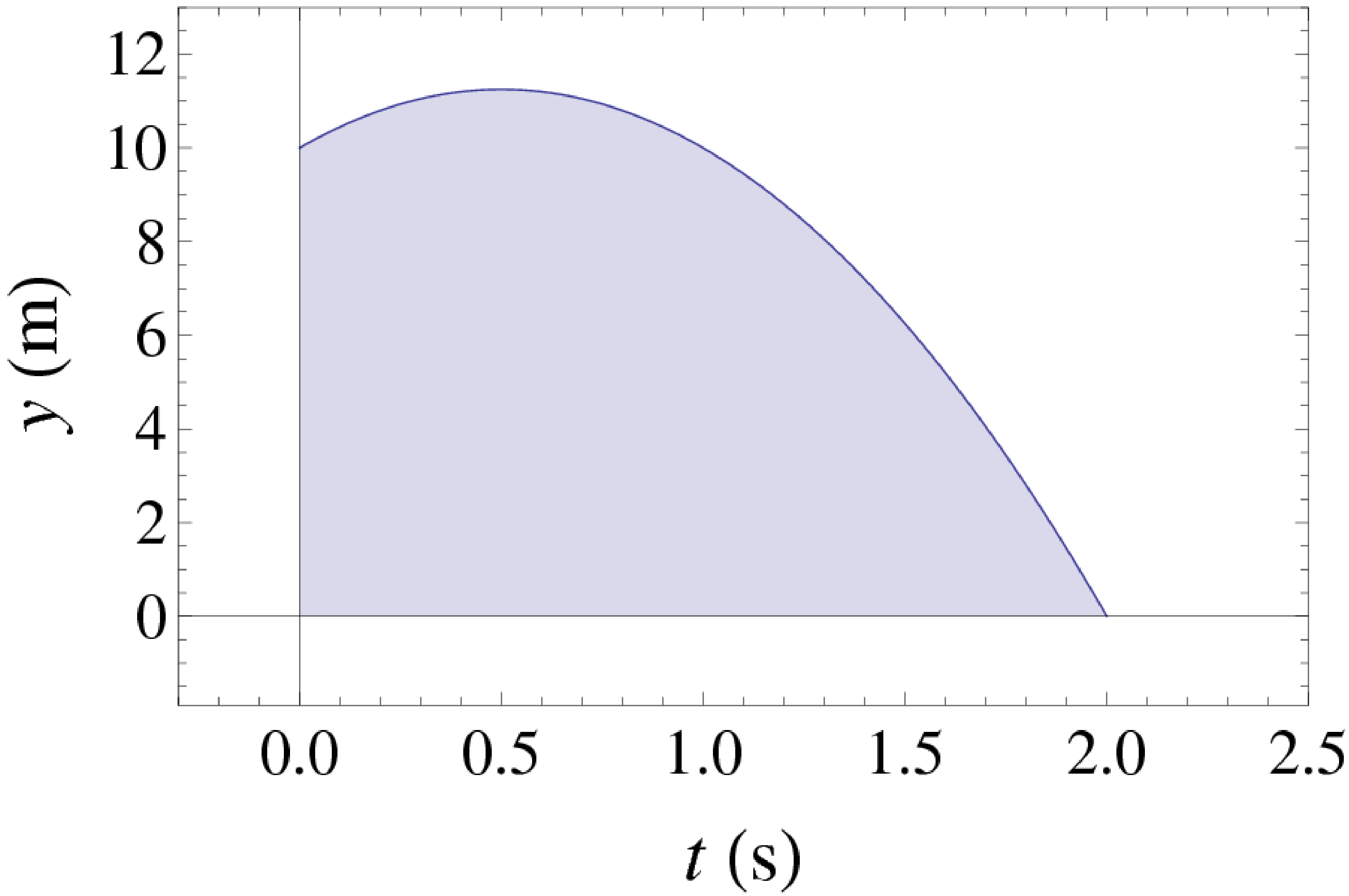
Take

$$g = -10 \text{ m/s}^2$$

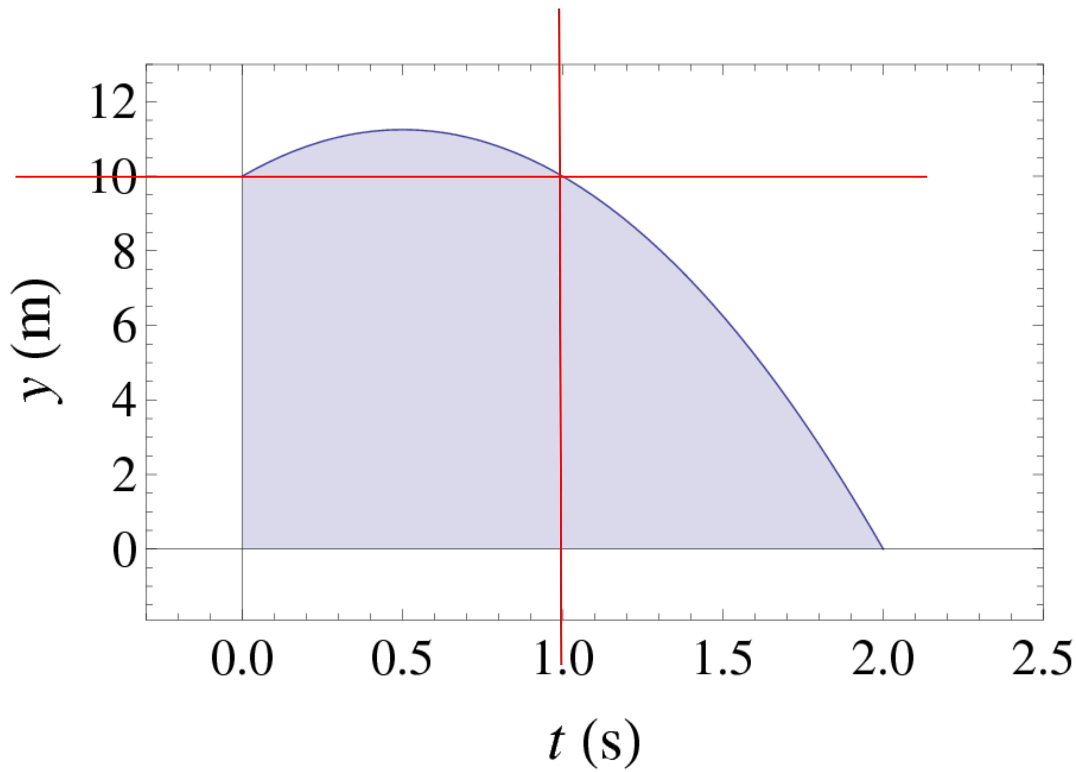
$$y(t) = -5t^2 + 5t + 10$$

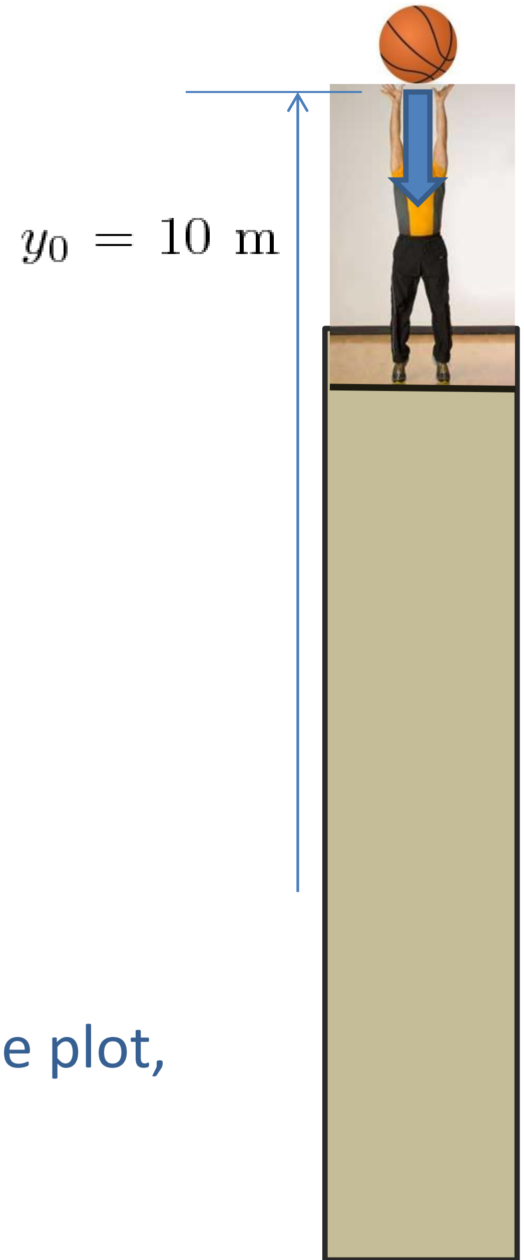
$$v_y(t) = -10t + 5$$

Using a computer it is easy to plot $y(t)$ as a function of t



(a) When is the object back at the same height of 10 m?





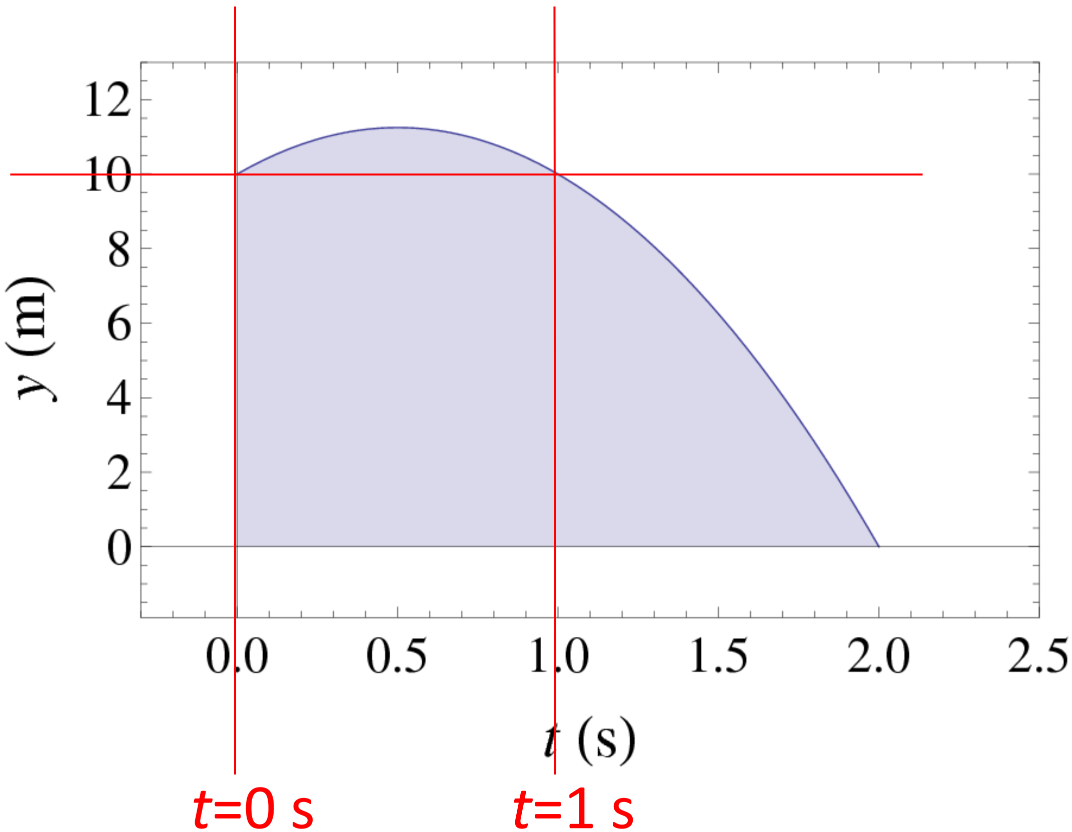
What is *t*?

(With a good figure you can read it from the plot, but we can also calculate it)

$$y(t) = 10 \quad \Rightarrow \quad -5t^2 + 5t + 10 = 10$$

This equation is easily solved giving

$$-5t^2 + 5t = 0 \quad \Rightarrow \quad -5t(t-1) = 0 \quad \Rightarrow \quad t = 0, 1 \ \text{ s.}$$



$$y(t) = \frac{1}{2}at^2 + v_0 t + y_0$$

Check result:

$$y(1) = \quad -5 \times 1^2 + 5 \times 1 + 10 = 10 \text{ m.}$$

What about the velocity?

