Introduction to Supersymmetry

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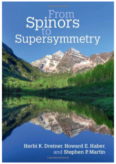
Pre-SUSY School 2023
July 11-14, 2023
University of Southampton

Topics:

- Why: Motivation for supersymmetry (SUSY)
- What: SUSY Lagrangians, SUSY breaking and the Minimal Supersymmetric Standard Model, superpartner decays
- How: to look for supersymmetry
- Where: SUSY might be hiding
- Who: Sorry, not covered.

For some more details:
A supersymmetry primer, `hep-ph/9709356, version 7, 2016`

Also, Herbi Dreiner and Howie Haber and I have a massive new book: From Spinors to Supersymmetry.



If you find corrections, please do let us know!

People have cited various reasons why the next extension of the Standard Model might involve **supersymmetry (SUSY)**.

Some of them:

- Unification of gauge interactions
- A light Higgs boson, $M_h = 125$ GeV
- A possible cold dark matter particle
- Mathematical elegance, beauty
  - ⋆ "That isn't really scientific."
    – Some internet pundits
  - ⋆ "Mathematical elegance and beauty are essential guides."
    – Einstein, Dirac, . . .

For me, the single compelling reason is the **Hierarchy Problem**.

An analogy: Coulomb self-energy correction to the electron's mass

A point-like electron would have infinite classical electrostatic energy.

Instead, suppose the electron is a solid sphere of uniform charge density and radius R. The energy stored in the electric field is:

$$E_{\rm Coulomb} = \frac{3e^2}{5R}$$

Interpreting this as a correction $\Delta m_e = E_{\rm Coulomb}/c^2$ to the electron mass:

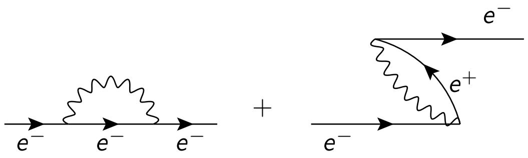
$$m_{e,\rm physical} = m_{e,\rm bare} + (1\,{\rm MeV}/c^2)\,\left(\frac{0.9\times 10^{-15}\,{\rm meters}}{R}\right).$$

A divergence arises if we try to take $R \to 0$.

Naively, we might expect $R \gtrsim 10^{-17}$ meters, to avoid having to tune the bare electron mass to better than 1%, for example:

$$0.511\,{\rm MeV}/c^2 = -100.000\,{\rm MeV}/c^2 + 100.511\,{\rm MeV}/c^2.$$

However, there is another important quantum mechanical contribution:



The virtual positron effect cancels most of the Coulomb contribution, leaving:

$$m_{e,\mathrm{physical}} = m_{e,\mathrm{bare}} \left[1 + \frac{3\alpha}{4\pi} \ln \left(\frac{\hbar/m_e c}{R} \right) + \ldots \right]$$

with $\hbar/m_e c = 3.9 \times 10^{-13}$ meters. Even if R is as small as the Planck length 1.6×10^{-35} meters, where quantum gravity effects become important, this is only a 9% correction.

The existence of a "partner" particle for the electron, the positron, cancels a dangerously huge contribution to its mass.

This “reason” for the positron’s existence can be understood from a **symmetry**: the Poincaré invariance of Einstein’s special relativity imposed on the quantum theory of electrons and photons (QED).
If we did not yet know about special relativity or the positron, we would have had three options:

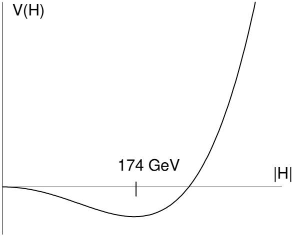
- Assume that the electron has structure at a measurable size $R \gtrsim 10^{-17}$ meters. **Conflicts with LEP e^+e^- collider measurements from last century.**
- Accept that the electron is pointlike or very small, $R \ll 10^{-17}$ meters, implying a mysterious fine-tuning between the bare mass and the Coulomb correction.
- Predict that the electron’s symmetry “partner”, the positron, must exist.

Today we know that the last option is the correct one.

The Hierarchy Problem

Potential for H, the complex scalar field that is the electrically neutral part of the Standard Model Higgs field:

$V(H) = m_H^2 |H|^2 + \lambda |H|^4$



From M_Z and G_{Fermi}, we need:

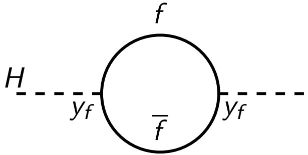
$$\langle H \rangle = \sqrt{-m_H^2/2\lambda} \approx 174 \text{ GeV}$$

For the physical Higgs mass $M_H = 2\sqrt{\lambda}\langle H \rangle + \ldots$ to be 125 GeV, we need:

$$\lambda \approx 0.126, \qquad m_H^2 \approx -(93\text{ GeV})^2$$

However, this appears fine-tuned (incredibly and mysteriously lucky!) when we consider the likely size of quantum corrections to m_H^2.

Contributions to m_H^2 from a Dirac fermion loop:



The correction to the Higgs squared mass parameter from this loop diagram is:

$$\Delta m_H^2 = \frac{y_f^2}{16\pi^2}\left[-2M_{\rm UV}^2 + 6m_f^2 \ln\left(M_{\rm UV}/m_f\right) + \ldots\right]$$

where y_f is the coupling of the fermion to the Higgs field H. $M_{\rm UV}$ should be interpreted as (at least!) the scale at which new physics enters to modify the loop integrations.

The lesson: m_H^2 is sensitive to the **largest** mass scales.

For example, some people believe that String Theory is responsible for modifying the high energy behavior of physics, making the theory finite. Compared to field theory, string theory modifies the Feynman integrations over Euclidean momenta:

$$\int dp\,[\ldots] \;\to\; \int dp\;e^{-p^2/M_{\rm string}^2}\,[\ldots]$$

Using this, one obtains from each Dirac fermion one-loop diagram:

$$\Delta m_H^2 \;\sim\; -\frac{y_f^2}{8\pi^2} M_{\rm string}^2 + \ldots$$

A typical guess is that $M_{\rm string}$ is comparable to $M_{\rm Planck} \approx 2.4 \times 10^{18}$ GeV.

These huge corrections make it difficult to understand how $-m_H^2$ could be as small as $(93\,\mathrm{GeV})^2$.

The Hierarchy Problem

We already know:

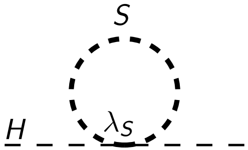
$$\frac{m_H^2}{M_{\rm Planck}^2} \approx -10^{-33}$$

Why should this be so small, if individual radiative corrections Δm_H^2 can be of order $M_{\rm Planck}^2$ or $M_{\rm string}^2$, multiplied by loop factors?

This applies even if String Theory is wrong and some other unspecified effects modify physics at $M_{\rm Planck}$, or any other very large mass scale, to make the loop integrals converge.

An incredible coincidence seems to be required to make the corrections to the Higgs squared mass cancel to give a much smaller number.

The Higgs mass is also quadratically sensitive to other scalar masses.
Suppose S is some heavy complex scalar particle that couples to the Higgs.

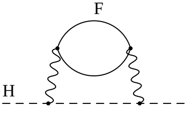


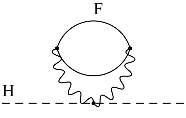
$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[M_{\mathrm{UV}}^2 - 2m_S^2 \ln\left(M_{\mathrm{UV}}/m_S\right) + \ldots\right]$$

Note that a scalar loop gives the **opposite sign** compared to a fermion loop.

In dimensional regularization, the terms proportional to M_{UV}^2 do not occur. However, this does *NOT* solve the problem, because the term proportional to m_S^2 is always there.

Indirect couplings of the Higgs to heavy particles still give a problem:





Here F is any heavy fermion that shares gauge quantum numbers with the Higgs boson. Its mass m_F does not come from the Higgs boson and can be arbitrarily large. One finds (C is a group-theory factor):

$$\Delta m_H^2 \;=\; C\left(\frac{g^2}{16\pi^2}\right)^2 \left[k M_{\rm UV}^2 + 48 m_F^2 \ln(M_{\rm UV}/m_F) + \ldots\right]$$

Here k depends on the choice of cutoff procedure (and is 0 in dimensional regularization). However, the m_F^2 contribution is always present.

More generally, any indirect communication between the Higgs boson and very heavy particles, or very high-mass phenomena in general, can give an unreasonably large contribution to m_H^2.

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The systematic cancellation of loop corrections to the Higgs mass squared requires the type of conspiracy that is better known to physicists as a **symmetry**.
Fermion loops and boson loops gave contributions with opposite signs:

$$\Delta m_H^2 = -\frac{y_f^2}{16\pi^2}(2M_{\rm UV}^2) + \ldots \qquad \textbf{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\rm UV}^2 + \ldots \qquad \textbf{(complex scalar)}$$

Supersymmetry, a symmetry between **fermions** and **bosons**, makes the cancellation not only possible, but automatic.
There are **two complex scalars** for every **Dirac fermion**, and $\lambda_S = y_f^2$.

Supersymmetry

A SUSY transformation turns a boson state into a fermion state, and vice versa. The operator Q that generates such transformations acts, schematically, like:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle.$$

This implies that Q must be an anticommuting spinor. This is an intrinsically complex object, so $Q^\dagger$ is also a distinct symmetry generator:

$$Q^\dagger|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q^\dagger|\text{Fermion}\rangle = |\text{Boson}\rangle.$$

The single-particle states of the theory fall into groups called **supermultiplets**, which are turned into each other by Q and $Q^\dagger$.

Fermion and boson members of a given supermultiplet are **superpartners** of each other.

Each supermultiplet contains equal numbers of fermion and boson degrees of freedom.

Types of supermultiplets

Chiral (or "Scalar" or "Matter" or "Wess-Zumino") supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

2 real spin-0 scalars = 1 complex scalar. ($n_B = 2$)

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or "Vector") supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

1 real spin-1 massless gauge vector boson. ($n_B = 2$)

The Standard Model photon γ, gluon g, and weak vector bosons $Z, W^\pm$ must fit into these.

Gravitational supermultiplet:

1 two-component Weyl fermion gravitino, helicity $\pm\frac{3}{2}$. ($n_F = 2$)

1 real spin-2 massless graviton. ($n_B = 2$)

How do the Standard Model quarks and leptons fit in?

Each quark or charged lepton is 1 Dirac = 2 Weyl fermions

Electron: $\Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$ $\leftarrow$ two-component Weyl LH fermion; $\leftarrow$ two-component Weyl RH fermion

Each of e_L and e_R is part of a chiral supermultiplet, so each has a complex, spin-0 superpartner, called $\widetilde{e}_L$ and $\widetilde{e}_R$ respectively. They are called the "left-handed selectron" and "right-handed selectron", although they carry no spin.

The conjugate of a right-handed Weyl spinor is a left-handed Weyl spinor. So, there are two left-handed chiral supermultiplets for the electron:

$$(e_L,\, \widetilde{e}_L) \quad \text{and} \quad (e_R^\dagger,\, \widetilde{e}_R^*).$$

The other charged leptons and quarks are similar. We do not need ν_R in the Standard Model, so only one neutrino chiral supermultiplet for each family:

$$(\nu_e,\, \widetilde{\nu}_e).$$

Chiral supermultiplets of the Minimal Supersymmetric Standard Model:

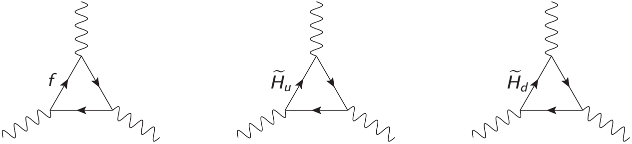
Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
($\times$3 families)	$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
($\times$3 families)	$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$

The superpartners of the Standard Model particles are written with a $\widetilde{\ }$. The scalar names are obtained by putting an "s" in front, so they are generically called **squarks** and **sleptons**, short for "scalar quark" and "scalar lepton".

The Standard Model Higgs boson requires two distinct chiral supermultiplets, H_u and H_d. The fermionic partners of the Higgs scalar fields are called **higgsinos**. There are two charged and two neutral Weyl fermion higgsino degrees of freedom.

Why do we need two Higgs supermultiplets? Two reasons:

1) Anomaly Cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \quad = \quad 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present. Otherwise, the electroweak gauge symmetry would not be allowed!

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge $+2/3$ quarks (u, c, t).
Only the H_d Higgs scalar can give masses to charge $-1/3$ quarks (d, s, b) and the charged leptons (e, μ, τ). We will show this later.

The vector bosons of the Standard Model live in gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad\widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

The spin 1/2 **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\widetilde{\ }$.

The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

However, the winos and the bino are not mass eigenstate particles; they mix with each other and with the higgsinos of the same charge.

If supersymmetry were an exact symmetry, then superpartners would have to be exactly degenerate with each other. For example,

$$\begin{aligned} &m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511\,\mathrm{MeV} \\ &m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u \\ &m_{\tilde{g}} = m_{\mathrm{gluon}} = 0 + \mathrm{QCD\ effects} \\ &\text{etc.} \end{aligned}$$

New particles with these masses are not present.

Supersymmetry is assumed to be spontaneously broken and therefore hidden, in the same way that the full electroweak symmetry $SU(2)_L \times U(1)_Y$ is hidden from very low-energy experiments.

A clue for SUSY breaking is given by our motivation in the Hierarchy Problem. The Higgs mass parameter gets corrections from each chiral supermultiplet:

$$\Delta m_H^2 = \frac{1}{16\pi^2}(\lambda_S - y_f^2)M_{\mathrm{UV}}^2 + \dots$$

If supersymmetry were exact and unbroken,

$$\lambda_S = y_f^2.$$

For SUSY to be a solution to the hierarchy problem, we must demand that this is still true even after SUSY is broken:

The breaking of supersymmetry must be "soft". This means that the part of the Lagrangian with **dimensionless** couplings remains supersymmetric.

The effective Lagrangian has the form:

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$$

where:

- $\mathcal{L}_{\rm SUSY}$ contains all of the gauge, Yukawa, and dimensionless scalar couplings, and preserves supersymmetry
- $\mathcal{L}_{\rm soft}$ violates supersymmetry, and contains only mass terms and couplings with *positive* mass dimension.

If $m_{\rm soft}$ is the largest mass scale in $\mathcal{L}_{\rm soft}$, then by dimensional analysis,

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(M_{\rm UV}/m_{\rm soft}) + \ldots \right],$$

where λ stands for dimensionless couplings. This is because Δm_H^2 must vanish in the limit $m_{\rm soft} \to 0$, in which SUSY is restored. Therefore, we might expect that $m_{\rm soft}$ should not be much larger than **roughly** 1000 GeV.

Without further justification, “soft” SUSY breaking might seem like a rather arbitrary requirement.

Fortunately, it arises naturally from the spontaneous breaking of SUSY, as we will see later.

Is there a good reason why the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far?

Yes! The reason is electroweak gauge invariance.