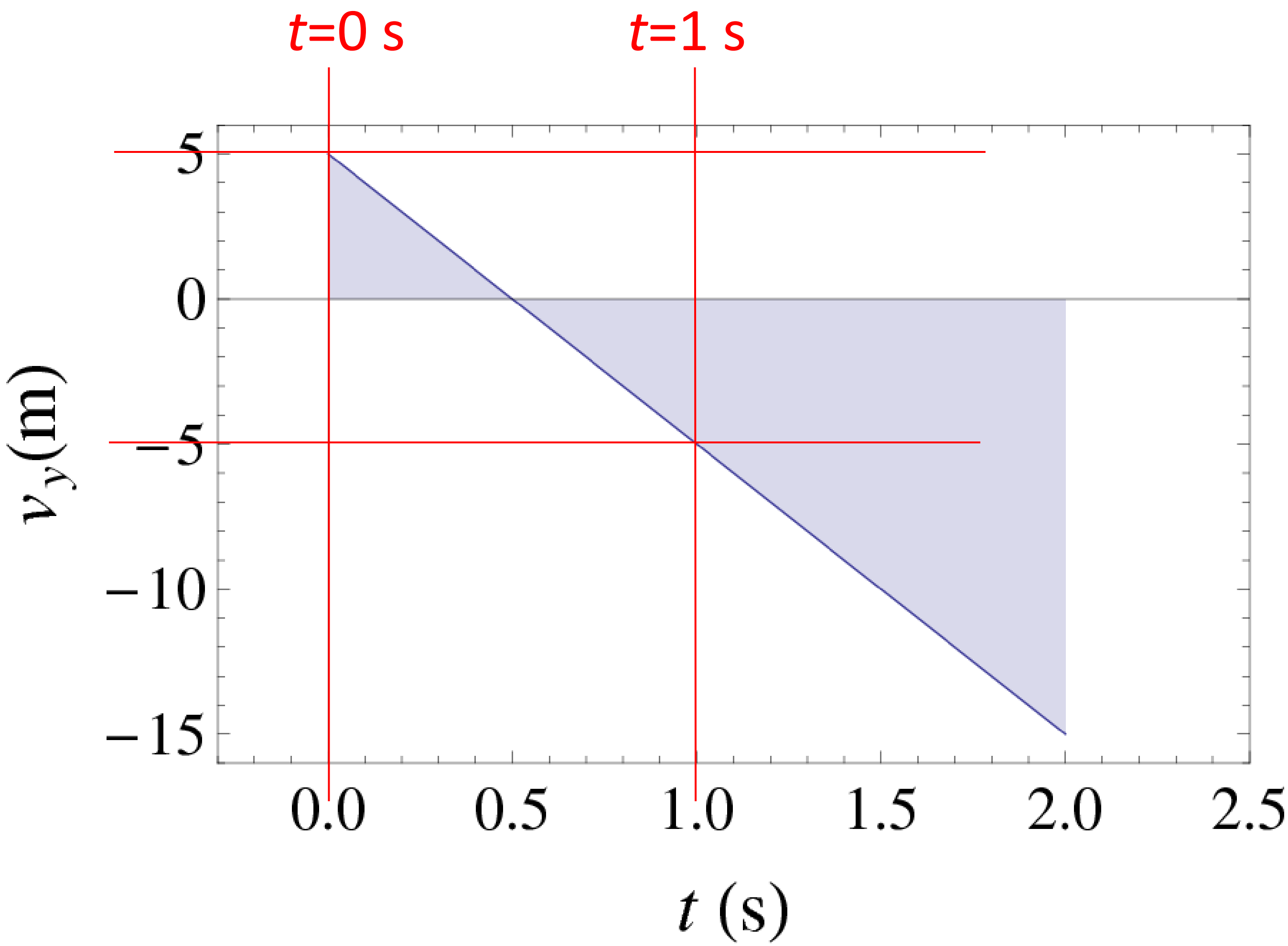
$$v_y(1) = -10 \times +5 = -5 \quad \mathrm{m/s}$$

Compare:

$$v_0 \ = \ 5 \ \ \mathrm{m/s}$$

Same size, but opposite sign!

We can also plot $v_y(t)$ as a function of t



(b) When does the object hit the ground?

Since we take the ground at $y = 0$, we have to solve the equation

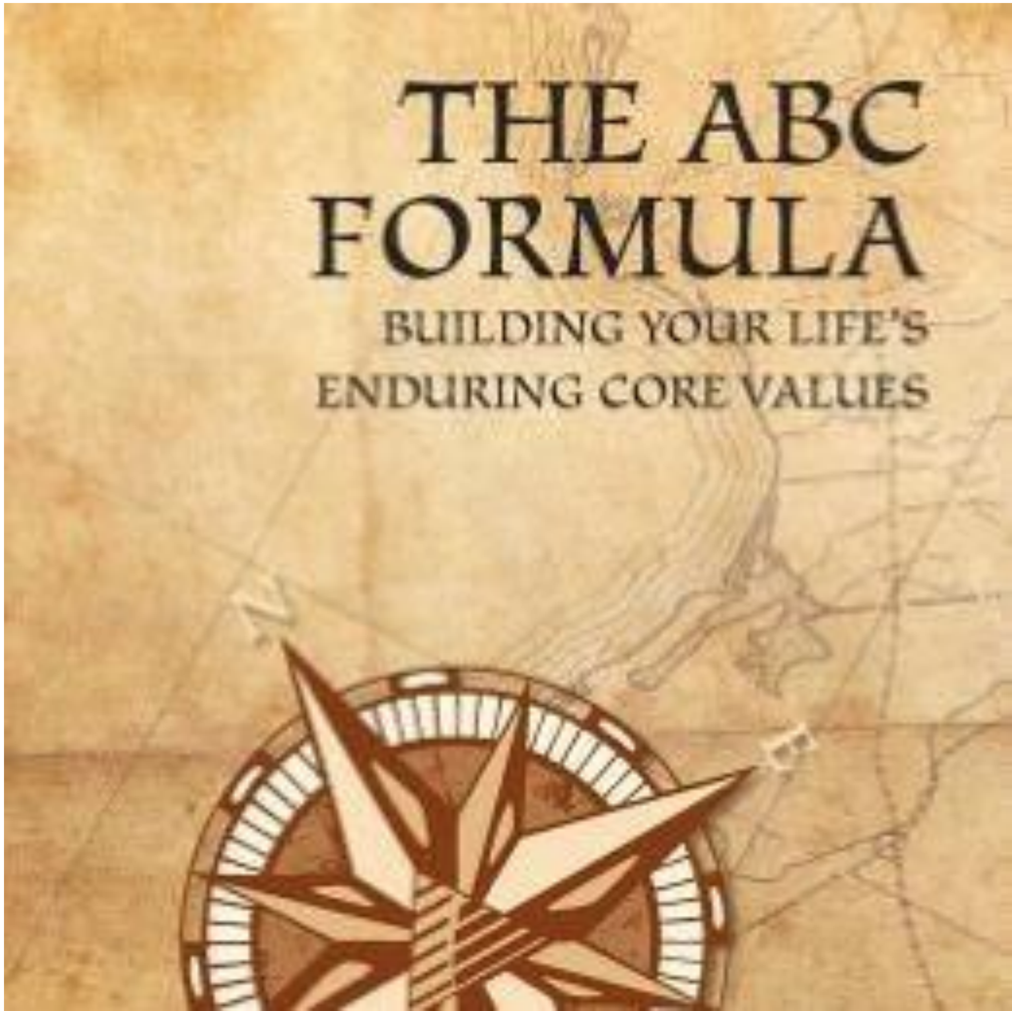
$$-5t^2 + 5t + 10 = 0.$$

Quadratic equations

$$at^2 + bt + c = 0.$$

Solutions with the abc-formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$



$$-5t^2 + 5t + 10 = 0.$$

$$t = \frac{-5 \pm \sqrt{5^2 - 4(-5)10}}{2 \times (-5)} = \frac{-5 \pm \sqrt{25 + 200}}{-10} = \frac{-5 \pm \sqrt{225}}{-10}$$

$$= \frac{-5 \pm 15}{-10} = \begin{cases} \frac{10}{-10} = -1 \\ \frac{-20}{-10} = 2 \end{cases}$$

Yet another t-shirt closer to your degree

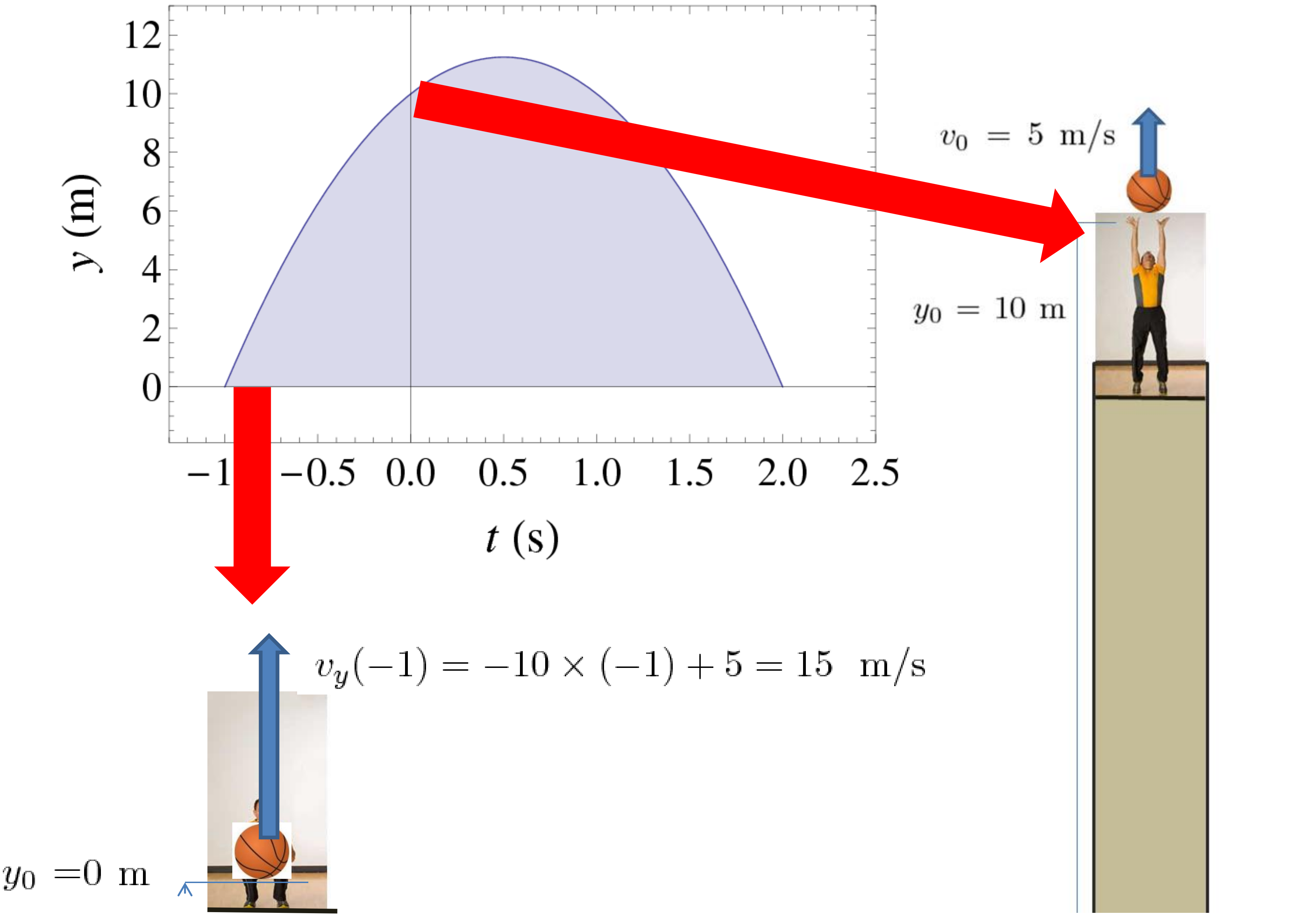
The Quadratic Formula ...

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For Quadratic Equations

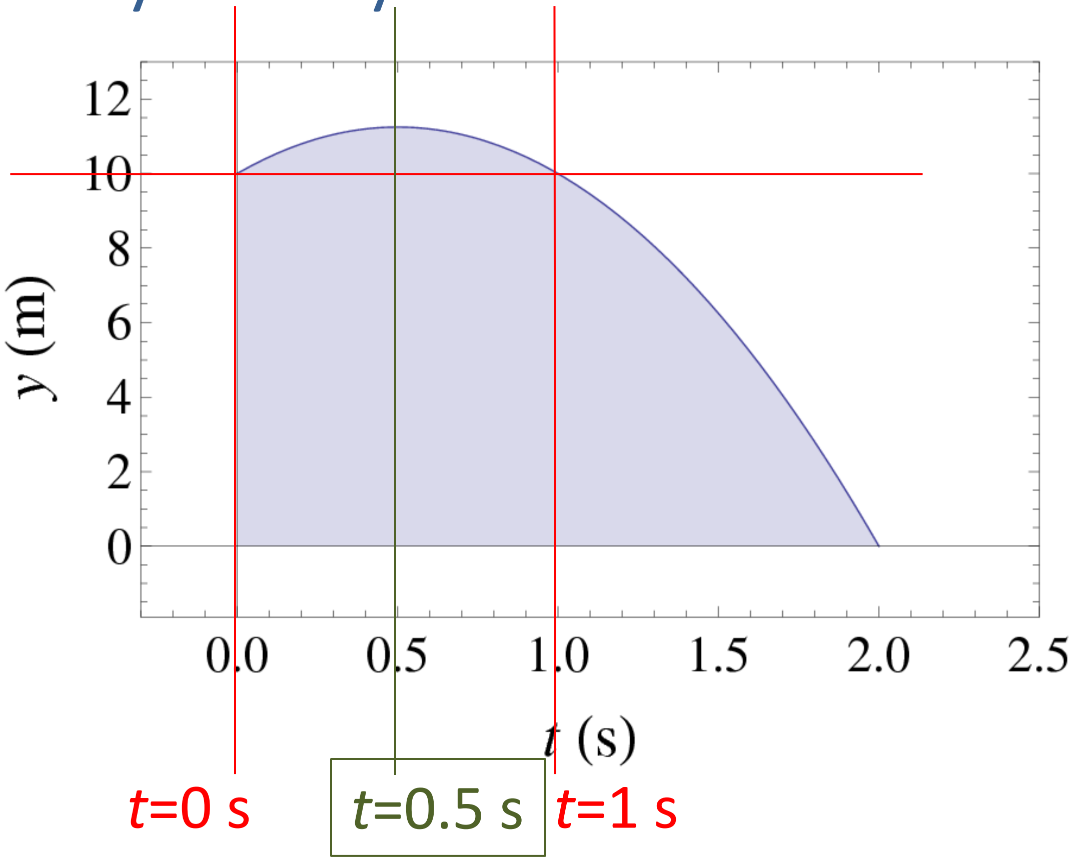
$ax^2 + bx + c = 0$

Why two values of *t*?



(c) When is the object at its maximum and what height does it reach?

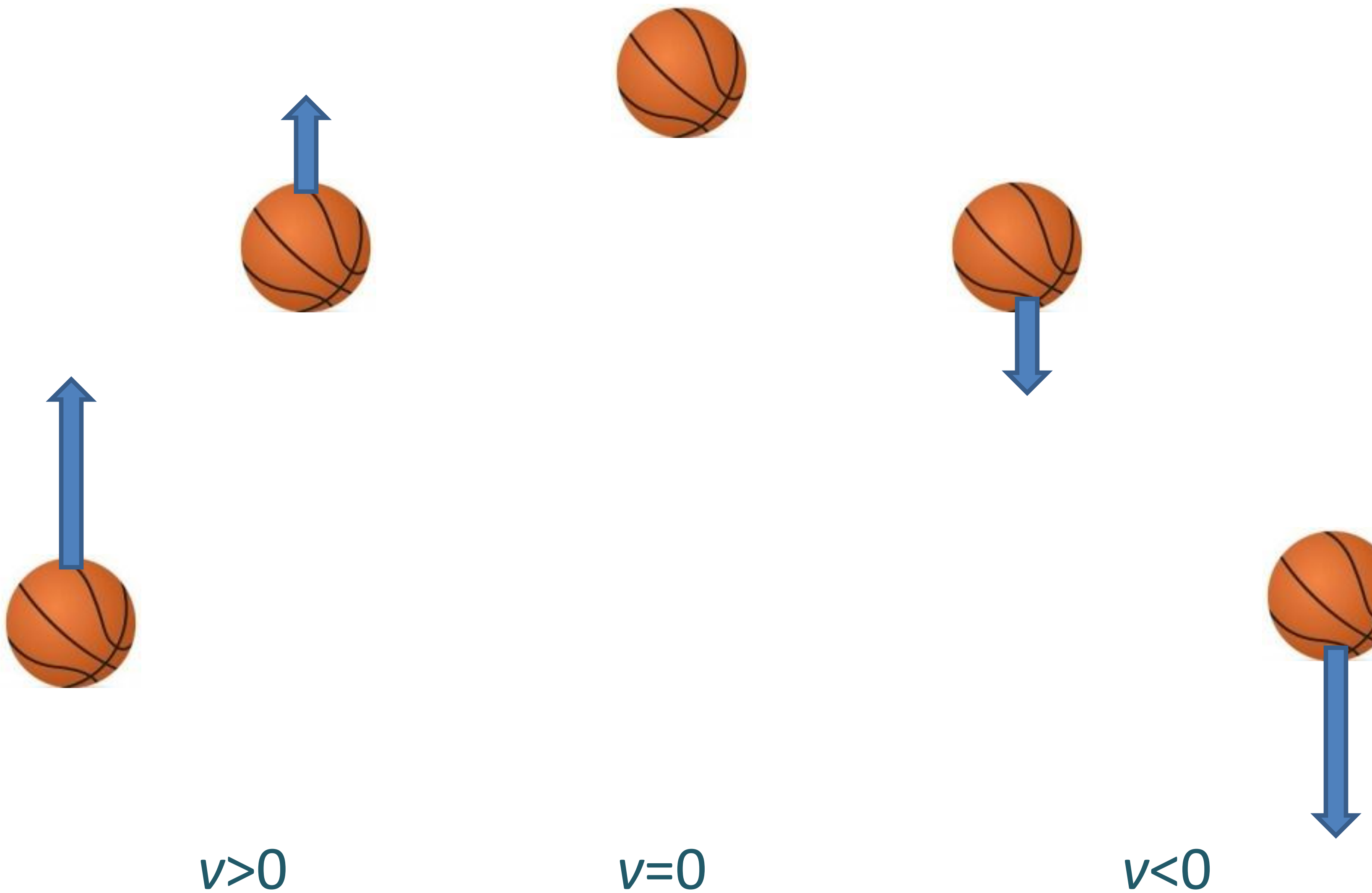
Method I: Symmetry



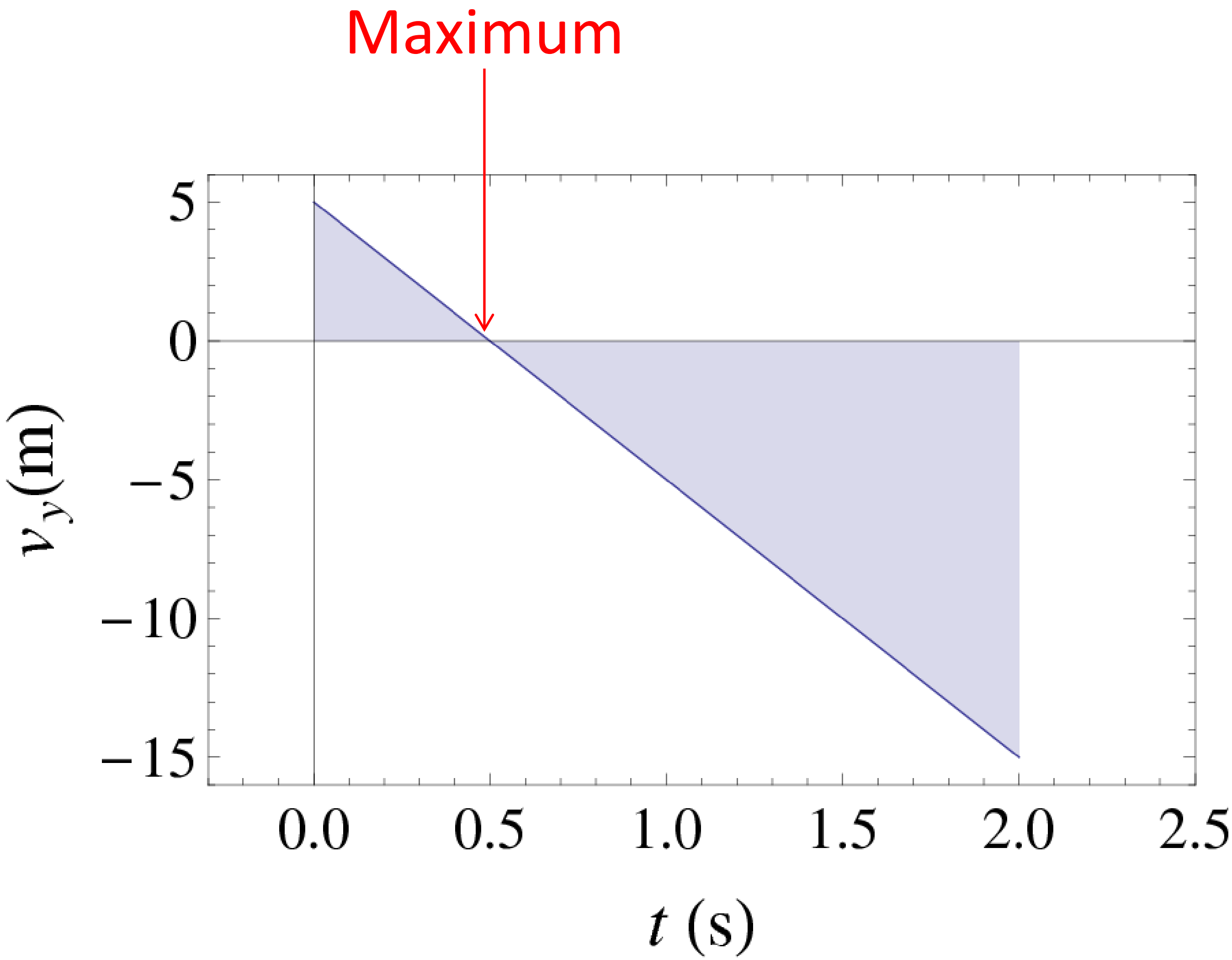
$$t_{\text{max}} = \frac{0+1}{2} = \frac{1}{2} = 0.5 \quad \text{s}$$

$$y(\frac{1}{2}) = -5\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + 10 = -\frac{5}{4} + \frac{5}{2} + 10 = \frac{5}{4} + 10 = 11\frac{1}{4}$$

What is a maximum?



Velocity is zero at the maximum!

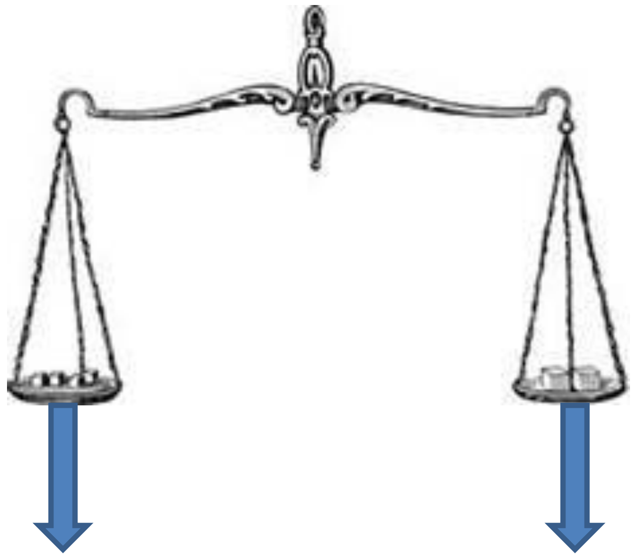


$$v_y(-1) = -10t + 5 = 0 \quad \Rightarrow \quad t = \frac{5}{10} = \frac{1}{2} \ \text{s.}$$

This is the same time as we found earlier!

Gravitational Force

Balance measures mass or, better, compares mass



$m_1g=m_2g$

Or

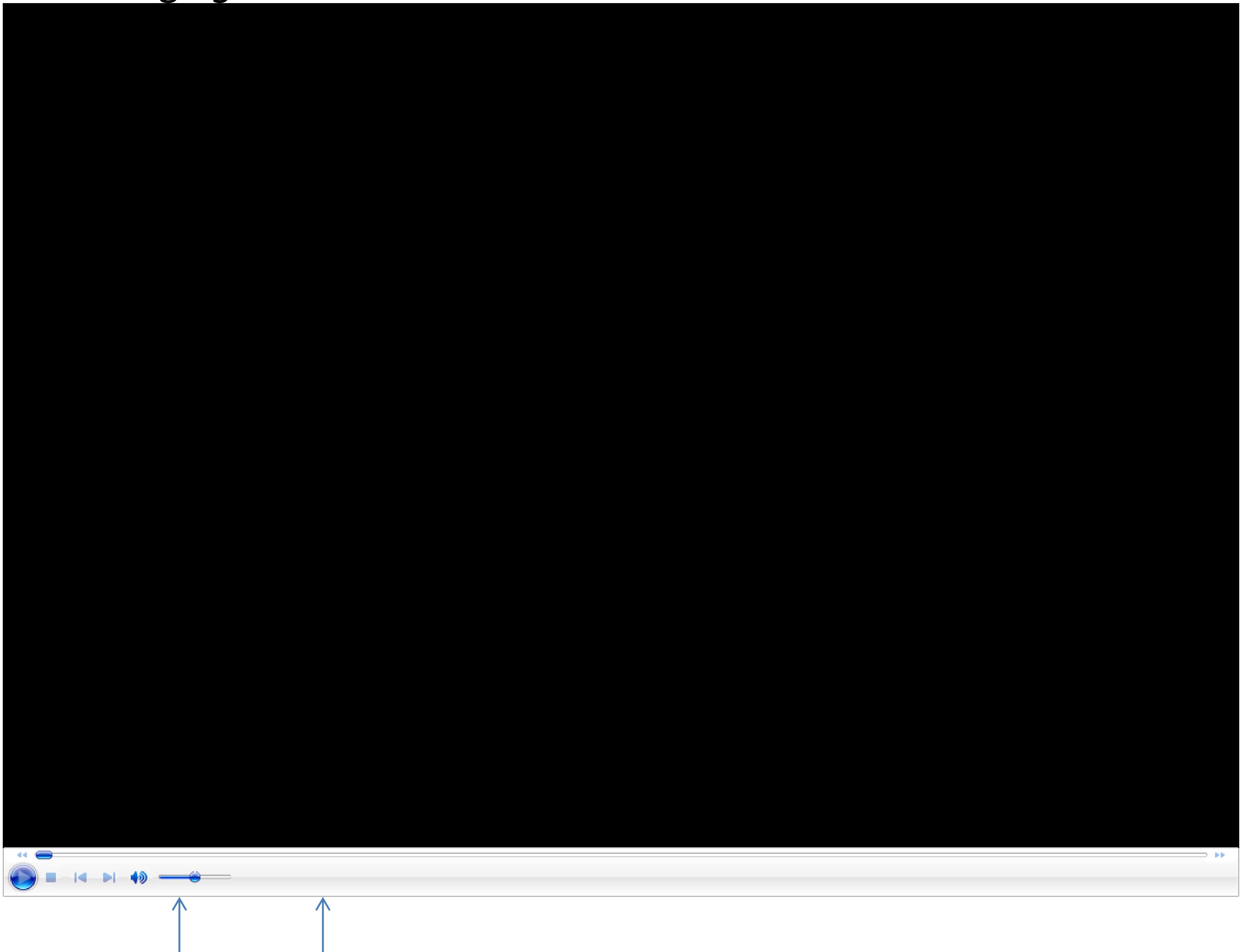
$m_1=m_2$

On the other hand,

F=mg or *m=F/g* if *g* is correct

So scale made for Earth does not work on the Moon

Although g is an acceleration do not confuse it with a



In space:

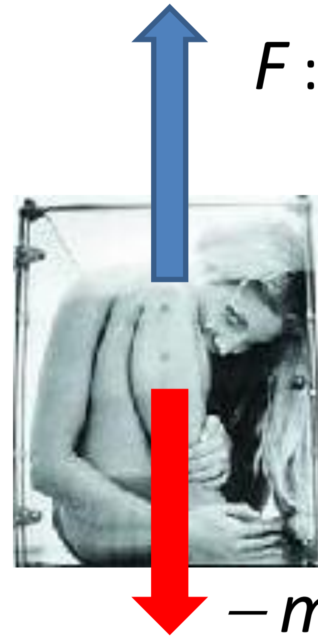
F : Force lifting person (on any arbitrary object)

$$F = ma$$

If acceleration is small then little force is needed.
In space, objects are weightless

On Earth:

F : force



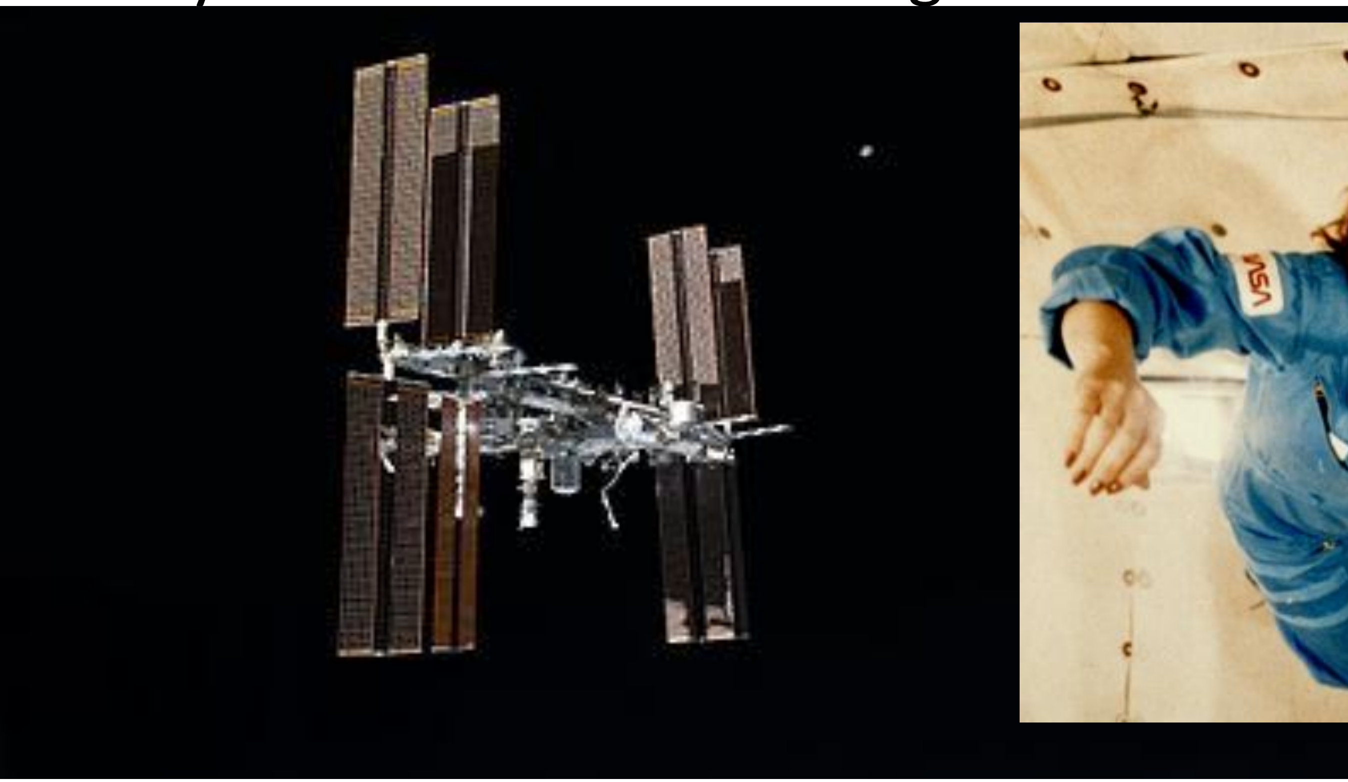
$-mg$: weight

$$F - mg = ma$$
$$F = m(g+a)$$

On Earth, one also has to overcome gravity.

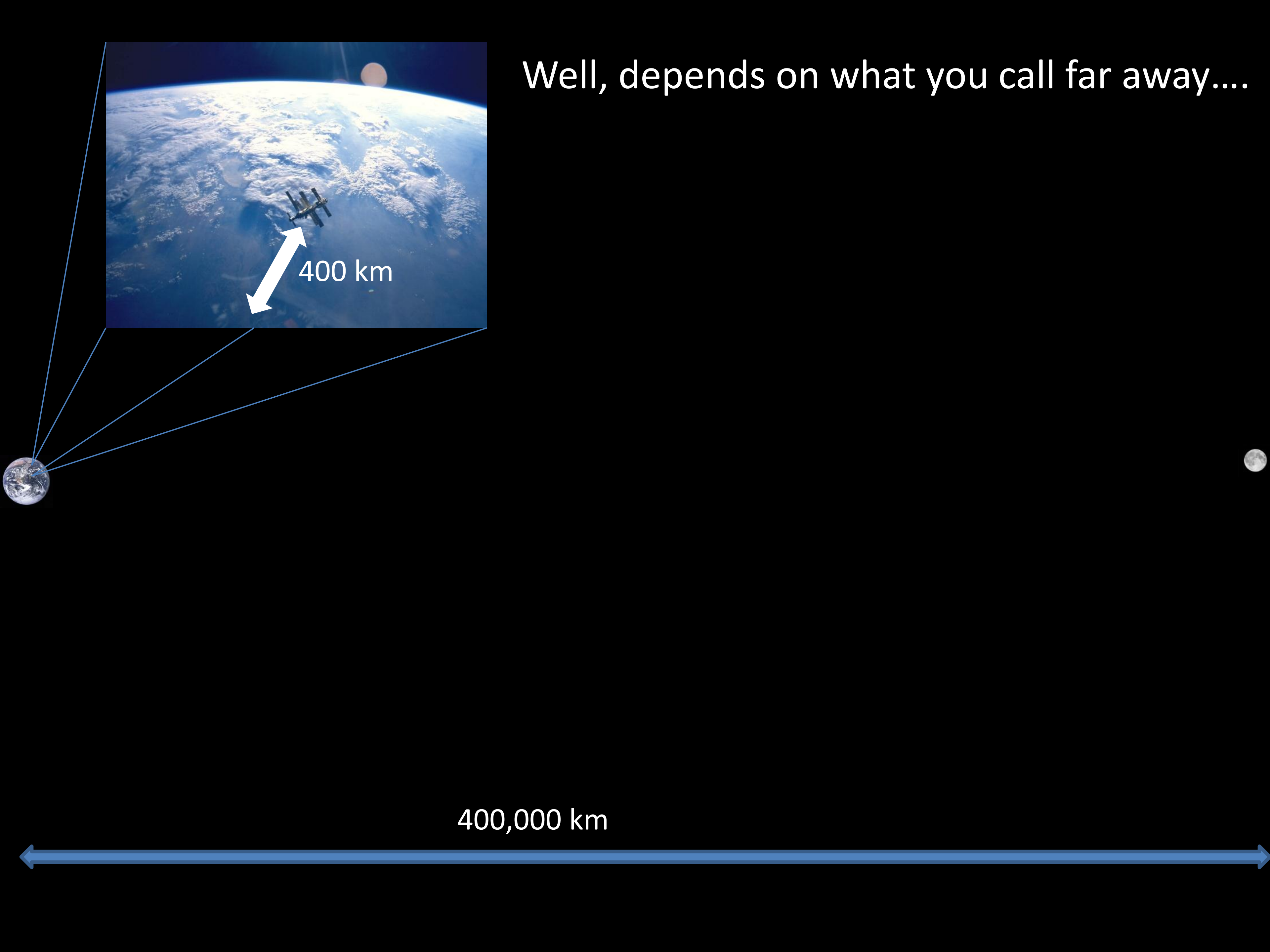
Question:

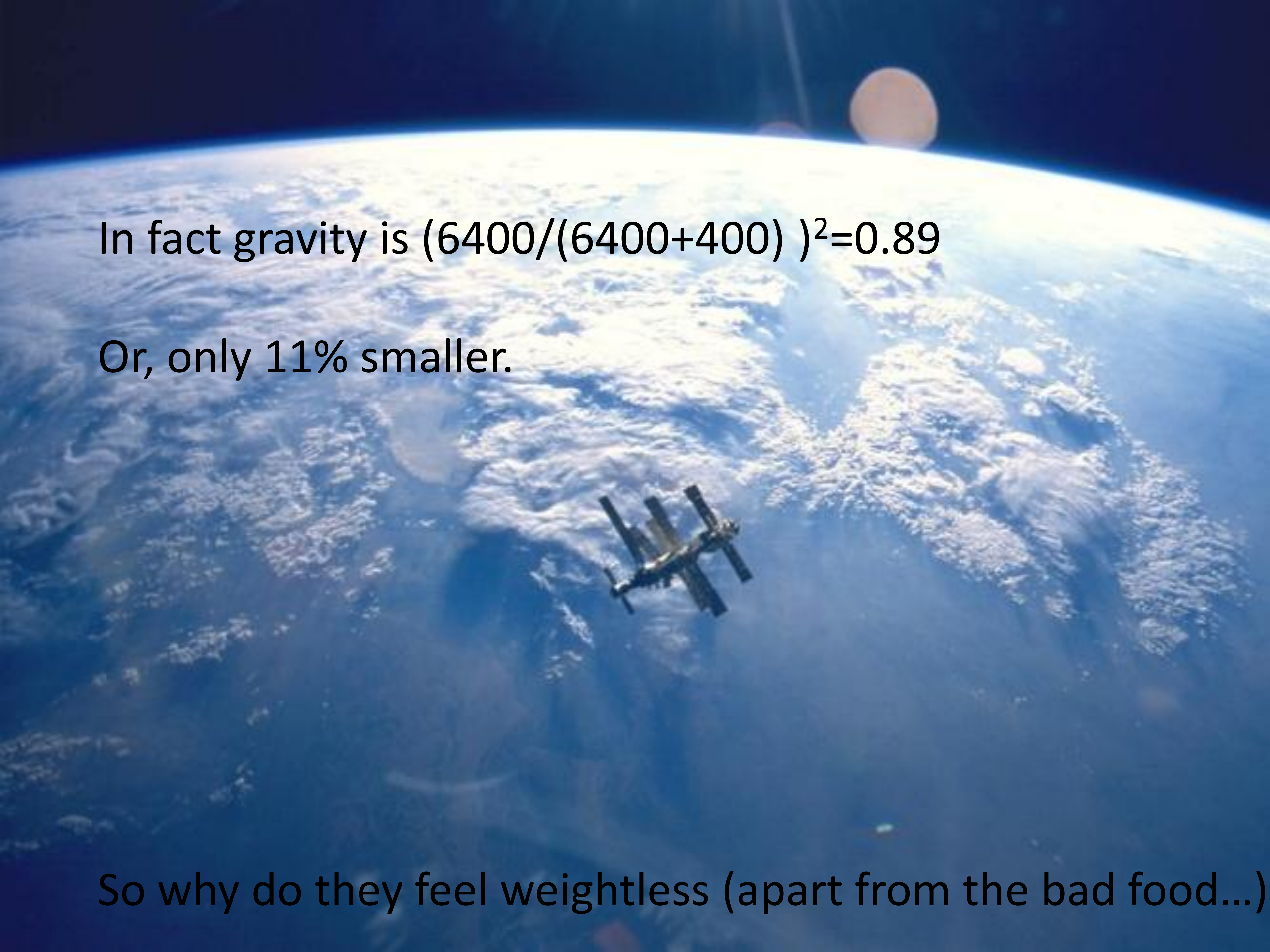
Why do astronauts feel weightless on the spacestation?



A. Because astronaut food really sucks, and they loose a lot of weight in no time….

B. Because the space station is so high up they kind'a run out of gravity….


Well, depends on what you call far away....
400 km
400,000 km



In fact gravity is $(6400/(6400+400))^2=0.89$

Or, only 11% smaller.

So why do they feel weightless (apart from the bad food…)

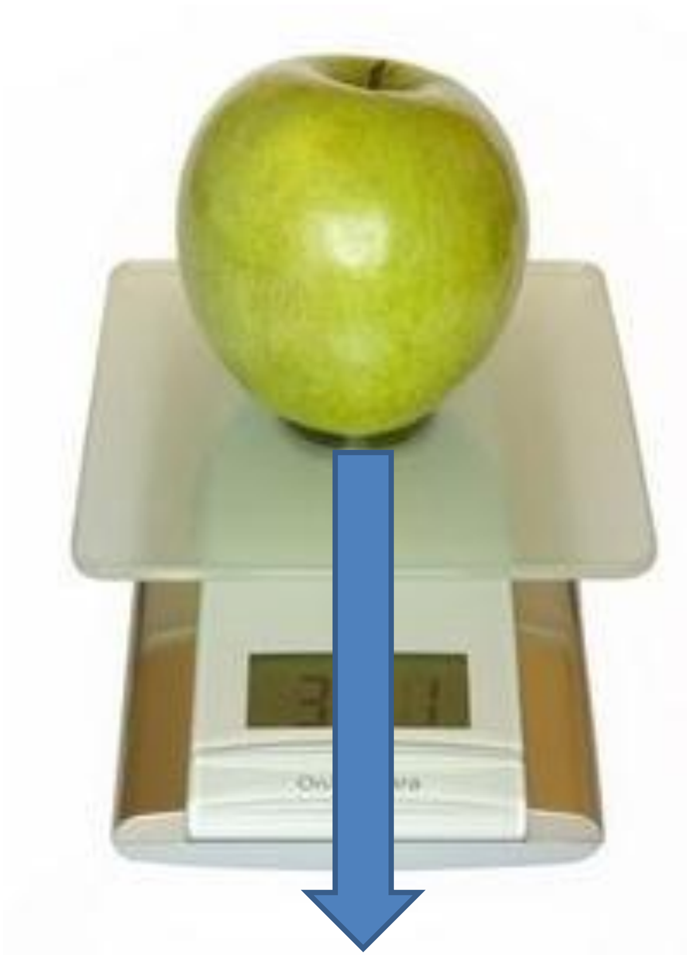
Let’s go back to a scale

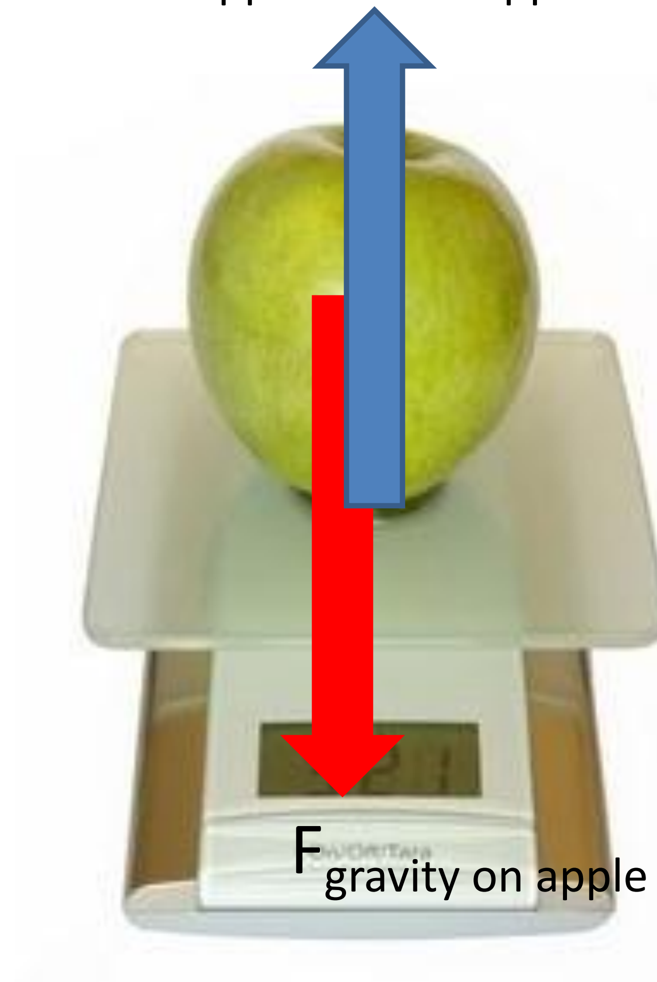
$F = -mg$

So that is the force you are applying to the scale

(we added a sign)