- All particles discovered as of 1995 (quarks, leptons, gauge bosons) would be exactly massless if the electroweak symmetry were not broken. So, their masses are at most of order $v = 174$ GeV, the electroweak breaking scale. **Gauge invariance required them to be light.**
- All of the particles in the MSSM that have **not** yet been discovered as of 2023 (squarks, sleptons, gauginos, Higgsinos, Higgs scalars) can get a mass even without electroweak symmetry breaking. **They are not required to be light by gauge invariance.**
- The lightest Higgs scalar is an exception; its mass of ~ 125 GeV is within (and near the upper end of) the range predicted by supersymmetry.

Two-component spinor language is natural and convenient for SUSY, because the supermultiplets are in one-to-one correspondence with the LH Weyl fermions.

More generally, two-component spinor language is more natural than four-component spinors for any theory of physics beyond the Standard Model, because parity violation is an essential truth.

Nature does not treat left-handed and right-handed fermions the same, and the higher we go in energy, the more essential this feature becomes.

Notations for two-component (Weyl) fermions

Left-handed (LH) two-component Weyl spinor: $\psi_\alpha \quad \alpha = 1, 2$
Right-handed (RH) two-component Weyl spinor: $\psi^\dagger_{\dot\alpha} \quad \dot\alpha = 1, 2$

The Hermitian conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor, and vice versa:

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha} \equiv \psi^\dagger_{\dot\alpha}$$

All spin-1/2 fermions in **any** theory can be defined in terms of a list of left-handed Weyl spinors, $\psi_{k\alpha}$ where k is a flavor index. With this convention, right-handed Weyl spinors always carry a dagger: $\psi^{\dagger k}_{\dot\alpha}$.

Pairs of spinor indices are suppressed when contracted like:

$${}^{\alpha}{}_{\alpha} \qquad \text{or} \qquad {}_{\dot\alpha}{}^{\dot\alpha}$$

Products of spinors are defined as:

$$\psi\xi \equiv \psi_\alpha \xi_\beta \epsilon^{\beta\alpha} \qquad \text{and} \qquad \psi^\dagger\xi^\dagger \equiv \psi^\dagger_{\dot\alpha}\xi^\dagger_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}$$

Since ψ and ξ are anti-commuting fields, the antisymmetry of $\epsilon^{\alpha\beta}$ implies:

$$\psi\xi = \xi\psi = (\psi^\dagger\xi^\dagger)^* = (\xi^\dagger\psi^\dagger)^*.$$

Instead of the gamma matrices used for Dirac spinors, define matrices $(\overline{\sigma}^\mu)^{\dot\alpha\beta}$ and $(\sigma^\mu)_{\alpha\dot\beta}$ according to:

$$\overline{\sigma}^0 = \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad \sigma^n = -\overline{\sigma}^n = (\vec{\sigma})_n \quad (\text{for } n = 1, 2, 3).$$

Then one has Lorentz-covariant vector quantities

$$\xi^\dagger\overline{\sigma}^\mu\psi \;=\; -(\xi\sigma^\mu\psi^\dagger)^*$$

which are used to construct kinetic terms and couplings to vectors.

Two Weyl spinors ξ, χ can form a 4-component Dirac or Majorana spinor:

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$$

In the 4-component formalism, the Dirac Lagrangian is:

$$\mathcal{L} \;=\; i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m\overline{\Psi}\Psi, \qquad \text{where} \qquad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix},$$

In the two-component fermion language, with spinor indices suppressed:

$$\mathcal{L} = i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi + i\chi^\dagger\overline{\sigma}^\mu\partial_\mu\chi - m(\xi\chi + \xi^\dagger\chi^\dagger),$$

up to a total derivative.

A Majorana fermion can be described in 4-component language in the same way by identifying $\chi = \xi$, and multiplying the Lagrangian by a factor of $\frac{1}{2}$ to compensate for the redundancy.

Another bit of notation. We write the Left-handed and Right-handed parts of a fermion's Dirac field as (for the electron for example):

$$
\begin{aligned}
e_L &\rightarrow e \\
e_R^\dagger &\rightarrow \bar{e}
\end{aligned}
$$

This avoids writing annoying L and R subscripts. Note, the bar is part of the name of the two-component fermion field! It is not a kind of conjugation.

Both e and $\bar{e}$ are **left**-handed fields.

For example, to describe the Standard Model fermions in 2-component notation:

$$\begin{aligned}\mathcal{L} &= iQ^{\dagger k}\overline{\sigma}^{\mu}D_{\mu}Q_k + i\bar{u}^{\dagger k}\overline{\sigma}^{\mu}D_{\mu}\bar{u}_k + i\bar{d}^{\dagger k}\overline{\sigma}^{\mu}D_{\mu}\bar{d}_k \\ &\quad + iL^{\dagger k}\overline{\sigma}^{\mu}D_{\mu}L_k + i\bar{e}^{\dagger k}\overline{\sigma}^{\mu}D_{\mu}\bar{e}_k\end{aligned}$$

with the family index $k = 1, 2, 3$ summed over. Color and weak isospin and spinor indices are suppressed, and D_{μ} is the appropriate Standard Model covariant derivative, for example,

$$\begin{aligned}D_{\mu}\begin{pmatrix}\nu_e \\ e\end{pmatrix} &= \left[\partial_{\mu} - i\frac{g}{2}W_{\mu}^{a}\tau^{a} + i\frac{g'}{2}B_{\mu}\right]\begin{pmatrix}\nu_e \\ e\end{pmatrix} \\ D_{\mu}\overline{e} &= \left[\partial_{\mu} - ig'B_{\mu}\right]\bar{e}\end{aligned}$$

with τ^{a} $(a = 1, 2, 3)$ equal to the Pauli matrices, and the gauge eigenstate weak bosons are related to the mass eigenstates by

$$\begin{aligned}W_{\mu}^{\pm} &= (W_{\mu}^{1} \mp W_{\mu}^{2})/\sqrt{2}, \\ \begin{pmatrix}Z_{\mu} \\ A_{\mu}\end{pmatrix} &= \begin{pmatrix}\cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W_{\mu}^{3} \\ B_{\mu}\end{pmatrix}.\end{aligned}$$

The simplest SUSY model: a free chiral supermultiplet

The minimum particle content for a SUSY theory is a complex scalar ϕ and its superpartner fermion ψ. We must at least have kinetic terms for each, so:

$$S = \int d^4x \left(\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion}\right)$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad\qquad \mathcal{L}_{\rm fermion} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

Note: I'm using $(-,+,+,+)$ metric.

A SUSY transformation should turn ϕ into ψ, so try:

$$\delta\phi = \epsilon\psi; \qquad\qquad \delta\phi^* = \epsilon^\dagger\psi^\dagger$$

where $\epsilon =$ infinitesimal, anticommuting, constant spinor, with dimension $[\text{mass}]^{-1/2}$, that parameterizes the SUSY transformation. Then we find:

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi.$$

We would like for this to be canceled by an appropriate SUSY transformation of the fermion field…

To have any chance, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and must contain one spacetime derivative. There is only one possibility, up to a multiplicative constant:

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi; \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha} \, \partial_\mu \phi^*$$

With this guess, one obtains:

$$\delta\mathcal{L}_{\rm fermion} = -\delta\mathcal{L}_{\rm scalar} + (\text{total derivative})$$

so the action S is indeed invariant under the SUSY transformation, justifying the guess of the multiplicative factor. This is called the free Wess-Zumino model.

Furthermore, if we take the commutator of two SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) = i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\phi$$

Since ∂_μ corresponds to the spacetime 4-momentum P_μ, This says that the commutator of two SUSY transformations is just a spacetime translation.

The fact that two SUSY transformations give back another symmetry (namely a spacetime translation) means that the SUSY algebra "closes".

If we do the same check for the fermion ψ:

$$\begin{aligned}\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) &= i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\psi_\alpha \\ &\quad + i\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) - i\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)\end{aligned}$$

The first line is expected, but the second line only vanishes on-shell (when the classical equations of motion are satisfied). However, we want SUSY to be a valid symmetry of the quantum theory (off-shell)!

To show that there is no problem, we introduce another bosonic spin-0 field, F, called an auxiliary field. Its Lagrangian density is:

$$\mathcal{L}_{\rm aux} = F^*F$$

Note that F has no kinetic term, and has dimensions $[\text{mass}]^2$, unlike an ordinary scalar field. It has the not-very-exciting equations of motion $F = F^* = 0$.

The auxiliary field F does not affect the dynamics, classically or in the quantum theory. But it does appear in **modified** SUSY transformation laws:

$$\begin{aligned}
\delta\phi &= \epsilon\psi \\
\delta\psi_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \boldsymbol{\epsilon_\alpha F} \\
\delta F &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi
\end{aligned}$$

Now the total Lagrangian

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F$$

is still invariant, and also one can now check:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}X) - \delta_{\epsilon_1}(\delta_{\epsilon_2}X) \;=\; i(-\epsilon_1\sigma^\mu\epsilon_2 + \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu X$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$, without using equations of motion. So in the "modified" theory, SUSY does close off-shell as well as on-shell.

The auxiliary field F is really just a book-keeping device to make this simple. We can see why it is needed by considering the number of degrees of freedom on-shell (classically) and off-shell (quantum mechanically):

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

Going on-shell eliminates half of the propagating degrees of freedom of the fermion, because the Lagrangian density is linear in time derivatives; the fermionic canonical momenta are not independent phase-space variables. The momentum conjugate to ψ is $\psi^\dagger$.

The auxiliary field will also plays an important role when we add interactions to the theory, and in gaining a simple understanding of SUSY breaking.

Summary so far: the Wess-Zumino model Lagrangian involves a scalar ϕ, a fermion ψ, and an auxiliary field F:

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F.$$

This describes a massless, non-interacting theory with supersymmetry.

Noether's Theorem: for every symmetry, there is a conserved current. In SUSY, the **supercurrent** J^μ_α is an anti-commuting 4-vector that also carries a spinor index.

By the usual Noether procedure, one finds for the supercurrent (and its conjugate $J^\dagger$), in terms of the variations of the fields δX for $X = (\phi, \phi^*, \psi, \psi^\dagger, F, F^*)$:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \quad \equiv \quad \sum_X \delta X \, \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu,$$

where K^μ satisfies $\delta\mathcal{L} = \partial_\mu K^\mu$. One finds:

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*; \qquad\qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi.$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0; \qquad\qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0,$$

as can be checked using the equations of motion.

From the conserved supercurrents one can construct the conserved charges:

$$Q_\alpha = \sqrt{2}\int d^3x\, J^0_\alpha; \qquad\qquad Q^\dagger_{\dot\alpha} = \sqrt{2}\int d^3x\, J^{\dagger 0}_{\dot\alpha},$$

As quantum mechanical operators, they satisfy:

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\,\delta X$$

for any field X. Let us also introduce the 4-momentum operator $P^\mu = (H, \vec{P})$, which satisfies:

$$[P_\mu, X] = i\partial_\mu X.$$

Now by using the canonical commutation relations of the fields, one finds:

$$\begin{aligned} \left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right] &= 2(\epsilon_1\sigma_\mu\epsilon_2^\dagger - \epsilon_2\sigma_\mu\epsilon_1^\dagger)\, P^\mu \\ \left[\epsilon Q + \epsilon^\dagger Q^\dagger,\, P^\mu\right] &= 0 \end{aligned}$$

This implies...

The SUSY Algebra

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu,$$
$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$$
$$[Q_\alpha, P^\mu] = [Q^\dagger_{\dot\alpha}, P^\mu] = 0$$

(The commutators turned into anti-commutators in the first two, when we extracted the anti-commuting spinors ϵ_1, ϵ_2.)

Masses and Interactions for Chiral Supermultiplets

The Lagrangian describing a collection of free, massless, chiral supermultiplets is

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i.$$

How do we make mass terms and interactions for these fields, while still preserving supersymmetry invariance?

Try to add to this a Lagrangian describing interactions:

$$\mathcal{L}_{\rm int} = \left(-\tfrac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + x^{ij} F_i F_j\right) + {\rm c.c.} + U$$

where, to be renormalizable, W^{ij}, W^i, x^{ij}, and U are polynomials in ϕ_i, ϕ^{*i} with degrees 1, 2, 0, and 4, respectively.

Now one can compute $\delta\mathcal{L}$ under the SUSY transformation, and require that it be a total derivative, so that the action $S = \int d^4x \mathcal{L}$ is invariant.

You can check that this works if, and only if, $x^{ij} = 0$ and $U = 0$, and:

$$\begin{aligned} W^{ij} &= \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} = M^{ij} + y^{ijk}\phi_k \\ W^{i} &= \frac{\partial W}{\partial \phi_i} = M^{ij}\phi_j + \tfrac{1}{2} y^{ijk}\phi_j\phi_k \end{aligned}$$

where we have defined a useful function:

$$W = \tfrac{1}{2} M^{ij}\phi_i\phi_j + \tfrac{1}{6} y^{ijk}\phi_i\phi_j\phi_k$$

called the **superpotential**. Must be holomorphic; it only depends on the ϕ_i, not on the ϕ^{*i}.

The superpotential W contains masses M^{ij} and couplings y^{ijk}, which must be symmetric under interchange of i, j, k.

Supersymmetry is very restrictive; you cannot just do anything you want!

The Lagrangian terms involving auxiliary fields F_i, F^{*i} are:

$$\mathcal{L} = F^{*i} F_i + W^i F_i + W_i^* F^{*i}$$

So the equations of motion are now:

$$F^{*i} \;=\; -W^i = M^{ij}\phi_j + \tfrac{1}{2} y^{ijk}\phi_j\phi_k,$$

This is still algebraic; no spacetime derivatives. By eliminating the auxiliary fields, we get the complete Lagrangian:

$$\begin{aligned} \mathcal{L} \;=\;& -\partial^\mu \phi^{*i}\partial_\mu \phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu \partial_\mu \psi_i \\ & -\tfrac{1}{2}\big(M^{ij}\psi_i\psi_j + y^{ijk}\phi_i\psi_j\psi_k\big) + \mathrm{c.c.} \\ & -V(\phi_i, \phi^{*i}) \end{aligned}$$

where the scalar potential is:

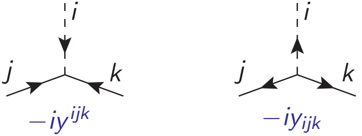
$$\begin{aligned} V(\phi_i, \phi^{*i}) \;=\;& M_{ik}M^{kj}\phi^{*i}\phi_j + \frac{1}{2} M^{in} y_{jkn}\phi_i\phi^{*j}\phi^{*k} \\ & + \frac{1}{2} M_{in} y^{jkn}\phi^{*i}\phi_j\phi_k + \frac{1}{4} y^{ijn} y_{kln}\phi_i\phi_j\phi^{*k}\phi^{*l} \end{aligned}$$

The superpotential $W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$ **determines all renormalizable non-gauge masses and interactions.**

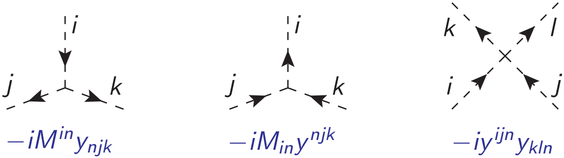
Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$.

The interaction Feynman rules for the chiral supermultiplets are:

Yukawa interactions:



Scalar interactions:



Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains physical fields:

- a gauge boson A_μ^a
- a gaugino λ_α^a.
- D^a, a real spin-0 auxiliary field with no kinetic term (non-propagating).

The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$, $1, 2, 3$ for $SU(2)_L$, and 1 for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i\lambda^{\dagger a} \overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a$$

where

$$\nabla_\mu \lambda^a \equiv \partial_\mu \lambda^a + g f^{abc} A_\mu^b \lambda^c.$$

The action is invariant under the SUSY transformation:

$$\delta A^a_\mu = -\tfrac{1}{\sqrt{2}}\big(\epsilon^\dagger \overline{\sigma}_\mu \lambda^a + \lambda^{\dagger a} \overline{\sigma}_\mu \epsilon\big),$$
$$\delta \lambda^a_\alpha = -\tfrac{i}{2\sqrt{2}}(\sigma^\mu \overline{\sigma}^\nu \epsilon)_\alpha\, F^a_{\mu\nu} + \tfrac{1}{\sqrt{2}}\epsilon_\alpha\, D^a,$$
$$\delta D^a = \tfrac{i}{\sqrt{2}}\big(\epsilon^\dagger \overline{\sigma}^\mu \nabla_\mu \lambda^a - \nabla_\mu \lambda^{\dagger a} \overline{\sigma}^\mu \epsilon\big).$$

Unlike in the non-gauged Wess-Zumino model, these supersymmetry transformations are non-linear, because there are gauge fields hidden inside the covariant derivatives.

This non-linearity can be eliminated by introducing even more auxiliary fields besides D^a; the most natural way to understand this is with **superfields** which we will talk about later. The current version is said to be in "Wess-Zumino gauge".

The auxiliary field D^a is needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell $(n_B = n_F = 2)$	2	2	0
off-shell $(n_B = n_F = 4)$	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, we must turn the ordinary derivatives into covariant ones:

$$\begin{aligned} \partial_\mu \phi_i &\rightarrow \nabla_\mu \phi_i = \partial_\mu \phi_i - igA^a_\mu (T^a \phi)_i \\ \partial_\mu \psi_i &\rightarrow \nabla_\mu \psi_i = \partial_\mu \psi_i - igA^a_\mu (T^a \psi)_i \end{aligned}$$

Must also add three new terms to the Lagrangian:

$$\begin{aligned} \mathcal{L} \;=\; & \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) \\ & + g(\phi^* T^a \phi) D^a . \end{aligned}$$

You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY and gauge transformations.

Consider the part of the Lagrangian involving the auxiliary fields D^a:

$$\mathcal{L} = \frac{1}{2} D^a D^a + g D^a (\phi^* T^a \phi)$$

The D^a obey purely algebraic equations of motion $D^a = -g(\phi^* T^a \phi)$, and so can be eliminated from the theory. The resulting scalar potential is:

$$\begin{aligned} V(\phi^{*i}, \phi_i) &= F^{*i} F_i + \frac{1}{2} D^a D^a \\ &= W_i^* W^i + \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2 \end{aligned}$$

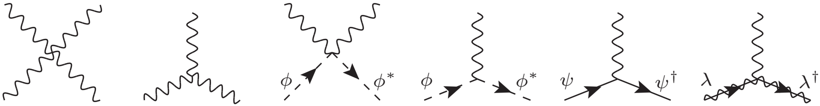
The two types of contributions to the scalar potential are called "F"-term and "D"-term. Note:

- Since V is a sum of squares, it is automatically ≥ 0.
- The scalar potential in SUSY theories is completely determined by the fermion masses, Yukawa couplings, and gauge couplings.

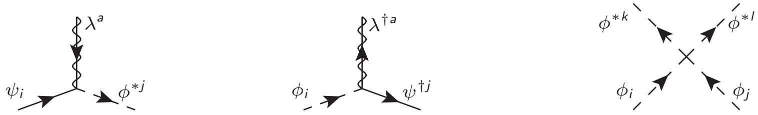
But both of these statements will be modified when we break SUSY.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



SUSY also predicts interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



$-i\sqrt{2}g_a(T^a)_i{}^j$ $\qquad$ $-i\sqrt{2}g_a(T^a)_i{}^j$ $\qquad$ $-ig_a^2(T_i^{ak}T_j^{al} + T_i^{al}T_j^{ak})$

These interactions are entirely determined by supersymmetry and the gauge group. Experimental measurements of the magnitudes of these couplings would provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown that there is still no quadratic sensitivity to M_{UV} in SUSY theories if we add these SUSY-breaking terms:

$$\begin{aligned}\mathcal{L}_{\mathrm{soft}} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right)\\ &\quad -(m^2)^i_j\phi^{*j}\phi_i\\ &\quad -\left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right),\end{aligned}$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

How to make a SUSY Model:

- Choose a gauge symmetry group.
  In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.
- Choose a superpotential W; must be invariant under the gauge symmetry.
  In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses, up to a single parameter and small loop corrections.
- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.
  Almost all unknowns and arbitrariness in the MSSM are here.

Let's do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$\boxed{W_{\rm MSSM} = \tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{y_e}\tilde{L}H_d + \mu H_u H_d}$$

H_u, H_d, $\tilde{Q}$, $\tilde{L}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\bar{e}}$ are the scalar fields appearing in the left-handed chiral supermultiplets.

$$Q = (u, d) \equiv (u_L, d_L), \qquad L = (e, \nu) \equiv (e_L, \nu_L),$$
$$\bar{e} \equiv e_R^\dagger, \qquad \bar{u} \equiv u_R^\dagger, \qquad \bar{d} \equiv d_R^\dagger$$

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, they are the same as in the Standard Model.

We need both H_u and H_d, because $\tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_d^*$ and $\tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_u^*$ are not analytic, and so not allowed in the superpotential.

In the approximation that only t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes (in $SU(2)_L$ components):

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+H_d^- - H_u^0H_d^0). \end{aligned}$$

The minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0\rangle = v_u$ and $\langle H_d^0\rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

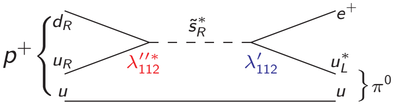
$$m_t = y_t v_u, \qquad m_b = y_b v_d, \qquad m_\tau = y_\tau v_d.$$

Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u$$
$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

These violate lepton number $(\Delta L = 1)$ or baryon number $(\Delta B = 1)$.

If both types of couplings were present, and of order 1, then the proton would decay in a fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential. One cannot directly require exact B and L conservation, since they are known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the renormalizable superpotential.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

All of the known Standard Model particles and the Higgs scalar bosons carry $P_R = +1$, while all of the squarks and sleptons and higgsinos and gauginos carry $P_R = -1$.

Matter parity and R-parity are **exactly equivalent**, because the product of $(-1)^{2S}$ for all of the fields in any interaction vertex that conserves angular momentum is always $+1$.

Consequences if R-parity is conserved

The particles with odd R-parity ($P_R = -1$) are the "supersymmetric particles" or "sparticles" or "superpartners".

Every interaction vertex in the theory has an even number of $P_R = -1$ particles. Then:

- The lightest particle with $P_R = -1$, called the "Lightest Supersymmetric Particle" or LSP, is absolutely stable. If the LSP is electrically neutral, it interacts only weakly, and so could be the non-baryonic dark matter required by cosmology and astrophysics.
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).
- Each sparticle other than the LSP must decay into a state with an odd number of LSPs (usually just one). The LSP escapes the detector, with a missing momentum signature.

Where does R-parity come from?

One way that matter parity could arise is as a surviving subgroup of a continuous gauge symmetry. For example, if $U(1)_{B-L}$ symmetry is gauged, and then broken at very high energy by a VEV of some field that carried an even integer value of $3(B-L)$, then matter parity will automatically be an exact symmetry of the MSSM.

There are alternatives to R-parity, for example **baryon triality**, a Z_3 discrete symmetry:

$$Z_3^B = e^{2\pi i(B-2Y)/3}$$

If Z_3^B is multiplicatively conserved, then the proton is absolutely stable, but the LSP is not.

Another possibility is that R-parity is spontaneously broken, by the VEV of some scalar field with $P_R = -1$.

The Soft SUSY-breaking Lagrangian for the MSSM

$$\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\; & -\tfrac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B}\right) + \text{c.c.} \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + \text{c.c.} \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_{\widetilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\widetilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\widetilde{\overline{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\widetilde{\overline{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\widetilde{\overline{e}}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - \left(bH_uH_d + \text{c.c.}\right).
\end{aligned}$$

The first line is masses for the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).

The second line consists of (scalar)^3 interactions.

The third line is (mass)^2 terms for the squarks and sleptons.

The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$M_1,\ M_2,\ M_3,\ \mathbf{a_u},\ \mathbf{a_d},\ \mathbf{a_e} \ \sim\ m_{\rm soft};$$
$$\mathbf{m^2_{\widetilde{Q}}},\ \mathbf{m^2_{\widetilde{L}}},\ \mathbf{m^2_{\widetilde{\overline{u}}}},\ \mathbf{m^2_{\widetilde{\overline{d}}}},\ \mathbf{m^2_{\widetilde{\overline{e}}}},\ m^2_{H_u},\ m^2_{H_d},\ b\ \sim\ m^2_{\rm soft}$$

where $m_{\rm soft}$ is not huge compared to 1 TeV.

The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals have been made.

The question can be answered experimentally by discovering the pattern of gaugino and squark and slepton masses.

Electroweak symmetry breaking and the Higgs bosons

Recall: in SUSY, there are two complex Higgs scalar doublets, (H_u^+, H_u^0) and (H_d^0, H_d^-), rather than one in the Standard Model.

In the Standard Model, there is only one physical Higgs boson, h.

In the Minimal Supersymmetric Standard Model, there are two neutral Higgs scalars h, H, one neutral pseudo-scalar A, and a charged Higgs boson H^+ and its anti-particle H^-.

Let us start by analyzing the scalar potential for the Higgs fields, to understand the vacuum expectation values (VEVs) and mass eigenstates.

The Higgs scalar potential is:

$$\begin{aligned} V &= \tfrac{1}{8}(g^2+g'^2)(|H_u^0|^2+|H_u^+|^2-|H_d^0|^2-|H_d^-|^2)^2 \\ &\quad +\tfrac{1}{2}g^2|H_u^+H_d^{0*}+H_u^0H_d^{-*}|^2 \\ &\quad +(|\mu|^2+m_{H_u}^2)(|H_u^0|^2+|H_u^+|^2)+(|\mu|^2+m_{H_d}^2)(|H_d^0|^2+|H_d^-|^2) \\ &\quad +b\,(H_u^+H_d^- - H_u^0H_d^0)+\text{c.c.}. \end{aligned}$$

The g^2 and g'^2 parts come from the D-terms.
The $|\mu|^2$ parts come from the F-terms.
The other terms come from the soft SUSY-breaking Lagrangian.

We must now minimize this potential, and show that it is compatible with the known electroweak symmetry breaking.

First, note that the freedom to do $SU(2)_L$ gauge transformations allows us to take H_u^+ at the minimum without loss of generality. Then $\partial V/\partial H_u^+ = 0$ also requires $H_d^- = 0$. So, at the minimum of the potential, $U(1)_{\rm EM}$ will be unbroken. We are left with...

$$
\begin{aligned}
V &= (|\mu|^2 + m^2_{H_u})|H_u^0|^2 + (|\mu|^2 + m^2_{H_d})|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{c.c.}) \\
&\quad + \tfrac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2.
\end{aligned}
$$

A redefinition of the phase of H_u^0 can absorb any phase in b, so take it real and positive. This implies that at the minimum, $H_u^0 H_d^0$ is also real and positive, so H_u^0 and H_d^0 have opposite phases. Because they have opposite weak hypercharges ($\pm\frac{1}{2}$), a $U(1)_Y$ gauge rotation can make them both real and positive at the minimum.

Must require that $H_u^0 = H_d^0 = 0$ is **not** the minimum. Then:

$$
b^2 > (|\mu|^2 + m^2_{H_u})(|\mu|^2 + m^2_{H_d}).
$$

Also, we need the potential to be bounded from below. This requires:

$$
2b < 2|\mu|^2 + m^2_{H_u} + m^2_{H_d}.
$$

If these conditions are met, then spontaneous electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{EM}}$ occurs.

The resulting Higgs VEVs can be parameterized:

$$
\begin{aligned}
&v_u = \langle H_u^0 \rangle, \\
&v_d = \langle H_d^0 \rangle, \\
&v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\mathrm{GeV})^2 \\
&\tan\beta = v_u/v_d .
\end{aligned}
$$

The quark and lepton masses are related to these VEVs and the superpotential Yukawa couplings by:

$$
y_t = \frac{m_t}{v\sin\beta}, \qquad y_b = \frac{m_b}{v\cos\beta}, \qquad y_\tau = \frac{m_\tau}{v\cos\beta}, \qquad \text{etc.}
$$

If we want the running Yukawa couplings to avoid getting non-perturbatively large up to very high scales, we need:

$$
1.5 \;\lesssim\; \tan\beta \;\lesssim\; 55.
$$

These bounds depend somewhat on other parameters, however, and get weaker if squarks are very heavy.

Define mass-eigenstate Higgs bosons: h, H, A, G, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G \\ A \end{pmatrix}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

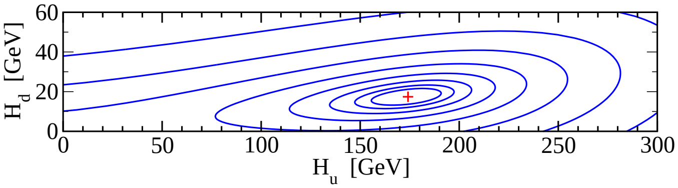
Now, expand the potential to second order in these fields to find:

$$\begin{aligned} m_A^2 &= 2b/\sin 2\beta \\ m_{h,H}^2 &= \tfrac{1}{2}\Big(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_Z^2 m_A^2 \cos^2 2\beta}\Big), \\ m_{H^\pm}^2 &= m_A^2 + m_W^2 \\ \tan 2\alpha &= [(m_A^2 + m_Z^2)/(m_A^2 - m_Z^2)] \tan 2\beta \end{aligned}$$

Note: only two independent parameters here, $\tan\beta$, m_A.

The Goldstone bosons have $m_G = m_{G^\pm} = 0$; they are absorbed by the $W^\pm, Z$ bosons to give them masses, as in the Standard Model.