$$F_{\text{scale on apple}} = -F_{\text{apple on scale}} = mg$$





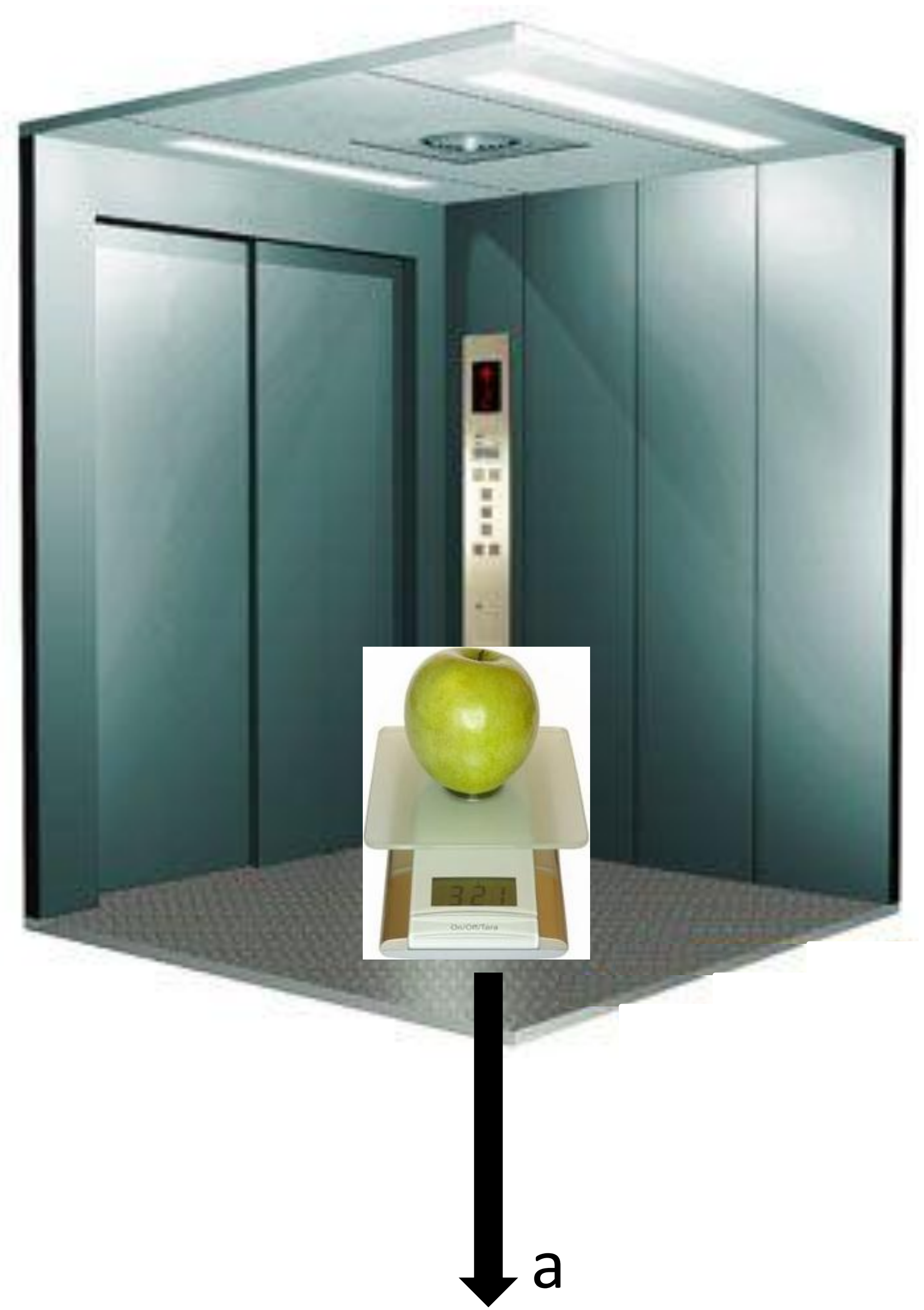
$$F_{\text{gravity on apple}} = -mg$$

$$F_{\text{apple on scale}} = -mg$$

$$F_{\text{gravity on apple}} + F_{\text{scale on apple}} = 0 = ma$$

or a=0 and the apple is at rest

Let’s put the whole thing in an accelerating elevator.



Note: it is accelerating not moving at a constant velocity

What does the scale indicate now?

$F_{\text{gravity on apple}} + F_{\text{scale on apple}} = -\,ma$

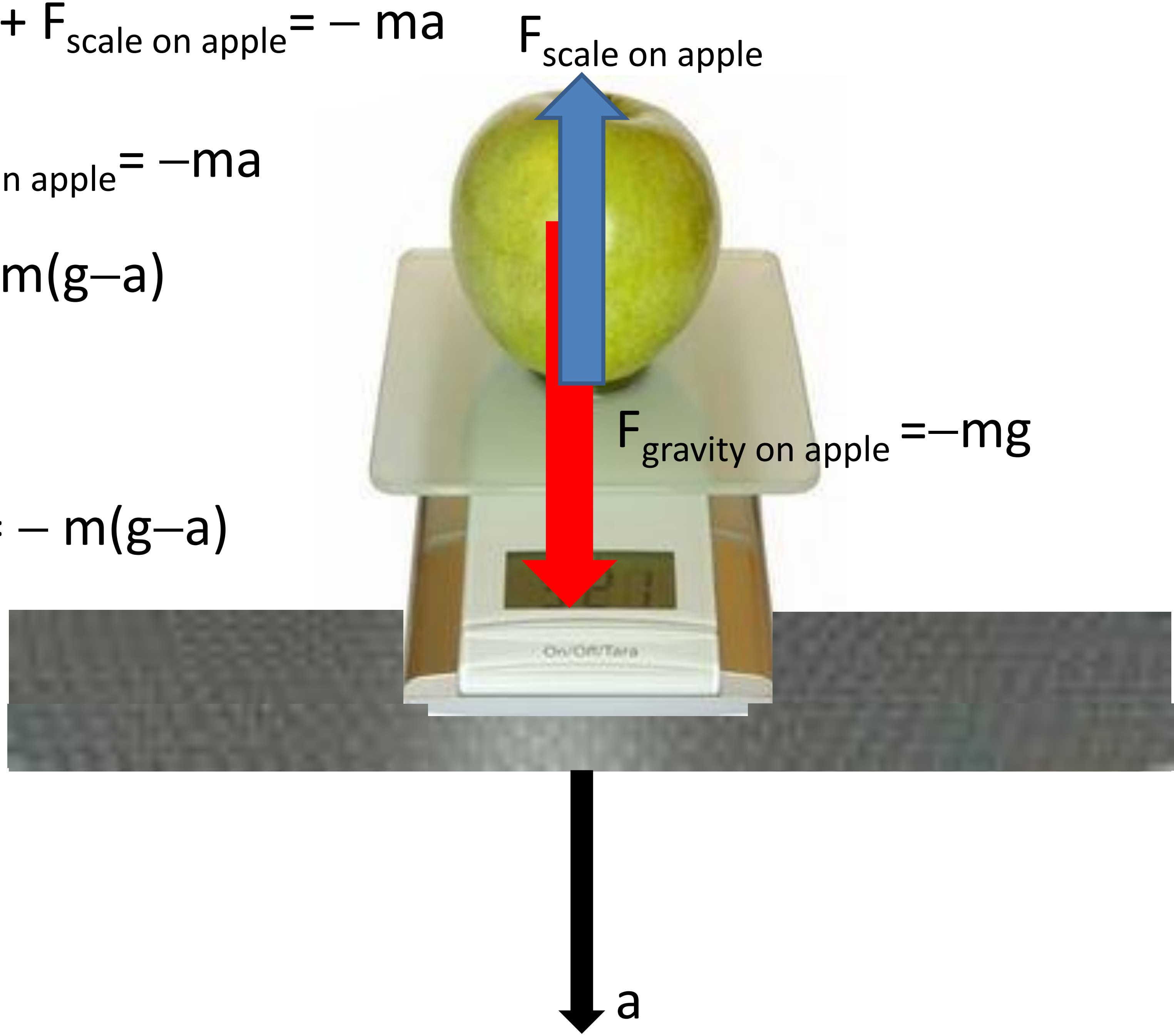
or

$-mg + F_{\text{scale on apple}} = -ma$

$F_{\text{scale on apple}} = m(g-a)$

and

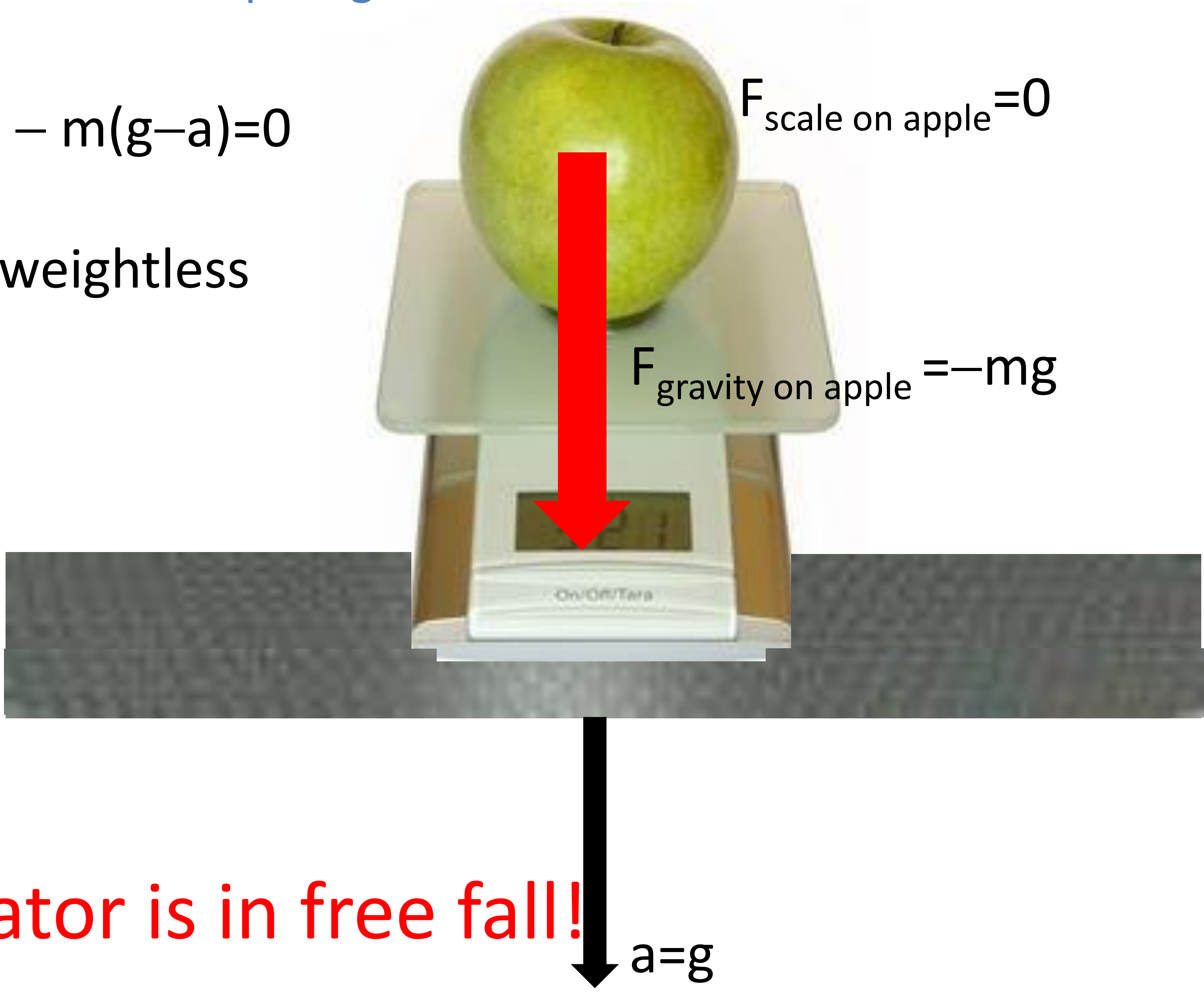
$F_{\text{apple on scale}} = -\,m(g-a)$



What if the acceleration equals g?