Typical contour map of the Higgs potential in SUSY:



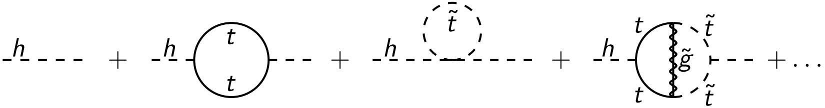
The Standard Model-like Higgs boson h corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level, it is easy to show from above that the lightest Higgs scalar would obey:

$$m_h < m_Z.$$

Naively, this disagrees with the recent discovery of $m_h = 125$ GeV.
However, taking into account loop effects, one can get the observed Higgs mass. . .

Radiative corrections to the Higgs mass in SUSY:

$$m_h^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level:	m_Z^2	pure electroweak
At one-loop:	$y_t^2 m_t^2$	top Yukawa comes in
At two-loop:	$\alpha_S y_t^2 m_t^2$	SUSYQCD comes in
At three-loop:	$\alpha_S^2 y_t^2 m_t^2$	more SUSYQCD

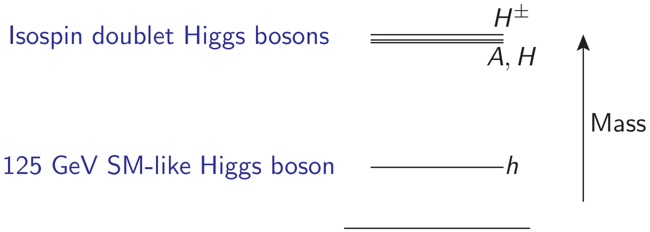
Even the three-loop corrections can add ± 1 GeV or so to m_h.

This is still much larger than the eventual experimental uncertainty expected at the LHC!

The decoupling limit for the Higgs bosons

If $m_A \gg m_Z$, then:

- h has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A, H, H^{\pm}$ form an isospin doublet, and are much heavier than h



Many (but not all) models of SUSY breaking approximate this decoupling limit.

Neutralinos

The neutral higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$, $\tilde{W}^0$) mix with each other because of electroweak symmetry breaking. In the gauge eigenstate basis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$,

$$\mathcal{L}_{\text{neutralino mass}} = -\tfrac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

The diagonal terms are just the gaugino masses in the soft SUSY-breaking Lagrangian. The $-\mu$ entries can be traced back to the superpotential. The off-diagonal terms come from the gaugino-Higgs-Higgsino interactions, and are always less than m_Z.

The physical neutralino mass eigenstates $\tilde{N}_i$ (another popular notation is $\tilde{\chi}_i^0$) are obtained by diagonalizing the mass matrix with a unitary matrix.

$$\tilde{N}_i \quad = \quad \mathbf{N}_{ij}\psi_j^0,$$

where

$$\begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix} = \mathbf{N}^* \mathbf{M} \mathbf{N}^{-1},$$

with $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$.

The lightest neutralino fermion, $\tilde{N}_1$, is a candidate for the cold dark matter required by cosmology and astrophysics, but there are now significant constraints on this.

Charginos

Similarly, the charged higgsinos $\tilde{H}_u^+, \tilde{H}_d^-$ and the charged winos $\tilde{W}^+, \tilde{W}^-$ mix to form **chargino** fermion mass eigenstates.

$$\mathcal{L}_{\text{chargino mass}} = -\tfrac{1}{2}(\psi^{\pm})^T \mathbf{M}_{\widetilde{C}} \psi^{\pm} + \text{c.c.}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix} \qquad \text{with} \qquad \mathbf{X} = \begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix}$$

The mass eigenstates $\widetilde{C}^{\pm}_{1,2}$ (many other sources use $\tilde{\chi}^{\pm}_{1,2}$) are related to the gauge eigenstates by two unitary 2×2 matrices **U** and **V** according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}.$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions.

The chargino mixing matrices are chosen so that

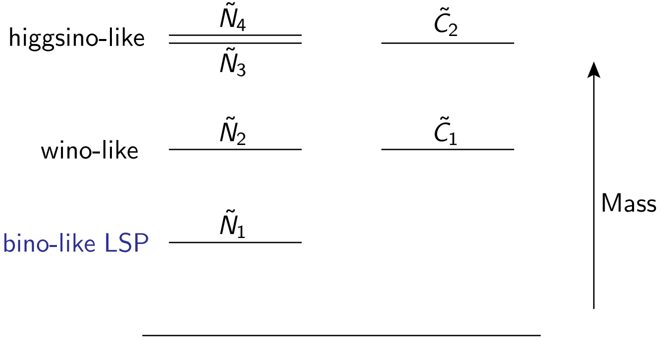
$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with positive real entries $m_{\widetilde{C}_i}$. In this case, one can solve for the tree-level $(\text{mass})^2$ eigenvalues in simple closed form:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} &= \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \\ &\mp\sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \end{aligned}$$

In many models of SUSY breaking, one finds that $M_2 \ll |\mu|$, so the lighter chargino is mostly wino with mass close to M_2, and the heavier is mostly higgsino with mass close to $|\mu|$.

A typical mass hierarchy for the neutralinos and charginos, assuming $m_Z \ll |\mu|$ and $M_1 \approx 0.5 M_2 < |\mu|$.



Although this was historically a very popular scenario, it is NOT guaranteed. The lightest states could easily be the higgsinos, or the winos.

The Gluino

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. So, at tree-level, its mass is the same as the corresponding parameter in the soft Lagrangian:

$$M_{\tilde{g}} = M_3.$$

However, the quantum corrections to this are quite large (again, because this is a color octet!). If one calculates the one-loop pole mass of the gluino, one finds:

$$M_{\tilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_s}{4\pi}\big[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\big]\Big)$$

where Q is the renormalization scale, the sum is over all 12 squark multiplets, and

$$A_{\tilde{q}} = \int_0^1 dx\, x \ln\big[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x) - i\epsilon\big].$$

This correction can be of order 5% to 25%, depending on the squark masses. It **increases** the gluino mass, compared to the tree-level value.

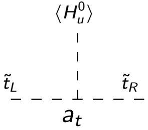
Squarks and Sleptons

To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

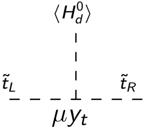
- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L$, $\widetilde{c}_L$, $\widetilde{t}_L$, $\widetilde{u}_R$, $\widetilde{c}_R$, $\widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L$, $\widetilde{s}_L$, $\widetilde{b}_L$, $\widetilde{d}_R$, $\widetilde{s}_R$, $\widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L$, $\widetilde{\mu}_L$, $\widetilde{\tau}_L$, $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$),
- a 3×3 matrix for sneutrinos ($\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$)

In many popular models, the first- and second-family squarks and sleptons are in 7 very nearly degenerate, unmixed pairs:
$(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{e}_L, \widetilde{\mu}_L)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$,
with mixing angles assumed small.

But, for the third-family squarks and sleptons, large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings are important. For the top squark:



and



The first diagram comes directly from the soft SUSY-breaking Lagrangian, and the second from the F-term contribution to the scalar potential. So, in the $(\tilde{t}_L, \tilde{t}_R)$ basis, the top squark (mass)^2 matrix is:

$$\begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + \Delta_{\tilde{t}_L} & a_t^* v_u - \mu y_t v_d \\ a_t v_u - \mu^* y_t v_d & m^2_{\tilde{\bar{u}}_3} + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix}.$$

The off-diagonal terms imply $\tilde{t}_L, \tilde{t}_R$ mixing.

Diagonalizing the top squark mass^2 matrix, one finds mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s_{\tilde{t}}^* \\ s_{\tilde{t}} & c_{\tilde{t}}^* \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

where $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$ by convention, and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$. If they are real, then $c_{\tilde{t}} = \cos\theta_{\tilde{t}} \quad \text{and} \quad s_{\tilde{t}} = \sin\theta_{\tilde{t}}$.

Similarly, mixing for the bottom squark and tau slepton states:

$$\begin{aligned} \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} &= \begin{pmatrix} c_{\tilde{b}} & -s_{\tilde{b}}^* \\ s_{\tilde{b}} & c_{\tilde{b}}^* \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}; \\ \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} &= \begin{pmatrix} c_{\tilde{\tau}} & -s_{\tilde{\tau}}^* \\ s_{\tilde{\tau}} & c_{\tilde{\tau}}^* \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \end{aligned}$$

To avoid flavor constraints, often **assume** for the first- and second-family squarks and sleptons that the mixing is small, due to small Yukawa and *a* terms. However, the mixing could be large if they are heavy.

The undiscovered particles in the MSSM:

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$\boldsymbol{h}$ H A $H^{\pm}$	H_u^0 H_d^0 H_u^+ H_d^-
squarks	0	-1	$\widetilde{u}_L$ $\widetilde{u}_R$ $\widetilde{d}_L$ $\widetilde{d}_R$	" "
			$\widetilde{s}_L$ $\widetilde{s}_R$ $\widetilde{c}_L$ $\widetilde{c}_R$	" "
			$\widetilde{t}_1$ $\widetilde{t}_2$ $\widetilde{b}_1$ $\widetilde{b}_2$	$\widetilde{t}_L$ $\widetilde{t}_R$ $\widetilde{b}_L$ $\widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L$ $\widetilde{e}_R$ $\widetilde{\nu}_e$	" "
			$\widetilde{\mu}_L$ $\widetilde{\mu}_R$ $\widetilde{\nu}_\mu$	" "
			$\widetilde{\tau}_1$ $\widetilde{\tau}_2$ $\widetilde{\nu}_\tau$	$\widetilde{\tau}_L$ $\widetilde{\tau}_R$ $\widetilde{\nu}_\tau$
neutralinos	1/2	-1	$\widetilde{N}_1$ $\widetilde{N}_2$ $\widetilde{N}_3$ $\widetilde{N}_4$	$\widetilde{B}^0$ $\widetilde{W}^0$ $\widetilde{H}_u^0$ $\widetilde{H}_d^0$
charginos	1/2	-1	$\widetilde{C}_1^{\pm}$ $\widetilde{C}_2^{\pm}$	$\widetilde{W}^{\pm}$ $\widetilde{H}_u^+$ $\widetilde{H}_d^-$
gluino	1/2	-1	$\widetilde{g}$	" "

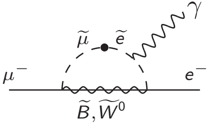
There are 105 new parameters associated with SUSY breaking in the MSSM.

How are we supposed to make any meaningful predictions in the face of this uncertainty?

Fortunately, we already have strong constraints on the MSSM soft terms, because of experimental limits on flavor and CP violation.

Hints of an Organizing Principle

For example, if there is a smuon-selectron mixing (mass)^2 term $\mathcal{L} = -m^2_{\tilde{\mu}_L^* \tilde{e}_L} \tilde{\mu}_L^* \tilde{e}_L$, and $\tilde{M} = \text{Max}[m_{\tilde{e}_L}, m_{\tilde{e}_R}, M_2]$, then this one-loop diagram gives the decay width:



$\mu^- \to e^- \gamma$

$$\Gamma(\mu^- \to e^- \gamma) = 5 \times 10^{-19}\,\text{eV} \left(\frac{m^2_{\tilde{\mu}_L^* \tilde{e}_L}}{\tilde{M}^2}\right)^2 \left(\frac{1000\ \text{GeV}}{\tilde{M}}\right)^4.$$

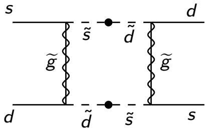
For comparison, the experimental limit is (from MEG at PSI):

$$\Gamma(\mu^- \to e^- \gamma) < 1.3 \times 10^{-22}\,\text{eV}.$$

So the amount of smuon-selectron mixing in the soft Lagrangian is limited:

$$\frac{m^2_{\tilde{\mu}_L^* \tilde{e}_L}}{\tilde{M}^2} < 0.016 \left(\frac{\tilde{M}}{1000\ \text{GeV}}\right)^2$$

Another example: $K^0 \leftrightarrow \overline{K^0}$ mixing:



$K^0 \leftrightarrow \overline{K^0}$

This constrains the flavor-violating SUSY breaking terms:
$\mathcal{L} = -m^2_{\tilde{d}_L^* \tilde{s}_L} \tilde{d}_L^* \tilde{s}_L - m^2_{\tilde{d}_R \tilde{s}_R^*} \tilde{d}_R \tilde{s}_R^*$.

Comparing these contributions with the observed Δm_{K^0} gives:

$$\frac{\mathrm{Re}[m^2_{\tilde{d}_L^* \tilde{s}_L} m^2_{\tilde{d}_R \tilde{s}_R^*}]^{1/2}}{\tilde{M}^2} \lesssim 0.002 \left(\frac{\tilde{M}}{1000 \text{ GeV}} \right)$$

where $\tilde{M}$ is the larger of the squark or gluino mass.

The experimental values of ϵ and ϵ'/ϵ in the effective Hamiltonian for the $K^0, \overline{K^0}$ system also give strong constraints on the amount of $\tilde{d}_L, \tilde{s}_L$ and $\tilde{d}_R, \tilde{s}_R$ mixing and CP violation in the soft terms.

Similarly:

The $D^0, \overline{D^0}$ system constrains $\tilde{u}_L, \tilde{c}_L$ and $\tilde{u}_R, \tilde{c}_R$ soft SUSY-breaking mixing.
The $B_d^0, \overline{B_d^0}$ system constrains $\tilde{d}_L, \tilde{b}_L$ and $\tilde{d}_R, \tilde{b}_R$ soft SUSY-breaking mixing.

To avoid experimental limits on flavor violation, the soft-SUSY breaking masses must be

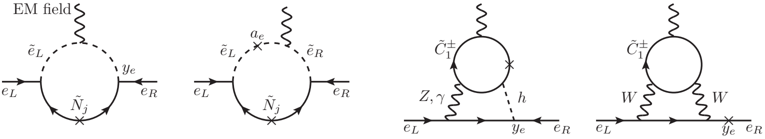
- **nearly flavor-bind**, or
- **aligned in flavor space**, or
- **very heavy (over 1000 GeV)** .

Direct limits from the LHC now suggest that the last option is at least part of the explanation.

Another challenge: the electric dipole moment of the electron.

Experimental limit is $d_e < 4.1 \times 10^{-30}$ e·cm, and the Standard Model prediction is negligible, $d_e^{\rm SM} \approx 10^{-38}$ e·cm.

SUSY has selectron-neutralino 1-loop and chargino 2-loop diagrams:



If superpartner masses are at the TeV scale, then the CP-violating phases must be less than the 1% level, with some model dependence. (See for example Cesarotti et al, 1810.07736.)

To avoid disaster: take selectrons very heavy (1-loop diagrams) and charginos very heavy (2-loop diagrams), OR CP-violating phases very small OR both.

The Flavor-Preserving Minimal Supersymmetric Standard Model

Idealized limit: the squark and slepton (mass)^2 matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space.

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{\tilde{u}}}} = m^2_{\bar{\tilde{u}}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{\tilde{d}}}} = m^2_{\bar{\tilde{d}}}\mathbf{1}, \qquad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}}\mathbf{1}, \qquad \mathbf{m^2_{\bar{\tilde{e}}}} = m^2_{\bar{\tilde{e}}}\mathbf{1}.$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will.

Also assume:

$$\mathbf{a_u} = A_{u0}\,\mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\,\mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\,\mathbf{y_e},$$

and no new CP-violating phases in a convention where b is real:

$$\mu,\, M_1,\, M_2,\, M_3,\, A_{u0},\, A_{d0},\, A_{e0} \;=\; \text{real}.$$

A modification of this (treating the third family squark and slepton masses differently) is sometimes called the "pMSSM" = phenomenological MSSM.

The Flavor-Preserving Minimal Supersymmetric Standard Model (continued)

The new parameters, besides those already in the Standard Model, are:

- M_1, M_2, M_3 (3 real gaugino masses)
- $m^2_{\tilde{Q}}$, $m^2_{\tilde{\bar{u}}}$, $m^2_{\tilde{\bar{d}}}$, $m^2_{\tilde{L}}$, $m^2_{\tilde{\bar{e}}}$ (5 squark and slepton mass^2 parameters)
- A_{u0}, A_{d0}, A_{e0} (3 real scalar^3 couplings)
- $m^2_{H_u}$, $m^2_{H_d}$, b, μ (4 real parameters)

So there are 15 real parameters in this model.
The parameters μ and $b \equiv B\mu$ are often traded for the known Higgs VEV $v = 174$ GeV, $\tan\beta$, and sign(μ).

Many SUSY breaking models are special cases of this.

However, these are Lagrangian parameters that run with the renormalization scale, Q. Therefore, one must also choose an "input scale" Q_0 where the flavor-independence holds.

What is the input scale Q_0 ?

Perhaps:

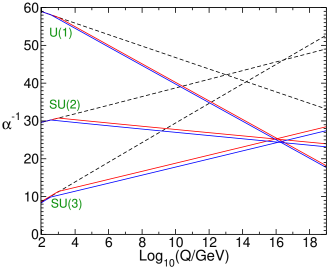
- $Q_0 = M_{\rm Planck}$, or
- $Q_0 = M_{\rm string}$, or
- $Q_0 = M_{\rm GUT}$, or
- Q_0 is some other scale associated with the type of SUSY breaking.

In any case, the SUSY-breaking parameters are picked at Q_0 as boundary conditions, then run them down to the weak scale using their renormalization group (RG) equations.

Flavor violation will remain small, because the Yukawa couplings of the first two families are small.

At the weak scale, use the renormalized parameters to predict physical masses, decay rates, cross-sections, dark matter relic density, etc.

A reason to be optimistic that this program can succeed: the SUSY unification of gauge couplings. The measured α_1, α_2, α_3 are run up to high scales using the RG equations of the Standard Model (dashed lines) and the MSSM (solid lines).



At one-loop order, the RG equations are:

$$\frac{d}{d(\ln Q)}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3)$$

with $b_a^{\rm SM}$=$(41/10, -19/6, -7)$ in the Standard Model, and $b_a^{\rm MSSM}$=$(33/5, 1, -3)$ in the MSSM because of the extra particles in the loops. The MSSM predicts unification at $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV.

If this hint is real, we might hope that a similar extrapolation for the soft SUSY-breaking parameters can also work.

Origins of SUSY breaking

Up to now, we have simply put SUSY breaking into the MSSM explicitly. For deeper understanding, how can SUSY spontaneously broken? This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad Q^\dagger_{\dot{\alpha}}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\Big(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\Big) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

In SUSY, the potential energy can be written as:

$$V = \sum_i |F_i|^2 + \tfrac{1}{2} \sum_a D^a D^a,$$

a sum of squares of auxiliary fields.

So, for spontaneous SUSY breaking, there must be a stable (or quasi-stable) state with at least one of $\langle F_i \rangle \neq 0$ or $\langle D^a \rangle \neq 0$.
Here, the auxiliary fields are given by **algebraic** equations:

$$F_i^* = -\frac{\partial W}{\partial \phi_i} \qquad \text{and} \qquad D^a = -g(\phi^\dagger T^a \phi).$$

Models of SUSY breaking where

- $\langle F_i \rangle \neq 0$ are called "O'Raifeartaigh models" or "F-term breaking models"
- $\langle D^a \rangle \neq 0$ are called "Fayet-Iliopoulis models" or "D-term breaking models"

F-term breaking is used in most known realistic models.

F-term breaking: the O'Raifeartaigh Model

The simplest example has 3 chiral supermultiplets, with:

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

Then the auxiliary fields are found from the algebraic equation $F_i^* = -\frac{\partial W}{\partial \phi_i}$:

$$F_1 = k - \frac{y}{2}\phi_3^{*2}, \qquad F_2 = -m\phi_3^*, \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*.$$

SUSY is necessarily broken because $F_1 = 0$ and $F_2 = 0$ are not compatible.

The minimum of $V(\phi_1, \phi_2, \phi_3)$ is at $\phi_2 = \phi_3 = 0$, with ϕ_1 not determined (classically). Quantum corrections fix the true minimum to be at $\phi_1 = 0$, where:

$$F_1 = k, \qquad V = k^2 > 0.$$

Note that ϕ_1 must be a **gauge singlet**. Otherwise, $k = 0$ to make W invariant.

F-term breaking (continued)

If you assume $m^2 > yk$ and expand the scalar fields around the minimum at $\phi_1 = \phi_2 = \phi_3 = 0$, you will find 6 real scalars with tree-level squared masses:

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk,\ m^2 + yk.$$

Meanwhile, there are 3 Weyl fermions with squared masses

$$0,\ m^2,\ m^2.$$

The fact that the fermions and scalars aren't degenerate is a clear sign that SUSY has indeed been spontaneously broken.

The 0 mass^2 eigenvalues belong to the complex scalar ϕ_1 and its superpartner ψ_1. The masslessness of ϕ_1 corresponds to the flat direction of the classical potential. It is lifted by quantum corrections at one loop, resulting in:

$$m^2_{\phi_1} = \frac{y^4 k^2}{48\pi^2 m^2}.$$

However, ψ_1 remains **exactly** massless, even including loop effects. **Why?**

The Goldstino ($\tilde{G}$)

In general, the spontaneous breaking of a global symmetry gives rise to a massless Nambu-Goldstone mode with the same quantum numbers as the broken symmetry generator.

Here, the broken generator is the fermionic charge Q_α, so the Nambu-Goldstone particle must be a massless, neutral, Weyl fermion, called the **Goldstino**. It is always the fermion that lives in the same supermultiplet with the auxiliary field that got a VEV to break SUSY.

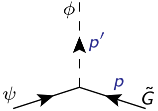
After SUSY breaking, you can show using Noether's Theorem that the Goldstino has an effective Lagrangian of the form (assuming F-term breaking for simplicity):

$$\mathcal{L}_{\text{Goldstino}} = i\widetilde{G}^\dagger \overline{\sigma}^\mu \partial_\mu \widetilde{G} + \frac{1}{\langle F \rangle}(J^\mu \partial_\mu \widetilde{G} + \text{c.c.})$$

where J^μ is the fermionic supercurrent, and contains products of all of the fields and their superpartners.

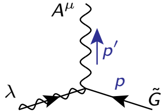
The Goldstino thus has **derivative** couplings to each particle-sparticle pair:

Fermion-Scalar-Goldstino couplings



$\frac{i}{\langle F \rangle}(p' \cdot \sigma)(p \cdot \overline{\sigma})$

Vector-Gaugino-Goldstino couplings



$\frac{i}{2\sqrt{2}\langle F \rangle}(p' \cdot \sigma \overline{\sigma}^\mu - \sigma^\mu p' \cdot \overline{\sigma})(p \cdot \sigma)$

These both grow with $\frac{1}{\langle F \rangle}$, so they are more important if the mass scale of SUSY breaking is smaller. (More on this later.)

The interactions are well-defined in the $\langle F \rangle \to 0$ limit, because the momenta in the numerator combine to give factors like $m_\phi^2 - m_\psi^2$ and $m_A^2 - m_\lambda^2$ on-shell, and these also vanish in the limit that there is no SUSY breaking.

The Goldstino is a consequence of spontaneously breaking **global** SUSY. Including gravity, SUSY becomes a local symmetry. The spinor ϵ_α used to define the SUSY transformations is no longer constant.

The resulting locally supersymmetric theory is **supergravity**. In unbroken supergravity, the graviton has a massless spin-$\frac{3}{2}$ partner (with only helicities $\pm\frac{3}{2}$) called the **gravitino**, with $P_R = -1$.

When local SUSY is spontaneously broken, the gravitino absorbs the would-be massless Goldstino as its helicity $\pm\frac{1}{2}$ components, and acquires a mass:

$$m_{3/2} \sim \frac{\langle F \rangle}{M_{\rm Planck}}$$

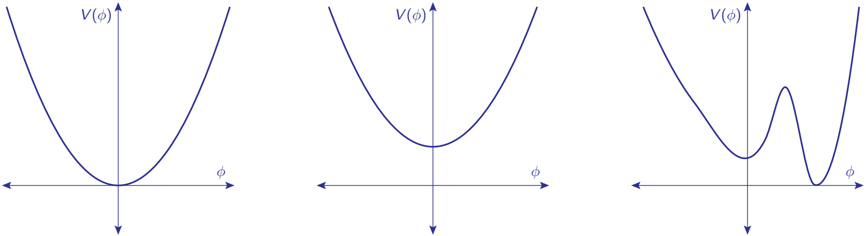
This follows by dimensional analysis, since $m_{3/2}$ must vanish if SUSY breaking is turned off ($\langle F \rangle \to 0$) or if gravity is turned off ($M_{\rm Planck} \to \infty$).

The gravitino inherits the couplings of the Goldstino it has eaten.

The O'Raifeartaigh model always breaks SUSY at the true minimum of the potential, for any values of the superpotential parameters.

Another possibility is that we live in a **meta-stable** vacuum with broken supersymmetry, and that supersymmetry is unbroken in the true vacuum.

Behavior of the scalar potential as a function of some order parameter ϕ:



SUSY unbroken　　SUSY broken　　meta-stable SUSY breaking

Meta-stable SUSY breaking is acceptable if the tunneling lifetime to decay from our SUSY-breaking vacuum (with $\phi = 0$ here) to the global minimum SUSY-preserving vacuum is longer than the age of the universe.

Intriligator, Seiberg, Shih arXiv:hep-th/0602239 showed that this can work in simple, uncontrived SUSY Yang-Mills models.

An even simpler example: adding a small term $\epsilon\phi_2^2$ to the O'Raifeartaigh superpotential turns it into a meta-stable SUSY breaking model. (Try it!)

Spontaneous Breaking of SUSY requires us to extend the MSSM

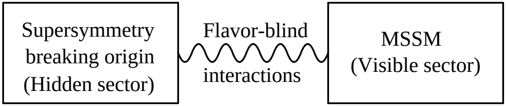
MSSM has no gauge-singlet chiral supermultiplet that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any **(scalar)-(gaugino)-(gaugino)** coupling that could turn into a gaugino mass term when a scalar gets a VEV.

We also have the clue that SUSY breaking could be nearly flavor-blind in order to not conflict with experiment.

This leads to the following general schematic picture of SUSY breaking...

Spontaneous SUSY breaking occurs in a "hidden sector" of particles with no (or tiny) direct couplings to the "visible sector" chiral supermultiplets of the MSSM.



However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly. As a bonus, if the mediating interactions are flavor-blind, then the soft SUSY-breaking terms of the MSSM will be also.

By dimensional analysis,

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M}$$

where M is a mass scale associated with the physics that mediates between the two sectors.

The O'Raifeartaigh model has the mass scale of supersymmetry breaking put in by hand, as the parameter $k = \sqrt{\langle F \rangle}$.

More plausible: **dynamical SUSY breaking**. The scale of $\langle F \rangle$ arises from some strong dynamics, set by the scale at which a new gauge theory gets strong:

$$\Lambda = e^{-8\pi^2/bg^2} M_{\mathrm{Planck}}$$

just as in QCD.

Then the field that breaks supersymmetry might be a composite made of strongly interacting fundamental fields.

Some great reviews on this subject:
Intriligator, Seiberg hep-ph/0702069
Dine, Mason hep-th/1012.2836
Poppitz, Trivedi hep-th/9803107
Shadmi, Shirman hep-th/9907225

Planck-scale Mediated SUSY Breaking (or, “gravity mediation”)

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since $m_{\rm soft}$ must vanish in the limit that SUSY breaking is turned off ($\langle F \rangle \to 0$), and in the limit that gravity becomes irrelevant ($M_P \to \infty$).

Since we think $m_{\rm soft} \sim 10^3$ GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11}\ \text{GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\mathcal{L}_{\text{PMSB}} = -\Big(\frac{x^a}{2M_P}F\lambda^a\lambda^a + \text{c.c.}\Big) - \frac{k_i^j}{M_P^2}FF^*\phi_i\phi^{*j}$$
$$-\Big(\frac{\alpha^{ijk}}{6M_P}F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P}F\phi_i\phi_j + \text{c.c.}\Big)$$

This is (part of) a fully supersymmetric Lagrangian that arises in supergravity.

When we replace F by its VEV $\langle F\rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian, with:

- Gaugino masses: $M_a = x^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious that these are flavor-blind!

A dramatically simplified parameter space is often called "Minimal Supergravity" (or "mSUGRA") or the "Constrained MSSM".

Assume only four parameters $m_{1/2}$, m_0^2, A_0, and B_0:

$$
\begin{aligned}
&M_3 = M_2 = M_1 = m_{1/2} \\
&\mathbf{m}^2_{\tilde{\mathbf{Q}}} = \mathbf{m}^2_{\tilde{\bar{\mathbf{u}}}} = \mathbf{m}^2_{\tilde{\bar{\mathbf{d}}}} = \mathbf{m}^2_{\tilde{\mathbf{L}}} = \mathbf{m}^2_{\tilde{\bar{\mathbf{e}}}} = m_0^2\,\mathbf{1} \\
&m^2_{H_u} = m^2_{H_d} = m_0^2 \\
&\mathbf{a_u} = A_0 \mathbf{y_u}, \qquad \mathbf{a_d} = A_0 \mathbf{y_d}, \qquad \mathbf{a_e} = A_0 \mathbf{y_e} \\
&b = B_0 \mu.
\end{aligned}
$$

The most important thing to know about mSUGRA is that it is almost certainly wrong!

These soft relations should be true at the renormalization scale $Q_0 = M_P$, and then run down to the weak scale, possibly including large threshold effects if there is a Grand Unified Theory.

However, it is traditional to use $Q_0 = M_{\rm GUT}$ instead, **because nobody knows how to extrapolate above $M_{\rm GUT}$.** (Not a very good reason!)