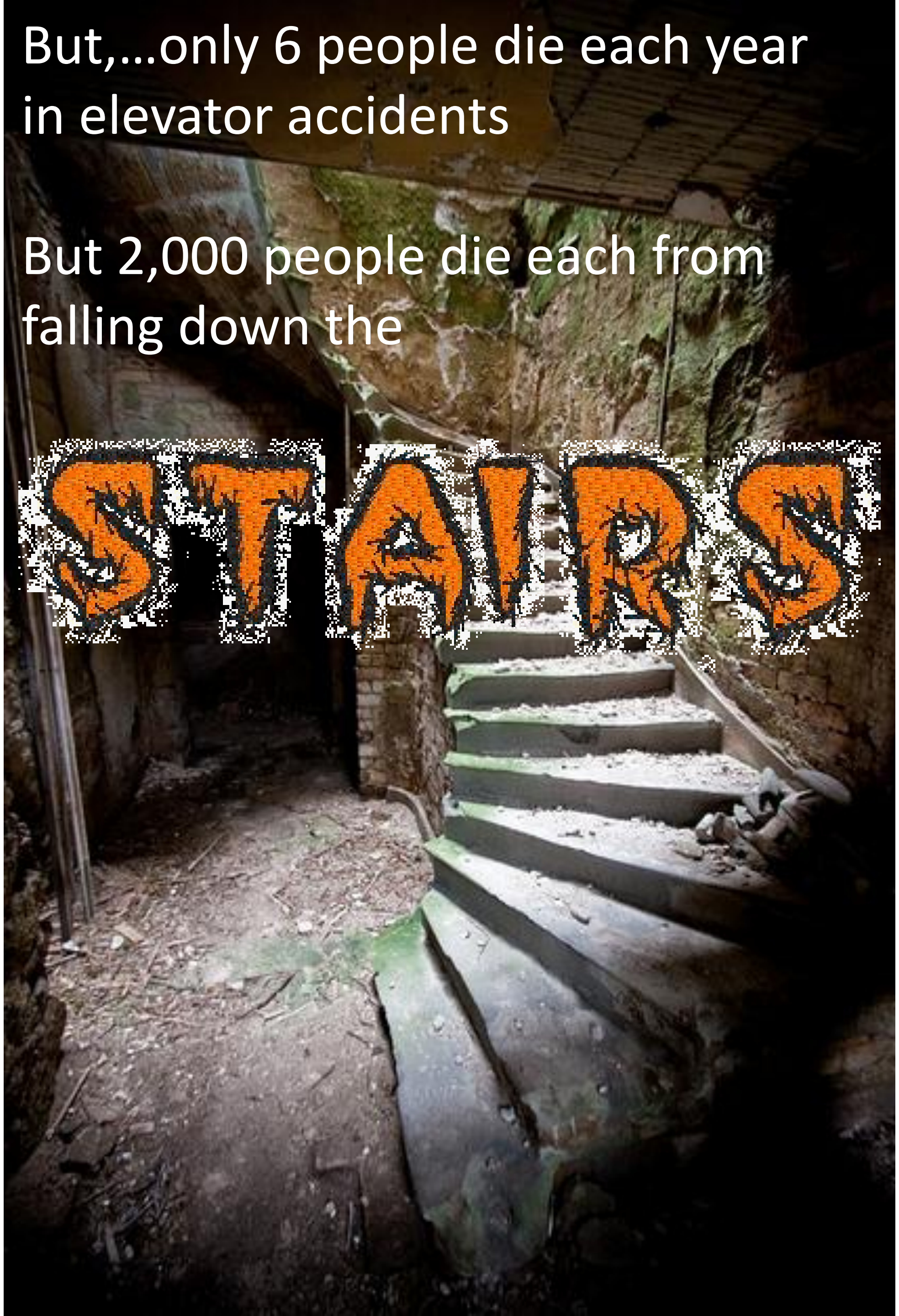
$F_{\text{apple on scale}} = -\,m(g-a) = 0$

The apple is weightless



The elevator is in free fall!

Elevators are fricking dangerous!


But,…only 6 people die each year in elevator accidents
But 2,000 people die each from falling down the
STAIRS

Then again, people don't care much for statistics…
Most dangerous animals in the USA!

Mwah…
Not so much

They just get a rather bad rap

1 death/year

0.1 deaths/year

0.5 deaths/year

0.3 deaths/year

On the other hand, these nasty looking animals….

20 deaths/year

22 deaths/year

31 deaths/year

53 deaths/year

And the winner is

130 deaths per year

Although the majority of them is car related.

still more people die from deer attacks than by sharks…


DEER
WEEK
Discovery
CHANNEL

So somebody in free fall feels weightless

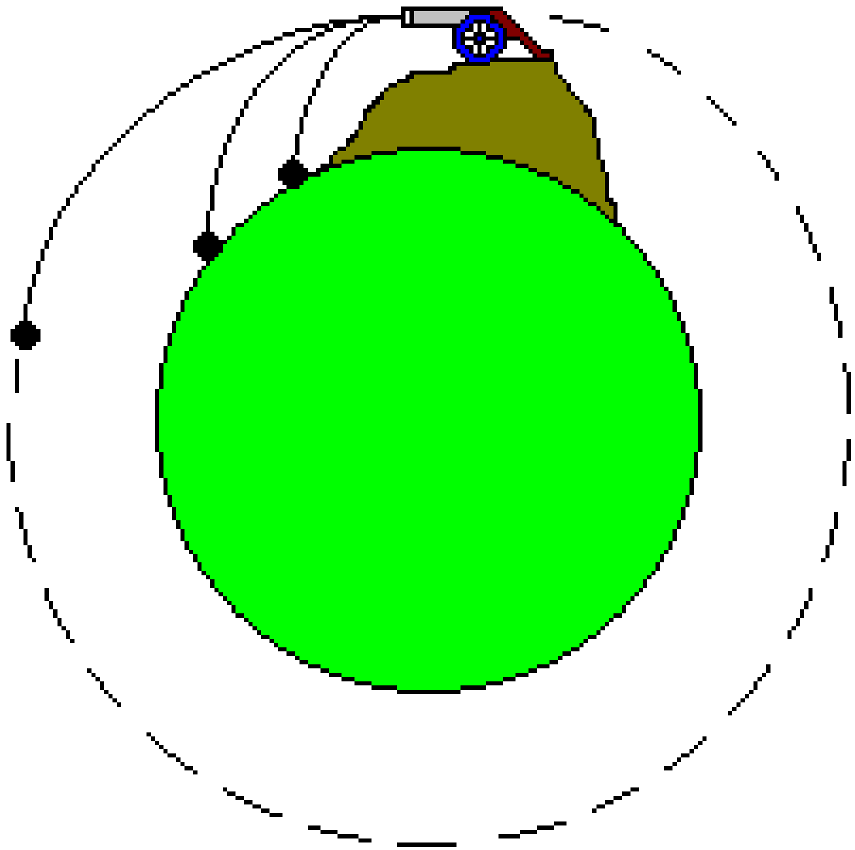
Note that this does not mean mass less

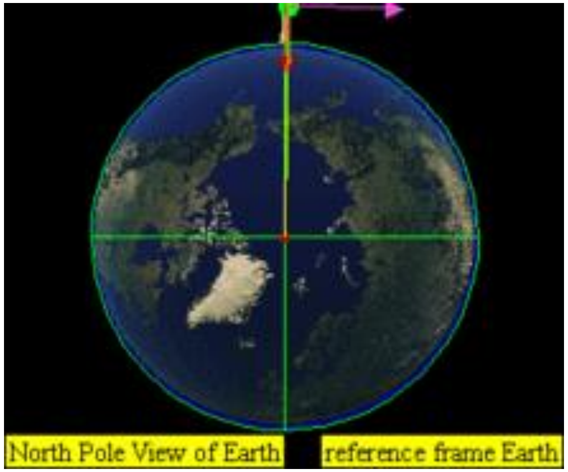
But this motion stops at some point

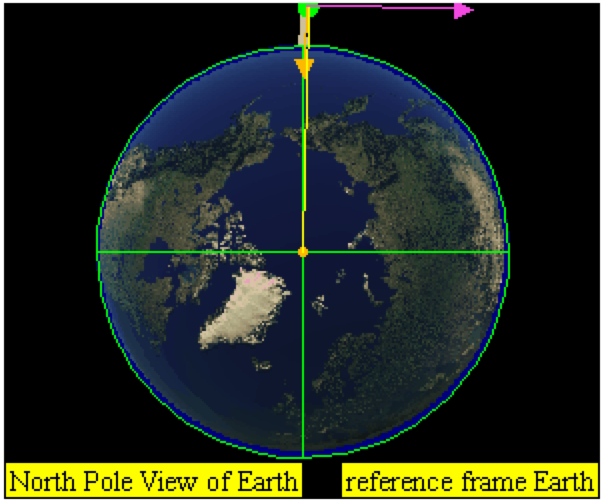
So how can a spaceship not hit the ground?

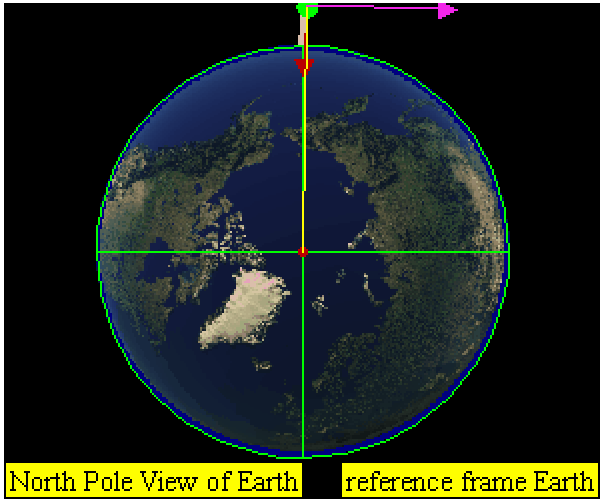
Spaceships are falling all the time without hitting the Earth

When the conditions are just right, it will keep going round

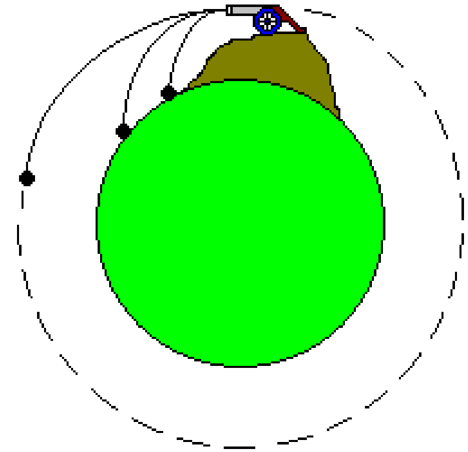



North Pole View of Earth
reference frame Earth


North Pole View of Earth
reference frame Earth


North Pole View of Earth
reference frame Earth

What is the speed necessary: v=28,000 km/h



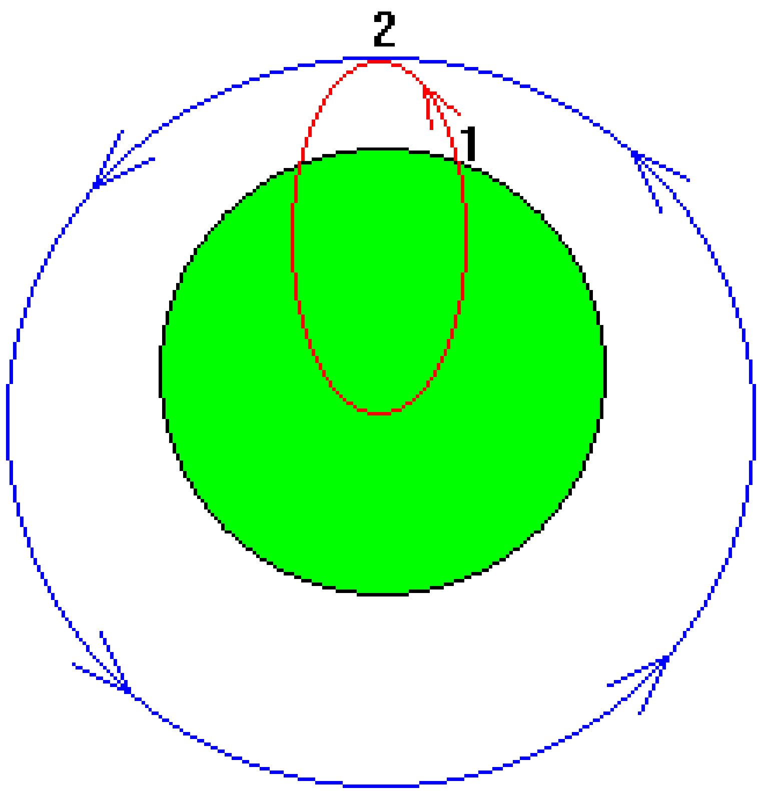
Jet liner v=900 km/h

V=1200 km/h

V=3500 km/h

X-15 rocket plane v=7200 km/h

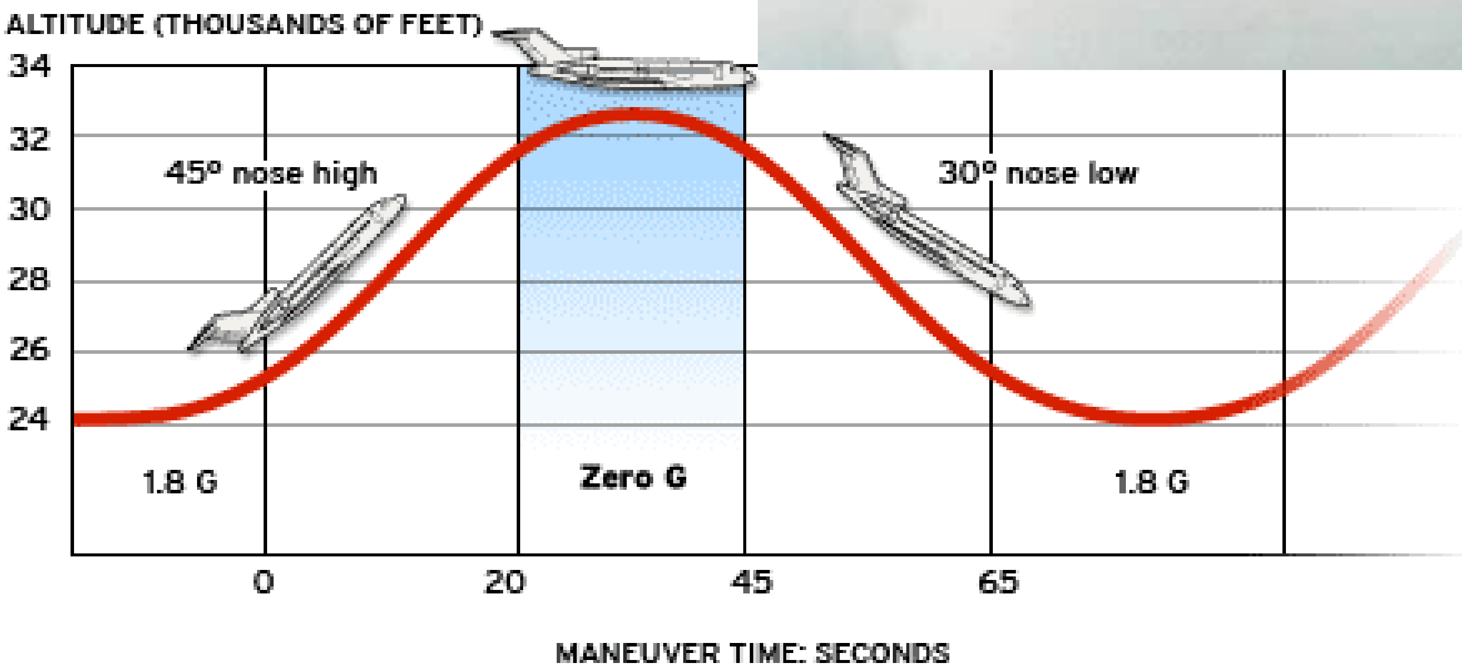
However, also an object that does not satisfy these conditions would “circle” around the center of the Earth, if that annoying surface was not there….