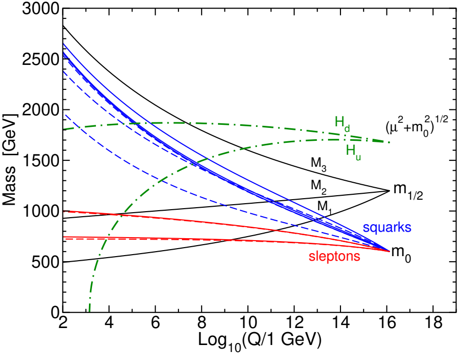
Renormalization Group Running for an mSUGRA model with
$m_{1/2} = 1200$ GeV, $m_0 = 600$ GeV, $A_0 = -1200$ GeV, $\tan\beta = 15$, $\mu > 0$

Gaugino masses M_1, M_2, M_3

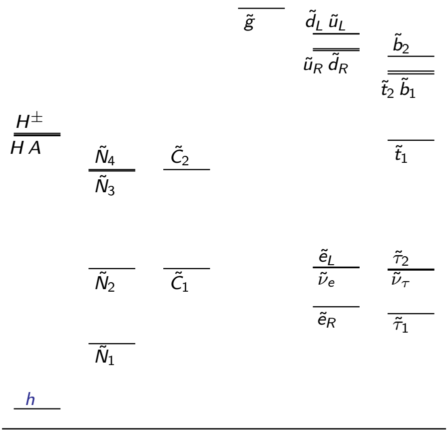
Slepton masses (dashed=stau)

Squark masses (dashed=stop)

Higgs: $(\mu^2 + m_{H_d}^2)^{1/2}$,
$(\mu^2 + m_{H_u}^2)^{1/2}$



Here is the resulting sparticle mass spectrum:



$M_{\text{gluino}} = 2600$ GeV

$M_{\text{squarks}} = 2350$ GeV

$M_{\text{LSP}} = 520$ GeV

$M_h = 121$ GeV

This model would be OK as of today, except. . . it predicts $M_h \approx 121$ GeV

Impact of the discovery $M_h = 125$ GeV in the MSSM

In the decoupling limit:

$$\begin{aligned} M_h^2 &= m_Z^2 \cos^2(2\beta) \\ &\quad + \frac{3}{4\pi^2} y_t^2 m_t^2 \left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + \sin^2(2\theta_{\tilde{t}}) f_1 - \sin^4(2\theta_{\tilde{t}}) f_2 \right] + \ldots \end{aligned}$$

where f_1 and f_2 are certain positive functions of $m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2}$.

To get $M_h = 125$ GeV, need

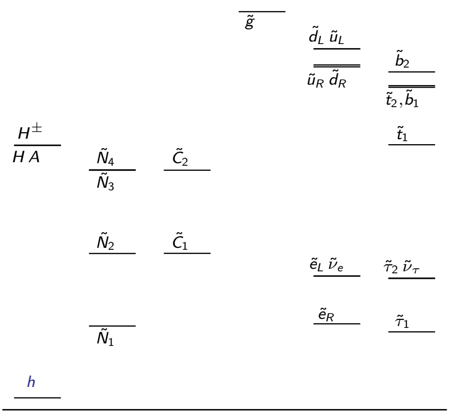
- heavy top squarks $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gg m_t^2$,

  and/or
- large stop mixing $\sin(2\theta_{\tilde{t}})$, in which case $\Big[\ldots \Big] \lesssim \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + 3$

  and/or
- some addition to the model.

The level-repulsion associated with large stop mixing suggests that one of the stop masses is much lighter than the other.

What's left for mSUGRA? Here's a model that survives LHC:



$M_{\tilde{g}} = 4200$ GeV

$M_{\rm squarks} = 3700$ GeV

$M_{\tilde{t}_1} \ll$ other squarks

$M_h = 125$ GeV

$M_{1/2} = 2000$ GeV, $\quad m_0 = 500$ GeV, $\quad A_0 = -2000$ GeV, $\quad \tan\beta = 15.$

To get $M_h = 125$ GeV, squarks and gluino out of reach of LHC.

Computer programs, including:

`SoftSUSY`, `SuSpect`, `SARAH`, `SPheno`, `FlexibleSUSY`, `ISASUSY`, `SuSeFLAV`, `FeynHiggs`, `SUSYHD`, `H3m`, `CPsuperH`, `NMSPEC`, `NMSSMCalc`, `NMSSMtools`, ...
can help you generate the superpartner and Higgs mass spectrum, given a choice of SUSY-breaking model parameters.

These can be interfaced to programs that produce cross-sections, decay rates, and Monte Carlo events:
`SDECAY`, `HDECAY`, `SUSY-HIT`, `PROSPINO`, `MadGraph/MadEvent`, `Pythia`, `ISAJET`, `HERWIG`, `WHIZARD`, `SHERPA`, `SUSYGEN`, `GRACE`, `CompHEP`, `CalcHEP`, ...

They can also be interfaced to programs that compute the abundance of dark matter and dark matter detection signals:
`micrOMEGAs`, `DarkSUSY`, `ISAReD`, ...

Superfields and Superspace

A geometric interpretation of supersymmetry can be given in **superspace**.

Generalize spacetime coordinates to super-coordinates:

$$x^\mu, \quad \theta_\alpha, \quad \theta^\dagger_{\dot\alpha}$$

Last two are constant, complex, anti-commuting ("Grassmann-odd"), two-component spinors with dimension $[\text{mass}]^{-1/2}$.

- 4 commuting coordinates, 4 anti-commuting coordinates.
- Component fields of a supermultiplet will be united into a single **superfield** $=$ a function on superspace.
- Supersymmetry transformations $=$ translations in superspace
- Elegant formulation, some calculations much nicer and easier

Warm-up: derivatives, integrals for a single anti-commuting variable η.

Since $\eta^2 = 0$, a power-series expansion terminates at first order, and a general function is linear in η:

$$f(\eta) = f_0 + \eta f_1.$$

Therefore:

$$\frac{df}{d\eta} = f_1.$$

To define the integration operation, take:

$$\int d\eta = 0, \qquad \int d\eta \; \eta = 1,$$

and impose linearity. This is called Berezin integration, and implies:

$$\int d\eta \, f(\eta) = f_1,$$

so differentiation and integration are the same thing!

Important properties of differentiation and integration:

Note that $d/d\eta$ anti-commutes with every Grassmann-odd object, so

$$\frac{d(\eta'\eta)}{d\eta} = -\frac{d(\eta\eta')}{d\eta} = -\eta'.$$

The Berezin integration obeys translation invariance:

$$\int d\eta\, f(\eta+\eta') = \int d\eta\, f(\eta)$$

and integration by parts:

$$\int d\eta\, \frac{df}{d\eta} = 0 \qquad \text{(Fundamental Theorem of Calculus!)}$$

Can define a delta function by:

$$\int d\eta\, \delta(\eta-\eta')f(\eta) = f(\eta')$$

which implies:

$$\delta(\eta-\eta') = \eta-\eta'.$$

Return to the superspace for 4 dimensions: a superfield can be expanded in a power series in anticommuting variables θ_α and $\theta^\dagger_{\dot\alpha}$. There are two of each, so the expansion ends after at most two θ and two $\theta^\dagger$.

So, a general (complex) Grassmann-even superfield is:

$$\begin{aligned} S(x,\theta,\theta^\dagger) &= a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c + \theta^\dagger\overline{\sigma}^\mu\theta v_\mu \\ &\quad + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d, \end{aligned}$$

where

$$a(x), \quad b(x), \quad c(x), \quad v_\mu(x), \quad d(x)$$

are $1+1+1+4+1=8$ complex bosonic component fields, and

$$\xi^\alpha(x), \quad \chi^\dagger_{\dot\alpha}(x), \quad \eta^\alpha(x), \quad \zeta^\dagger_{\dot\alpha}(x)$$

are $2+2+2+2=8$ complex fermionic component fields.

However, this superfield S is too general; it has too many components to be a chiral supermultiplet or a vector supermultiplet.

Differentiation in superspace (compare the η toy example):

$$\frac{\partial}{\partial\theta^\alpha}(\theta^\beta) = \delta_\alpha^\beta, \qquad \frac{\partial}{\partial\theta^\alpha}(\theta^\dagger_{\dot\beta}) = 0, \qquad \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}(\theta^\dagger_{\dot\beta}) = \delta^{\dot\alpha}_{\dot\beta}, \qquad \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}(\theta^\beta) = 0.$$

Integration over superspace:

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}, \qquad d^2\theta^\dagger = -\frac{1}{4}d\theta^\dagger_{\dot\alpha} d\theta^\dagger_{\dot\beta}\epsilon^{\dot\alpha\dot\beta},$$

defined so that:

$$\int d^2\theta\, \theta\theta = 1, \qquad \int d^2\theta^\dagger\, \theta^\dagger\theta^\dagger = 1.$$

The first one just picks out the coefficient of $\theta\theta$, and the second picks out the coefficient of $\theta^\dagger\theta^\dagger$.

Integration by parts works just as you would hope:

$$\int d^2\theta \frac{\partial}{\partial\theta^\alpha}(\text{anything}) = 0, \qquad \int d^2\theta^\dagger \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}(\text{anything}) = 0,$$

Supersymmetry transformations the superspace way:

Define linear differential operators that act on superfields:

$$\hat{Q}_\alpha = i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \qquad \hat{Q}^{\dagger\dot{\alpha}} = i\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - (\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu.$$

Then an infinitesimal SUSY transformation on S, parameterized by ϵ, $\epsilon^\dagger$, is:

$$\begin{aligned}
\sqrt{2}\,\delta_\epsilon S &= -i(\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger)S \\
&= \Big(\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}}\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i\big[\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\overline{\sigma}^\mu\theta\big]\partial_\mu\Big)S \\
&= S(x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\overline{\sigma}^\mu\theta,\, \theta+\epsilon,\, \theta^\dagger+\epsilon^\dagger) - S(x^\mu,\, \theta,\, \theta^\dagger),
\end{aligned}$$

This is just a translation in superspace, with:

$$\begin{aligned}
\theta^\alpha &\rightarrow \theta^\alpha + \epsilon^\alpha, \\
\theta^\dagger_{\dot{\alpha}} &\rightarrow \theta^\dagger_{\dot{\alpha}} + \epsilon^\dagger_{\dot{\alpha}}, \\
x^\mu &\rightarrow x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\overline{\sigma}^\mu\theta.
\end{aligned}$$

Exercise: you can show that

$$\begin{aligned} \left\{\hat{Q}_\alpha,\, \hat{Q}^\dagger_{\dot{\beta}}\right\} &= 2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu \;=\; -2\sigma^\mu_{\alpha\dot{\beta}}\hat{P}_\mu, \\ \left\{\hat{Q}_\alpha,\, \hat{Q}_\beta\right\} &= 0, \qquad\qquad \left\{\hat{Q}^\dagger_{\dot{\alpha}},\, \hat{Q}^\dagger_{\dot{\beta}}\right\} = 0. \end{aligned}$$

Here, the differential operator generating spacetime translations is

$$\hat{P}_\mu = -i\partial_\mu.$$

This is the SUSY algebra again!
However, the hatted objects $\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot{\alpha}}, \hat{P}^\mu$ here are differential operators acting on functions in superspace, conceptually different from the corresponding unhatted objects $Q_\alpha, Q^\dagger_{\dot{\alpha}}, P^\mu$ in the morning lecture, which were operators acting on the Hilbert space of states. The correspondence between them, for a quantum operator X in the Heisenberg picture that is also a function of superspace, is:

$$\begin{aligned} \left[X,\, \epsilon Q + \epsilon^\dagger Q^\dagger\right] &= (\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger)X, \\ \left[X,\, P_\mu\right] &= \hat{P}_\mu X. \end{aligned}$$

Goal: define a Lagrangian in terms of superfields and their derivatives.
Problem: the obvious derivatives of a superfield,

$$\frac{\partial S}{\partial \theta_\alpha} \qquad \text{and} \qquad \frac{\partial S}{\partial \theta^\dagger_{\dot\alpha}}$$

are not themselves superfields; they don't transform correctly! The SUSY transformation of the derivative is not the derivative of the SUSY transformation:

$$\delta_\epsilon \left(\frac{\partial S}{\partial \theta_\alpha} \right) \neq \frac{\partial}{\partial \theta_\alpha} \delta_\epsilon S$$

Instead, need to define **chiral and anti-chiral covariant derivatives**:

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \theta^\dagger)_\alpha \partial_\mu, \qquad \overline{D}^{\dot\alpha} = \frac{\partial}{\partial \theta^\dagger_{\dot\alpha}} - i(\overline{\sigma}^\mu \theta)^{\dot\alpha} \partial_\mu$$

Note these look very similar to $\hat{Q}$ and $\hat{Q}^\dagger$, but have different minus signs and i's.

The crucial feature of chiral and anti-chiral covariant derivatives is:

$$\delta_\epsilon (D_\alpha S) = D_\alpha (\delta_\epsilon S), \qquad \delta_\epsilon \left(\overline{D}_{\dot\alpha} S\right) = \overline{D}_{\dot\alpha} (\delta_\epsilon S)$$

for any superfield S.

Thus $D_\alpha S$ and $\overline{D}_{\dot\alpha} S$ are both superfields, unlike the ordinary derivatives $\partial S/\partial\theta_\alpha$ and $\partial S/\partial\theta_{\dot\alpha}$.

They still obey integration by parts:

$$\int d^2\theta \, D_\alpha(\text{anything}) = 0 \qquad \text{and} \qquad \int d^2\theta^\dagger \, \overline{D}_{\dot\alpha}(\text{anything}) = 0$$

and the useful identities:

$$D_\alpha D_\beta D_\gamma \,(\text{anything}) = 0 \qquad \text{and} \qquad \overline{D}_{\dot\alpha} \overline{D}_{\dot\beta} \overline{D}_{\dot\gamma} \,(\text{anything}) = 0.$$

An aside: why do we use $\dagger$ to conjugate $\hat{Q}$, but $\overline{}$ to conjugate D ?
Answer: they denote different kinds of conjugation.

- The dagger on $\hat{Q}^\dagger$ represents Hermitian conjugation in the same sense that $\hat{P} = -i\partial_\mu$ is an Hermitian differential operator on an inner product space.
- The bar on $\overline{D}$ represents conjugation in the same sense that ∂_μ is a real differential operator (**without** the $-i$)

Recall, from undergraduate QM, using integration by parts,

$$\int d^4x\, \psi^*(x)\, \hat{P}\, \phi(x) \;=\; \left(\int d^4x\, \phi^*(x)\, \hat{P}\, \psi(x)\right)^*$$

Similarly, for integration on superspace:

$$\int d^4x \int d^2\theta \int d^2\theta^\dagger\, T^*\, \hat{Q}^\dagger_{\dot{\alpha}} S \;=\; \left(\int d^4x \int d^2\theta \int d^2\theta^\dagger\, S^*\, \hat{Q}_\alpha\, T\right)^*$$

In contrast, the identity

$$\overline{D}_{\dot{\alpha}} S^* \;=\; (D_\alpha S)^*$$

is just analogous to the equation

$$(\partial_\mu \phi)^* \;=\; \partial_\mu \phi^*.$$

To describe a chiral supermultiplet, use the anti-chiral derivative to impose:

$$\overline{D}_{\dot{\alpha}}\Phi = 0$$

A superfield Φ that obeys this is a **chiral superfield**.

To solve the constraint, define

$$y^\mu \;\equiv\; x^\mu + i\theta^\dagger\overline{\sigma}^\mu\theta,$$

and use the superspace coordinates.

$$y^\mu, \qquad \theta_\alpha, \qquad \theta^\dagger_{\dot{\alpha}}$$

In these coordinates:

$$D_\alpha \;=\; \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\mu\theta^\dagger)_\alpha\frac{\partial}{\partial y^\mu}, \qquad \text{and} \qquad \overline{D}_{\dot{\alpha}} \;=\; -\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}}.$$

The last says that a chiral superfield is a function of y^μ and θ only, but not $\theta^\dagger$. Therefore, the expansion of a chiral superfield is just:

$$\Phi \;=\; \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y).$$

Exactly the correct degrees of freedom for a chiral supermultiplet!

Going back to the original x^μ, θ, $\theta^\dagger$ coordinates:

$$\begin{aligned}\Phi &= \phi(x) + i\theta^\dagger\overline{\sigma}^\mu\theta\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x)\\ &\quad -\frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\overline{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x),\end{aligned}$$

Now, using the $\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger$ superfield form of the SUSY transformation, you can check that:

$$\begin{aligned}\delta_\epsilon\phi &= \epsilon\psi,\\ \delta_\epsilon\psi_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F,\\ \delta_\epsilon F &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi,\end{aligned}$$

exactly agreeing with what was found in the component language.

More things you can check:

- Any analytic function of chiral superfields is also a chiral superfield.
- The complex conjugate of a chiral superfield, Φ^*, is an antichiral superfield, obeying $D_\alpha \Phi^* = 0$.
- If Φ is any chiral superfield, then $\int d^4x \int d^2\theta \, \Phi$ is invariant under a SUSY transformation (trivial: integration by parts!)

The usual Wess-Zumino model Lagrangian is:

$$\mathcal{L} = \int d^2\theta d^2\theta^\dagger \, \Phi^*\Phi + \left(\int d^2\theta \, W(\Phi) + \text{c.c.} \right)$$

The first term contains the kinetic terms, and W is the superpotential containing the masses and non-gauge interactions.

Four equivalent ways of writing the chiral supermultiplet kinetic term, called a "D-term":

$$\int d^2\theta d^2\theta^\dagger \, \Phi^*\Phi \;=\; \Phi^*\Phi\Big|_{\theta\theta\theta^\dagger\theta^\dagger} = \;-\frac{1}{4}\int d^2\theta \, \Phi\overline{DD}\Phi^* \;=\; [\Phi^*\Phi]_D$$

Three equivalent ways of writing the superpotential mass/interaction part, called an "F-term":

$$\int d^2\theta \, W(\Phi) \;=\; W(\Phi)\Big|_{\theta\theta} = \; [W(\Phi)]_F$$

Can use the same notations for non-renormalizable contributions to the Lagrangian:

$$\left[\Phi^*\Phi^2\right]_D \qquad \text{and} \qquad \left[\Phi^4\right]_F$$

For example, in the MSSM, the term

$$\frac{1}{M}\,[QQQL]_F$$

is a non-renormalizable superpotential interaction term that violates both baryon number and lepton number (but not R-parity!)

What about gauge fields and interactions?

Define a **vector superfield** by imposing a reality constraint on the general case:

$$V(x,\theta,\theta^\dagger) = \left[V(x,\theta,\theta^\dagger)\right]^*$$

The component field expansion for this is a special case of the general superfield:

$$\begin{aligned} V(x,\theta,\theta^\dagger) \quad = \quad & a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* + \theta^\dagger\overline{\sigma}^\mu\theta A_\mu \\ & +\theta^\dagger\theta^\dagger\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger) + \theta\theta\theta^\dagger(\lambda^\dagger - \frac{i}{2}\overline{\sigma}^\mu\partial_\mu\xi) \\ & +\theta\theta\theta^\dagger\theta^\dagger(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a). \end{aligned}$$

Here A_μ is the usual gauge field for a $U(1)$ gauge group, λ is the gaugino, D is the auxiliary field, which we've already met.

The other component fields a, ξ, b are **additional** auxiliary fields that have no dynamics and can be "gauged away".

The action is invariant under **supergauge transformations**:

$$V \rightarrow V + i(\Omega^* - \Omega)$$

where Ω is a chiral superfield gauge transformation parameter. In components:

$$\begin{aligned}
a &\rightarrow a + i(\phi^* - \phi), \\
\xi_\alpha &\rightarrow \xi_\alpha - i\sqrt{2}\psi_\alpha, \\
b &\rightarrow b - iF, \\
A_\mu &\rightarrow A_\mu + \partial_\mu(\phi + \phi^*), \\
\lambda_\alpha &\rightarrow \lambda_\alpha, \\
D &\rightarrow D.
\end{aligned}$$

If we use this freedom to get rid of a, ξ, b, then we are said to be in **Wess-Zumino gauge**, and:

$$V(x, \theta, \theta^\dagger) = \theta^\dagger \overline{\sigma}^\mu \theta A_\mu + \theta^\dagger \theta^\dagger \theta \lambda + \theta\theta\theta^\dagger \lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D.$$

Can still do ordinary gauge transformations parameterized by ϕ, while remaining in Wess-Zumino gauge.

There is also a **field-strength** chiral superfield, which contains the usual gauge field strength $F_{\mu\nu}$ as one of its components.
Define:

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V$$

Then can show that in Wess-Zumino gauge (up to total derivative terms):

$$\mathcal{L} = \frac{1}{4}\int d^2\theta\, \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{c.c.} = \frac{1}{2}D^2 + i\lambda^\dagger \overline{\sigma}^\mu \partial_\mu \lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

is the usual Lagrangian for the gauge field, gaugino, and auxiliary field D.

As before, this Lagrangian is manifestly invariant under the SUSY transformation (defined using $\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger$), which you can show just by integrating by parts in superspace.

To couple the gauge field to a chiral superfield with gauge charge q and gauge coupling g, just modify the kinetic term:

$$\mathcal{L} = \int d^2\theta d^2\theta^\dagger \; \Phi^* \boldsymbol{e^{2gqV}} \Phi.$$

This might look non-renormalizable, because the exponential has arbitrarily many terms in its expansion. But, in Wess-Zumino gauge, the exponential series soon terminates, because:

$$\begin{aligned} V^2 &= -\frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger A_\mu A^\mu, \\ V^n &= 0 \qquad\qquad (\text{for all } n \geq 3) \end{aligned}$$

Can also show that this Lagrangian is invariant under $U(1)$ gauge transformations:

$$\Phi \rightarrow e^{2igq\Omega}\Phi.$$

Non-abelian gauge fields require slightly more complicated expressions, but are conceptually very similar.

I won't go through the details, because I think we've hit (or perhaps greatly exceeded) the limit of what can be absorbed from slides in lectures like this, unless you've already seen this before.

Gauge-Mediated SUSY Breaking (GMSB) models

The idea: SUSY breaking is transmitted from a hidden sector by the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions. This makes them automatically flavor-blind!

To do this, introduce new, heavy, chiral supermultiplets, called **messengers**, which couple to $\langle F \rangle$ and also to the MSSM gauge bosons and gauginos.

If typical messenger particle masses are $M_{\rm mess}$, the MSSM soft terms are:

$$m_{\rm soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\rm mess}}$$

The $\alpha_a/4\pi$ is a one-loop factor for diagrams involving gauge interactions. This follows by dimensional analysis, since $m_{\rm soft}$ must vanish as $\langle F \rangle \to 0$, or as the messengers become very heavy.

Note that $\sqrt{\langle F \rangle}$ can be as low as 10^4 GeV, if $M_{\rm mess}$ is comparable.

This is much lower than in Planck-scale Mediated SUSY Breaking. So, these are also sometimes called "low-scale SUSY breaking" models.

GMSB models typically predict that the gravitino (which has absorbed the Goldstino) is the LSP. This is because:

$$m_{\widetilde{G}} \sim \frac{\langle F \rangle}{M_P} \quad \ll \quad m_{\rm soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\rm mess}}$$

provided that $M_{\rm mess} \ll M_P$. In fact, $m_{\widetilde{G}}$ can be as low as 0.1 eV, for $\sqrt{\langle F \rangle} \sim 10^4$ GeV.

The lightest of the MSSM superpartner states is often called the Next-to-Lightest Supersymmetric Particle (NLSP).

The NLSP need not be neutral, since it can decay into its Standard Model partner and the goldstino/gravitino.

Minimal Gauge-Mediated SUSY Breaking model

For a minimal model, take a set of new chiral supermultiplets q, $\overline{q}$, ℓ, $\overline{\ell}$ that transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3}); \quad \overline{q} \sim (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}); \quad \ell \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}); \quad \overline{\ell} \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2}).$$

These supermultiplets contain messenger quarks $\psi_q, \psi_{\overline{q}}$ and scalar quarks $q, \overline{q}$ and messenger leptons $\psi_\ell, \psi_{\overline{\ell}}$ and scalar leptons $\ell, \overline{\ell}$.

These particles get very large masses by coupling to a gauge-singlet chiral supermultiplet S through a superpotential:

$$W_{\rm mess} = y_2 S \ell \overline{\ell} + y_3 S q \overline{q}.$$

The scalar component of S and its auxiliary field are both assumed to acquire VEVs, denoted $\langle S \rangle$ and $\langle F_S \rangle$ respectively.

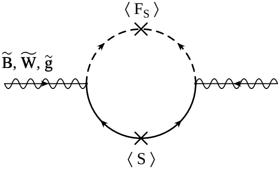
The chiral supermultiplet S might be composite, and $\langle F_S \rangle \neq 0$ might come from an O'Raifeartaigh model, or from some more complicated dynamical mechanism.

Minimal Gauge-Mediated SUSY Breaking model (continued)

The effect of SUSY breaking is to split the messenger masses:

$$\ell, \overline{\ell} : \qquad m^2_{\text{fermions}} = |y_2 \langle S \rangle|^2 , \qquad m^2_{\text{scalars}} = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle| ;$$
$$q, \overline{q} : \qquad m^2_{\text{fermions}} = |y_3 \langle S \rangle|^2 , \qquad m^2_{\text{scalars}} = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle| .$$

The SUSY-breaking apparent here is transmitted to the MSSM gauginos through one-loop graphs. The results are:

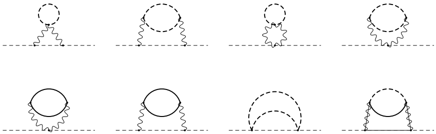


$$M_a = \frac{\alpha_a}{4\pi} \Lambda, \qquad \text{where} \qquad \Lambda \equiv \frac{\langle F_S \rangle}{\langle S \rangle}.$$

The MSSM gauge bosons do not get such a mass shift, since they are protected by gauge invariance. So SUSY breaking has been succesfully communicated to the MSSM.

Minimal Gauge-Mediated SUSY Breaking model (continued)

The MSSM scalars do not get any masses at 1-loop order, but do at 2-loops from these Feynman diagrams:



The result for each MSSM scalar ϕ can be written:

$$m_\phi^2 = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3^\phi + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2^\phi + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1^\phi \right], \qquad \text{where}$$

$$C_3^\phi = \begin{cases} 4/3 & \text{for } \phi = \widetilde{Q}_i, \widetilde{\bar{u}}_i, \widetilde{\bar{d}}_i; \\ 0 & \text{for } \phi = \widetilde{L}_i, \widetilde{\bar{e}}_i, H_u, H_d \end{cases}$$

$$C_2^\phi = \begin{cases} 3/4 & \text{for } \phi = \widetilde{Q}_i, \widetilde{L}_i, H_u, H_d; \\ 0 & \text{for } \phi = \widetilde{\bar{u}}_i, \widetilde{\bar{d}}_i, \widetilde{\bar{e}}_i \end{cases}$$

$$C_1^\phi = 3Y_\phi^2/5 \quad \text{for each } \phi \text{ with weak hypercharge } Y_\phi.$$

These squared masses are positive (fortunately!).

The Minimal GMSB model can be generalized by putting N identical copies of the messenger sector. The results then become:

$$M_a = \frac{\alpha_a}{4\pi} N\Lambda, \qquad \text{(gauginos)}$$
$$m_\phi^2 = 2N\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3^\phi + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2^\phi + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1^\phi \right], \qquad \text{(scalars)}$$

The parameters of this model framework are just:

- $N =$ number of messengers,
- $M_{\rm mess} =$ typical messenger mass scale,
- $\Lambda =$ effective SUSY-breaking order parameter
- μ, which can be traded for $\tan\beta$ and $\mathrm{sign}(\mu)$

These models can be further generalized by including more exotic messengers, perhaps with widely varying masses.

GMSB model predictions

The scalar^3 terms $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$, arise first at two-loop order, and are suppressed by an additional factor of $\alpha_a/4\pi$ compared to gaugino masses. So it is an excellent approximation to set them $= 0$.

Because gaugino masses arise at one loop, and scalar squared masses arise at two loops, they are roughly comparable:

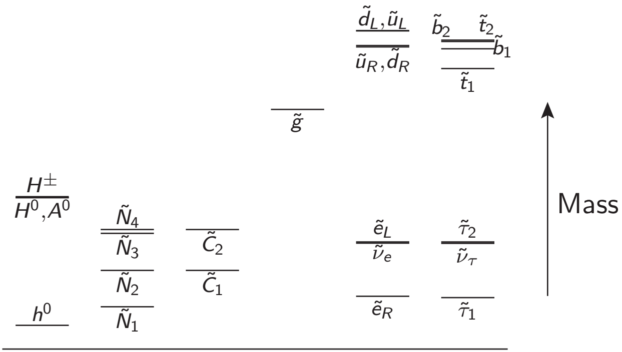
$$M_a, m_\phi \sim \frac{\alpha}{4\pi}\Lambda.$$

However, note that the gaugino masses scale like N, while the scalar masses scale like $\sqrt{N}$.

For $N = 1$, a bino-like neutralino will be the NLSP.
For $N \geq 2$, a stau will be the NLSP.

The above predictions for gaugino and scalar masses hold at the renormalization scale $Q_0 = M_{\rm mess}$. They must be run down to the electroweak scale. This generates non-zero $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$, and modifies the other predictions.

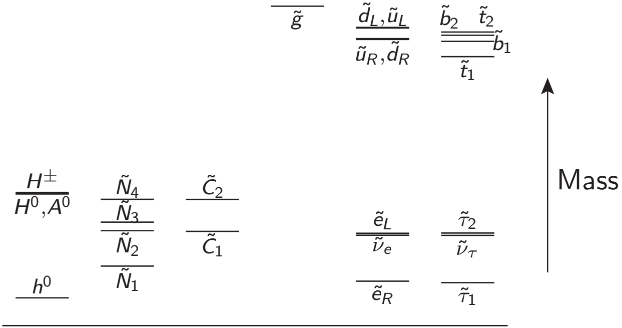
A sample sparticle mass spectrum for Minimal GMSB ($N = 1$)



The NLSP is a neutralino.