Free fall can be simulated on a plane:

- Without air resistance (drag) every fall would be a free fall
- However, one needs to compensate for drag to make it free fall
- About 25 seconds of free fall (followed by 25 seconds of 2g!)

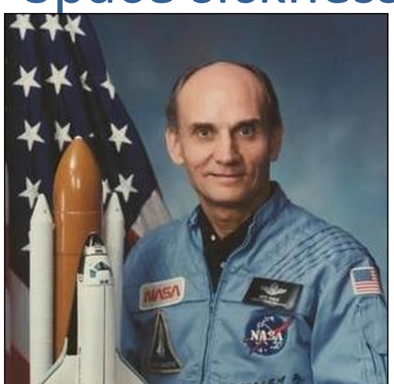
$$h = \frac{1}{2} g t^2$$



Weightless if fun…., I guess
With all kinds of unusual effects

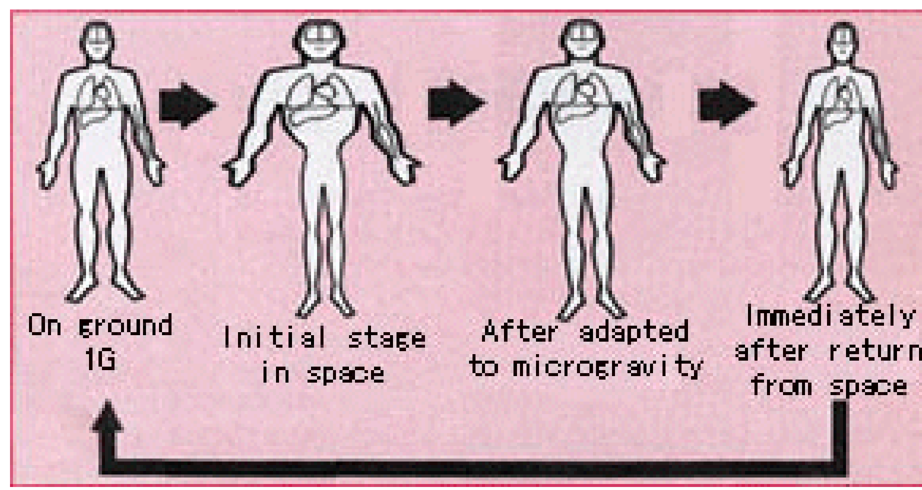
Giant frizzle, ok that's Earth, but this weightless frizzle

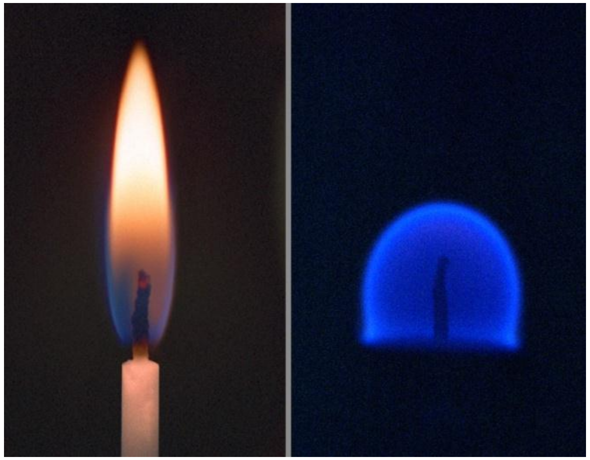
Space sickness, fluid redistribution



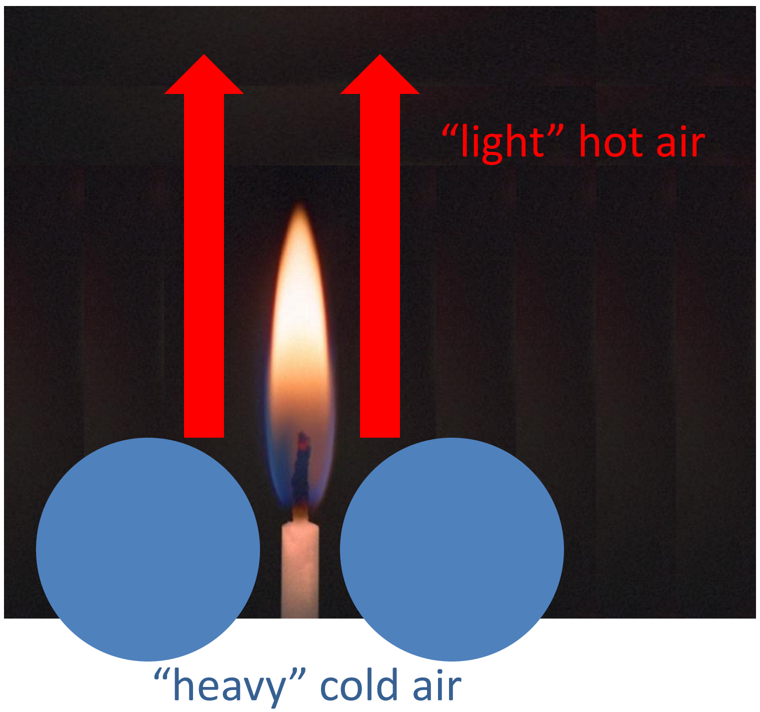
Senator Jake Garn, 1985

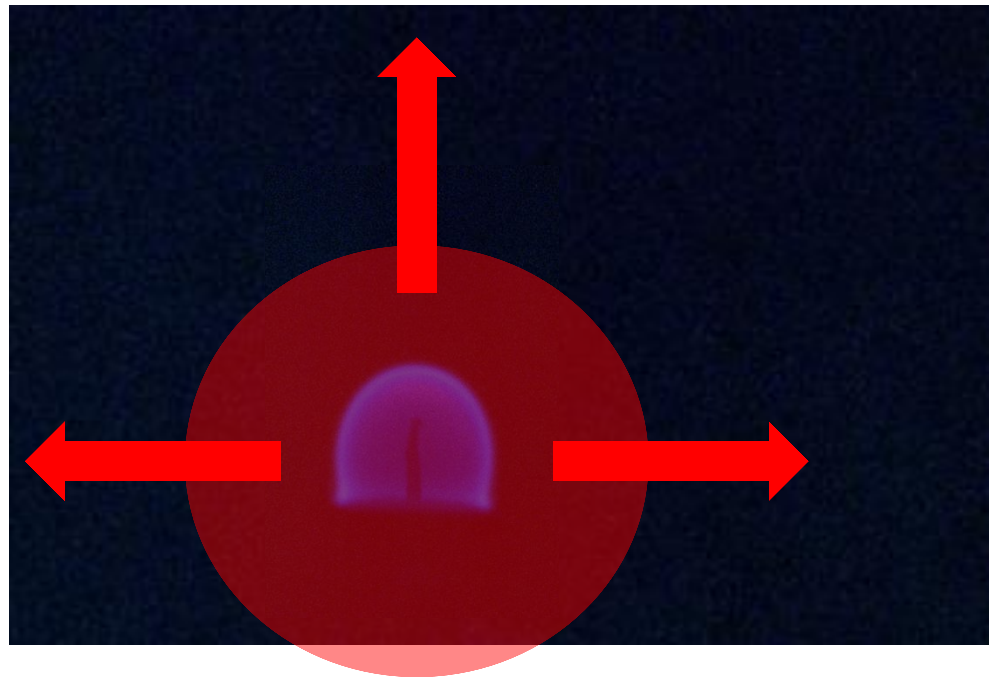
1 Garn=really sick





Flames get a little confused…





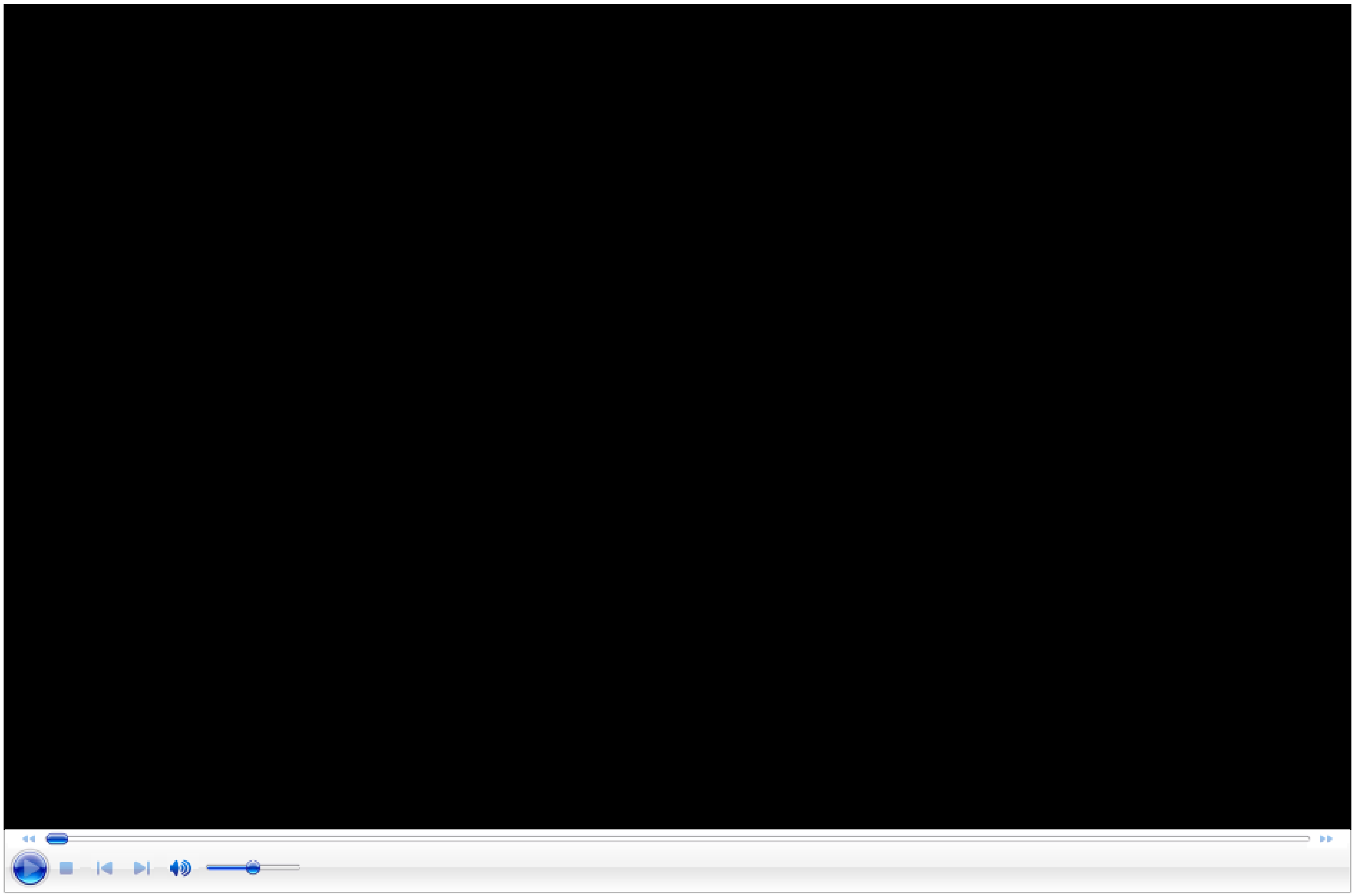
In gravity:

Shape determined by convection

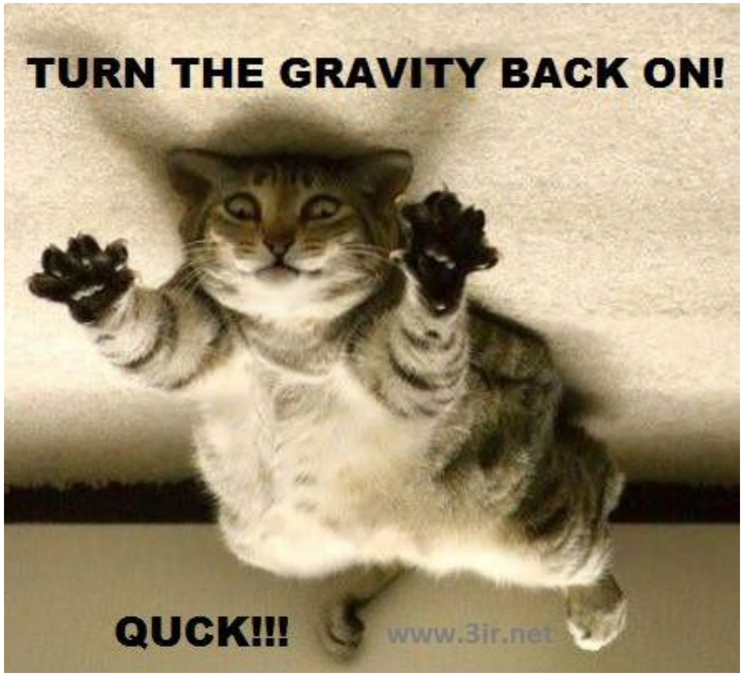
Zero gravity:

There is no light vs. heavy air

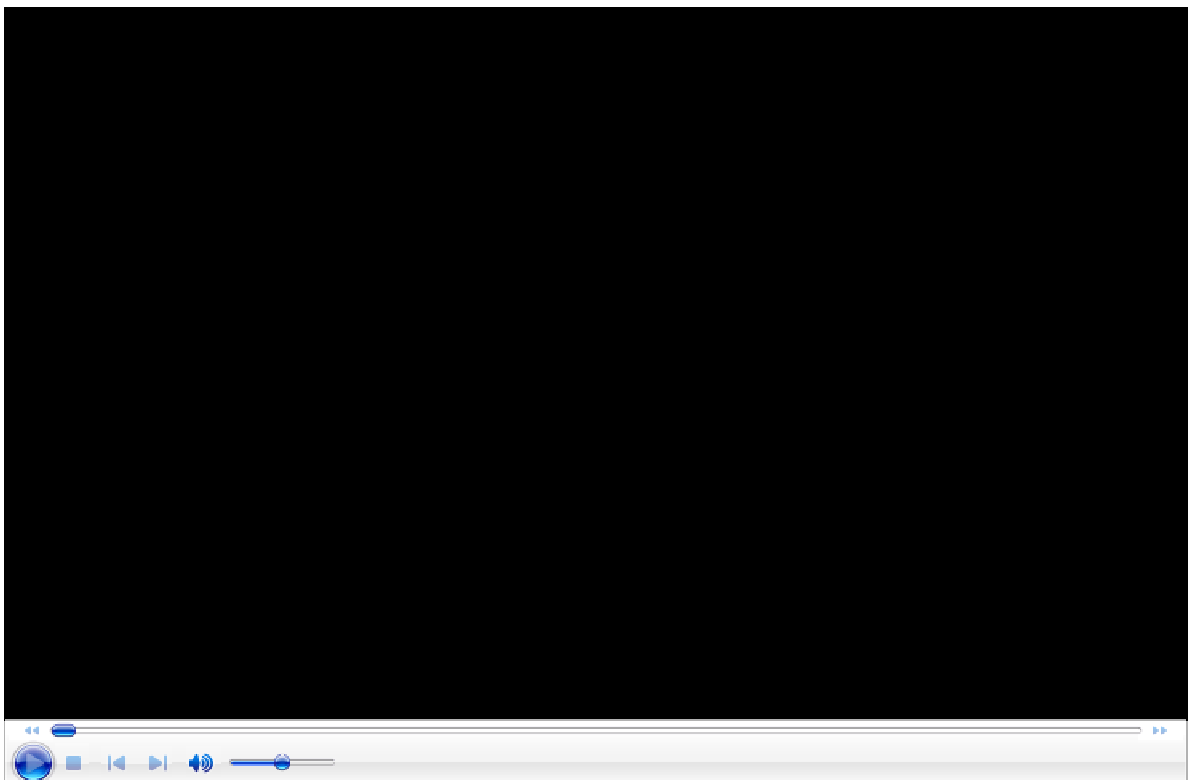
Cats get confused as was demonstrated experimentally:



Or maybe they're just some dog people trying to torture cats in the name of science



Cohesion becomes more important.



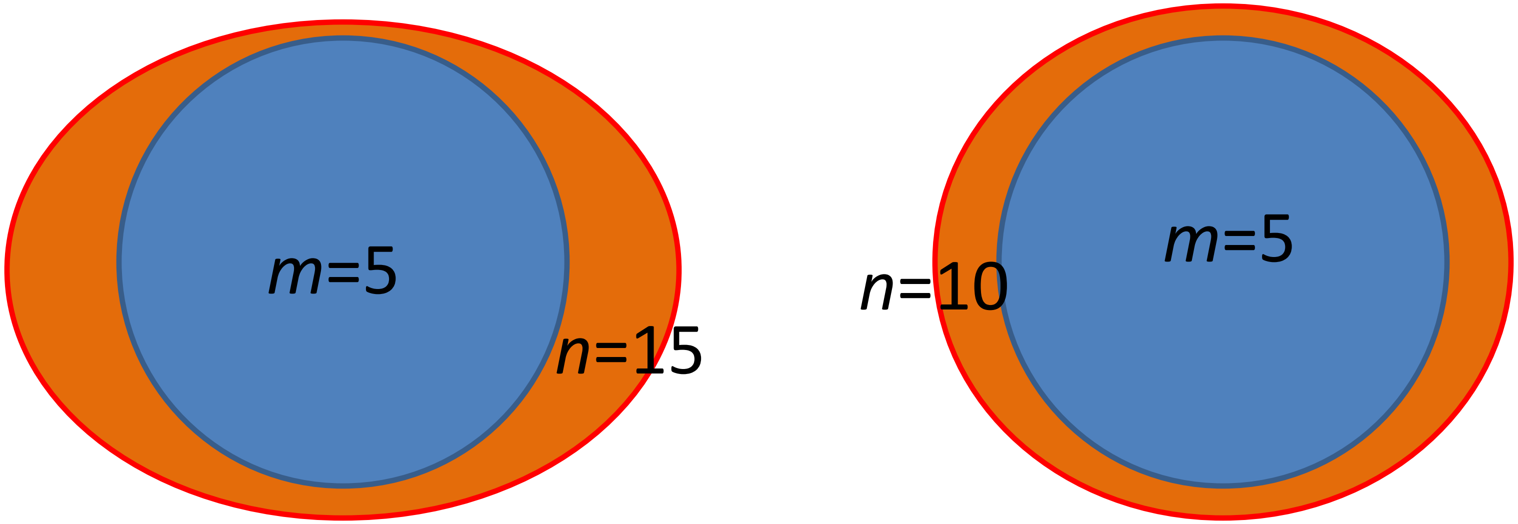
But *F=mg* or gravitation is really weird….

1. What is this mysterious force pulling down?

Are there invisible strings?
Do we need some substance to mediate the force?


2. *F=mg*: How does the Earth know my mass?

3. Why does gravity work on the same quantity that determines the inertia



Why can't two objects of the same mass react differently to gravity?

$n\ g = m\ a$ | $n\ g = m\ a$
$15\ g = 5\ a$ | $10\ g = 5\ a$

$a = 3\ g$ | $a = 2\ g$

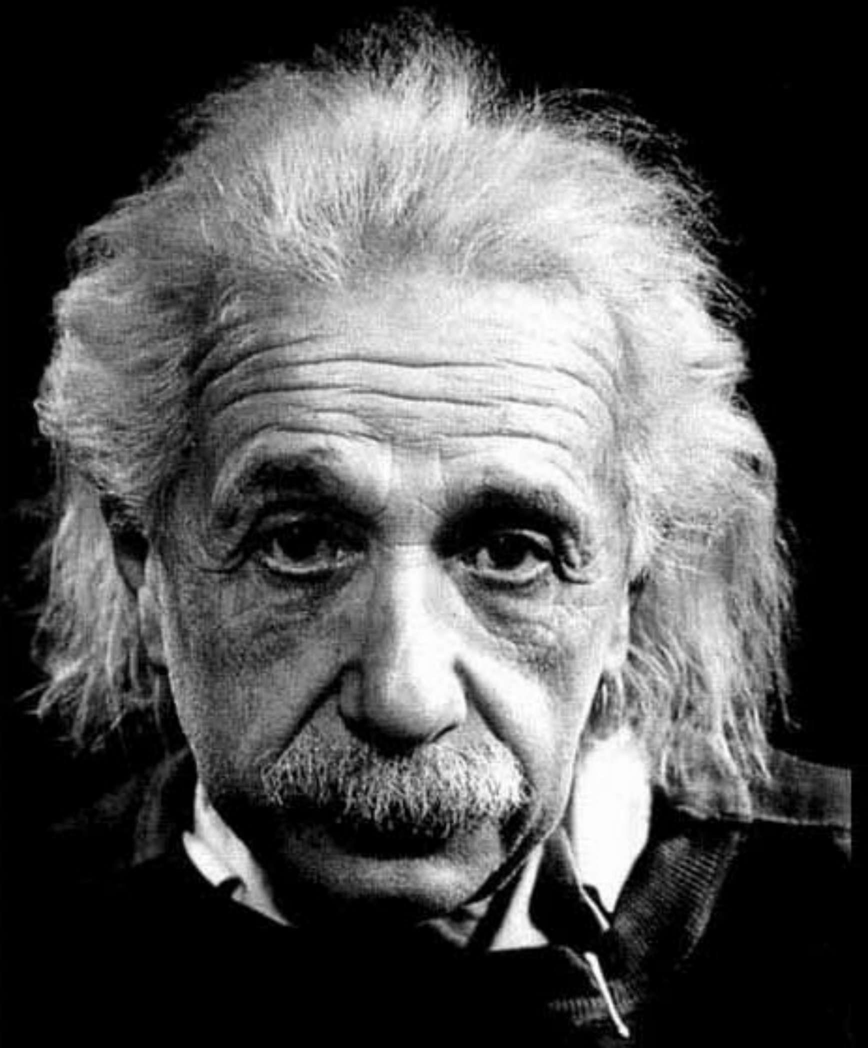
But no! a is always equal to g !

"Everything should be made as simple as possible, but not simpler."

Albert Einstein

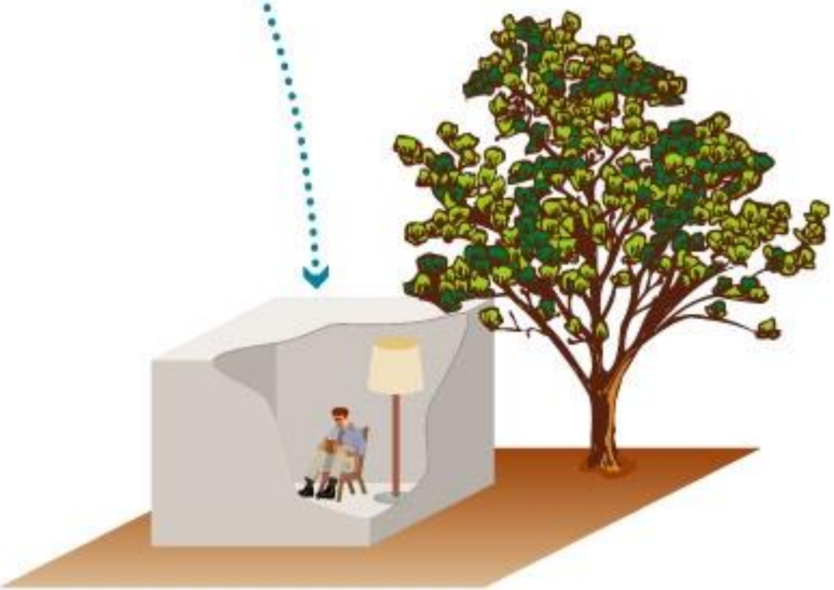
Such questions led Einstein to the theory of general relativity

Einstein around 1915 when he published the theory of general relativity

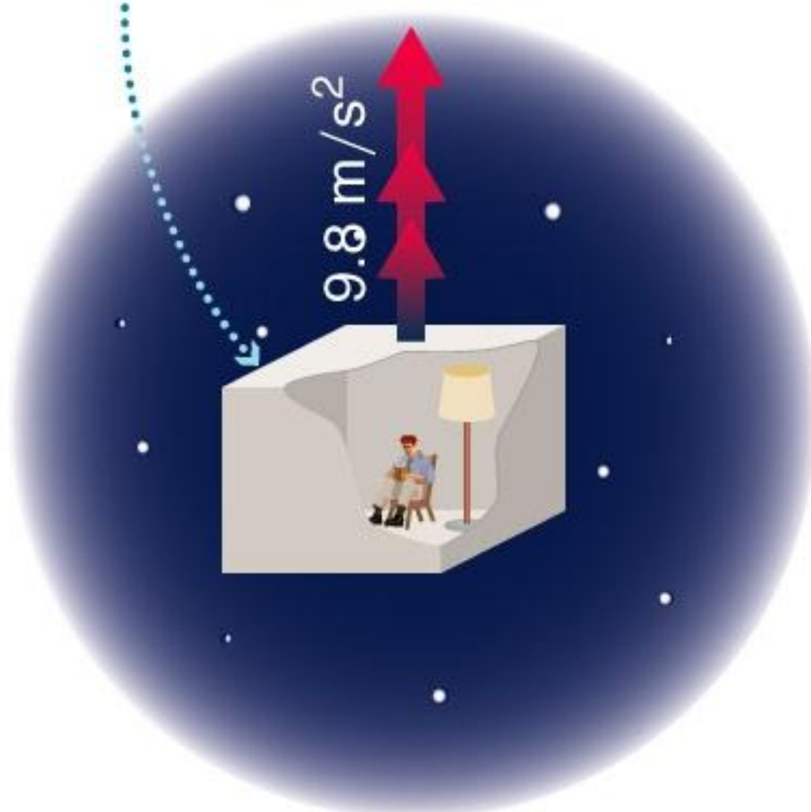


The Equivalence Principle

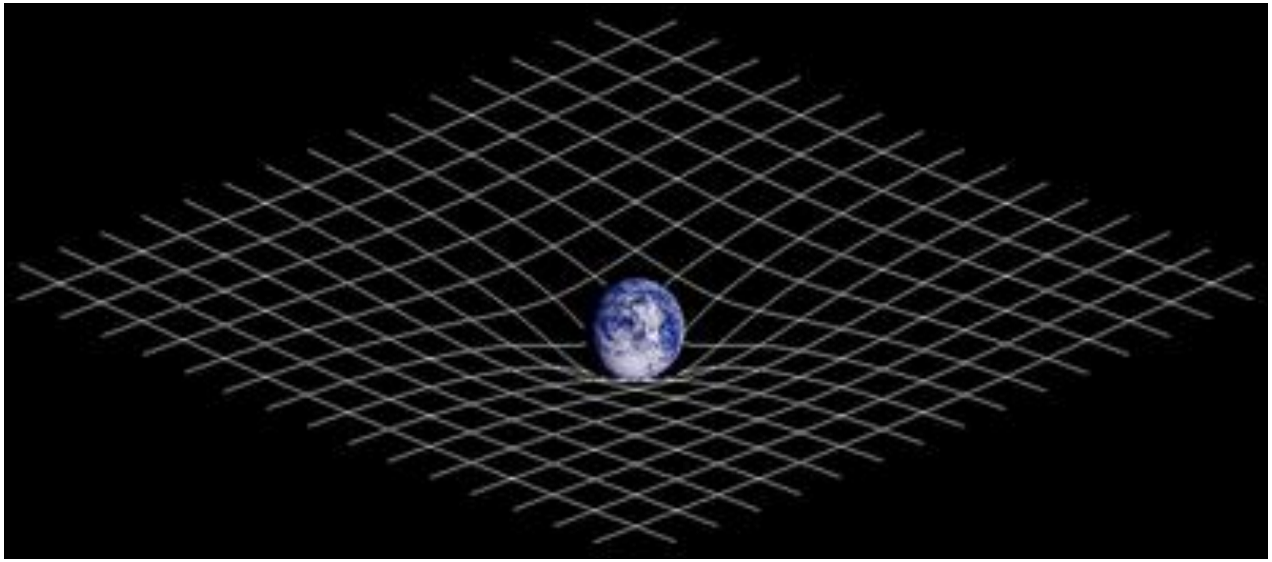
You cannot tell the difference between being in a closed room on Earth . . .



. . . . and being in a closed room accelerating through space at 1g.



It shows the intimate relation between gravitation and acceleration



Why: distortions of the space-time continuum….