To get $M_h = 125$, the squarks and gluinos must be so heavy that they cannot be produced at the LHC. This can be avoided by introducing extra vectorlike quark supermultiplets.

A sample sparticle mass spectrum for non-minimal GMSB ($N = 3$)



The NLSP is a stau ($\tilde{\mu}_R$ and $\tilde{e}_R$ are not much heavier).

Anomaly Mediated SUSY Breaking

Soft terms are determined by the renormalization group quantities (beta functions and anomalous dimensions) as:

$$
\begin{aligned}
M_a &= (\beta_{g_a}/g_a) m_{3/2} \qquad \text{(gaugino masses)} \\
(m^2)_i^j &= -\tfrac{1}{2} \frac{d\gamma_i^j}{d(\ln Q)} m_{3/2}^2 \qquad \text{(scalar masses)}
\end{aligned}
$$

These are flavor-blind, to a good approximation, because they are dominated by gauge couplings.

Unfortunately, in the simplest version, the MSSM sleptons are predicted to have negative squared mass!

Perhaps the simplest fix for the tachyonic slepton problem in Anomaly Mediated SUSY Breaking is to simply add a common m_0^2 for each scalar.

Then the parameters of the model are just

- $m_{3/2}$ (AMSB SUSY breaking scale)
- m_0 (ad hoc scalar squared mass)
- μ or equivalently, $\tan\beta$ and sign(μ)

The most striking feature is the ratio of gaugino masses:

$$M_1 : M_2 : M_3 \; = \; 3.3 : 1 : 10,$$

so that the wino is the lightest gaugino.

The lightest superpartners could be either nearly degenerate winos with

$$m_{\tilde{N}_1}, m_{\tilde{C}_1^\pm} \;\approx\; M_2$$

or higgsinos with

$$m_{\tilde{N}_1}, m_{\tilde{N}_2}, m_{\tilde{C}_1^\pm} \;\approx\; \mu.$$

The mass difference between $\tilde{C}_1^+$ and $\tilde{N}_1$ could be hundreds of MeV, so that the dominant decay is to a soft pion:

$$\tilde{C}_1^+ \to \pi^+ \tilde{N}_1$$

The Lightest SUSY Particle as Cold Dark Matter

Experimental cosmology and astrophysics implies cold dark matter with density:

$$\Omega_{\text{CDM}} h^2 = 0.12 \qquad\qquad (\text{WMAP, Planck, } \ldots)$$

where $h \approx 0.7$ is the Hubble constant in units of 100 km/(sec Mpc).
A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.12$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

As a crude estimate, a weakly interacting particle that annihilates in collisions with a characteristic mass scale M will have

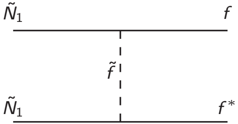
$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \Big(\frac{150 \text{ GeV}}{M} \Big)^2$$

So, a stable, weakly interacting particle with mass roughly of order the weak scale is a candidate. In particular, a neutralino LSP ($\tilde{N}_1$) may do it, if R-parity is conserved.

Contributions to annihilation cross section $\langle \sigma v \rangle$ for a neutralino LSP

To have a viable SUSY model with a $\tilde{N}_1$ LSP, it must not have too large a relic density Ωh^2, which means that $\langle \sigma v \rangle$ must not be too small. Let us examine the main contributions to the annihilation:

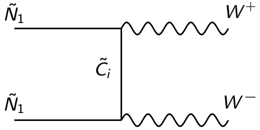
1) Annihilation through t-channel slepton and squark exchange.



When $\tilde{N}_1$ is mostly bino, as in many mSUGRA models, this is often the dominant process. However, to be efficient enough, the slepton masses ($m_{\tilde{e}_R}$, $m_{\tilde{\mu}_R}$, and $m_{\tilde{\tau}_1}$) must not be too large.

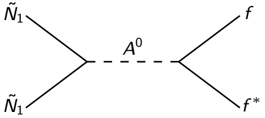
In fact, for slepton masses > 100 GeV that are not ruled out by the old LEP2 e^+e^- collider, the cross-section is usually too small, and so Ωh^2 typically comes out much too large.

2) Annihilation through t-channel chargino exchange.



This diagram can provide sufficient annihilation of mostly-bino LSPs even if all squarks and sleptons are heavy, but relies on a significant higgsino or wino content of $\tilde{N}_1$. Direct searches for dark matter strongly constrain this.

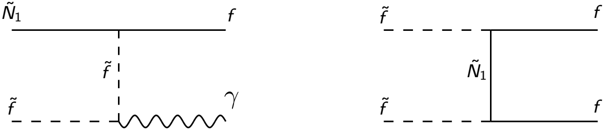
3) Resonant annihilation through an s-channel neutral Higgs boson.



This process is s-wave for the pseudo-scalar Higgs A^0, and p-wave suppressed for h^0 and H^0. Very efficient near resonance $m_{\tilde{N}_1} \approx m_{A^0}/2$, especially for large $\tan\beta$, because the $A^0 b\overline{b}$ coupling is proportional to $m_b \tan\beta$.

4) Co-annihilation of neutralinos with sleptons (or top squarks).

If one or more sleptons is only slightly heavier that $\widetilde{N}_1$, then they will coexist in thermal equilibrium in the early universe. This can increase the efficiency of annihilation in Standard Model states, for example through $\tilde{N}_1 \tilde{f} \to f\gamma$ and $\tilde{f}\tilde{f} \to ff$:



This co-annihilation effect is only important if significant numbers of $\tilde{f}$ are in thermal equilibrium with $\tilde{N}_1$, which requires $(m_{\tilde{f}} - m_{\tilde{N}_1})/m_{\tilde{N}_1} \lesssim 1/20$.

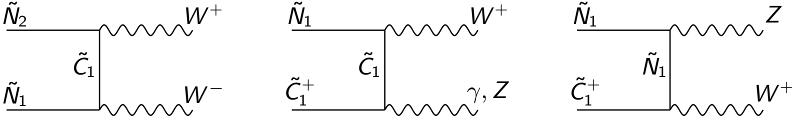
This is a motivation for the "compressed" superpartner mass spectrum, with the LSP not much lighter than other superpartners.

5) Co-annihilation of higgsino-like or wino-like multiplets

Nearly pure winos, $\tilde{N}_1, \tilde{C}_1^\pm$ with $\Delta M > 160$ MeV, can exist together in thermal equilibrium in early universe.

Same for nearly pure higgsinos $\tilde{N}_1, \tilde{N}_2, \tilde{C}_1^\pm$ with $\Delta M >$ few hundred MeV.

Then co-annihilation processes like these can efficiently annihilate them:



To get $\Omega h^2 = 0.12$, need:

- $M_{\tilde{H}} \approx \mu \approx 1.1$ TeV for higgsinos

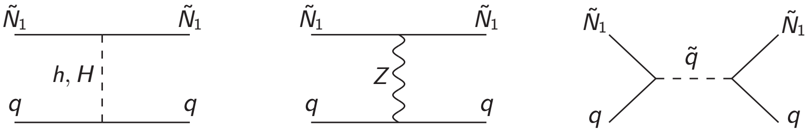
  or

- $M_{\tilde{W}} \approx M_2 \approx 2.8$ TeV for winos.

For smaller masses, annihilation is more efficient and the thermal source for dark matter is not enough.

Direct Detection of $\tilde{N}_1$ Dark Matter LSPs

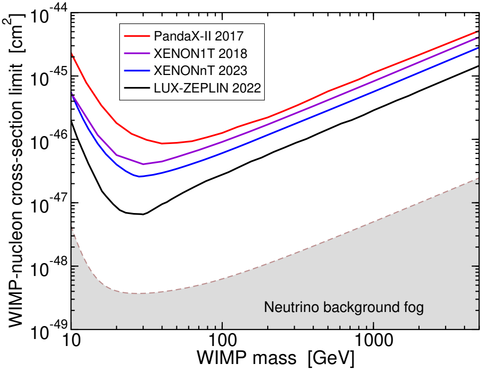
Dark matter neutralinos moving through a detector can recoil from nuclei:



The suppression due to small quark Yukawa couplings to the Higgs scalars in the first diagram can be overcome by the coherent effect of many nucleons. Proportional to atomic weight, $A = 131$ for a Xenon target.

Typical recoil energies are only $E \sim 100$ keV. The predicted event rates are very low (a few per kilogram of detector per day, or less). This depends on the mixing matrix of $\tilde{N}_1$, and also on the local density and velocity distribution of dark matter, which is not perfectly known.

Strongest current limits on neutralino dark matter from the LUX-ZEPLIN experiment, 2207.03764:



For $m_{\rm LSP} > 100$ GeV, the bound on the spin-independent cross-section is

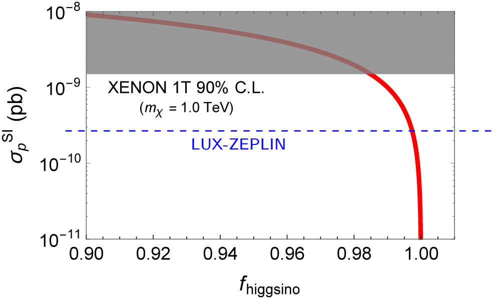
$$\sigma_{\rm SI} \;<\; 2.7 \times 10^{-10}\,\text{pb}\,\left(\frac{m_{\rm LSP}}{1000\,\rm GeV}\right)$$

Eventually, the experiments will hit the "neutrino fog", where astrophysical neutrino backgrounds greatly reduce the sensitivity to dark matter.

Implications of dark matter direct detection

Mixed gaugino-higgsino LSPs are often ruled out, but holes exist in parameter space where destructive interference means a loss of sensitivity.

Nearly pure higgsino (mass 1.1 TeV) or wino (mass 2.8 TeV) can escape detection if the mixing is small enough. For example:



Kowalska and Sessolo, 1802.04097

f_{higgsino} = higgsino content of LSP

Recall: nearly pure higgsinos have $\tilde{N}_1$, $\tilde{N}_2$, $\tilde{C}_1^{\pm}$ separated by few hundred MeV. Nearly pure winos similarly have $\tilde{N}_1$, $\tilde{C}_1^{\pm}$ separated by 160 MeV.

Difficult challenges for detection at the LHC!

Indirect detection of Dark Matter LSPs

Neutralino LSPs in the centers of the Sun and the Galaxy can annihilate to Standard Model particles with high energies, which can then be seen directly or indirectly.

$$\tilde{N}_1\tilde{N}_1 \quad\rightarrow\quad e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-,\ \nu\overline{\nu},\ q\bar{q},\ ZZ,\ W^+W^-$$

For example, ν_μ produced (either directly or indirectly) in $\tilde{N}_1\tilde{N}_1$ annihilation can travel to Earth and then undergo a charged current interaction leading to detection of upward-going muons. Neutrino telescopes are indirect dark matter detectors.

A striking signal: one-loop diagrams give

$$\tilde{N}_1\tilde{N}_1 \rightarrow \gamma\gamma,\ Z\gamma$$

Monochromatic photon lines have $E_\gamma = m_{\tilde{N}_1}$. Winos with masses $\lesssim$ 3 TeV ruled out by H.E.S.S. and Fermi, but only **if** density near galactic core is large. Cherenkov Telescope Array will have even better sensitivity to photons from dark matter annihilation.

Why no MSSM mass spectrum is "excluded" by dark matter

- Too much LSP dark matter ($\Omega h^2 > 0.12$)? No problem!
  - Introduce a lighter singlet $\tilde{S}$, so that $\tilde{N}_1 \to S f \overline{f}$. Then: $\Omega_{\tilde{S}} = \frac{m_{\tilde{S}}}{m_{\tilde{N}}} \Omega_{\tilde{N}}$. Also, $\tilde{S}$ could have very small couplings, avoiding direct and indirect searches.
  - Entropy from some other particle(s) decaying late, but only to Standard Model states, not $\tilde{N}_1$.
- Too little LSP dark matter ($\Omega h^2 < 0.12$)? No problem!
  - Non-thermal sources (for example, late particle decays) for $\tilde{N}_1$
  - Dark matter is something else (axions!)

SUSY at colliders

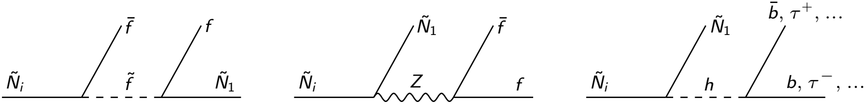
- The most important interactions for producing sparticles are gauge interactions, and interactions related to gauge interactions by SUSY.
- The production rate is known, up to mixing of sparticles, because SUSY predicts the dimensionless couplings.
- The LSPs are neutral and extremely weakly interacting, so they carry away energy and momentum, if R-parity conserved.
- At hadron colliders, the component of the momentum along the beam is unknown, so only the energy component in particles transverse to the beam is observable. So one may look for "missing transverse energy", E_T^{miss}.

Superpartner decays:

1) Neutralino decays
2) Chargino decays
3) Gluino decays
4) Squark decays (especially stops)
5) Slepton decays
6) Decays to the goldstino/gravitino

1) Neutralino Decays

If R-parity is conserved and $\tilde{N}_1$ is the LSP, then it cannot decay. For the others, the decays are of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, Z boson, or Higgs boson h) might be on-shell, if that two-body decay is kinematically allowed.

The visible decays are generally either:

$$\tilde{N}_i \rightarrow q\bar{q}\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$
$$\tilde{N}_i \rightarrow \ell^+\ell^-\tilde{N}_1 \qquad \text{(seen in detector as } \ell^+\ell^- + \not{E}\text{)}$$

Some SUSY signals rely on leptons in the final state. This is more likely if sleptons are relatively light.

If $\tilde{N}_i \rightarrow \tilde{N}_1 h$ is kinematically open, then it often dominates. Historically, called the "spoiler mode", because it spoils the leptonic signals. But, recently experimentalists have learned to exploit this!

2) Chargino Decays

Charginos $\tilde{C}_i$ also have decays of weak-interaction strength:

Again, the intermediate boson (squark or slepton $\tilde{f}$, or W boson) might be on-shell, if that two-body decay is kinematically allowed.

Chargino decays are typically either:

$$\tilde{C}_i^{\pm} \to q\bar{q}'\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$
$$\tilde{C}_i^{\pm} \to \ell^{\pm}\nu\tilde{N}_1 \qquad \text{(seen in detector as } \ell^{\pm} + \not{E}\text{)}$$

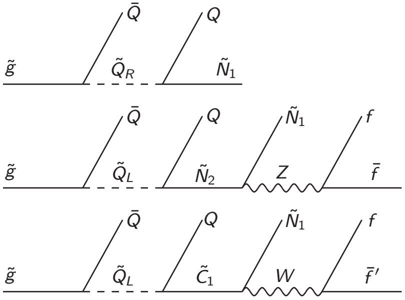
Again, leptons in final state are more likely if sleptons are relatively light. For both neutralinos and charginos, a relatively light, mixed $\tilde{\tau}_1$ can lead to enhanced τ's in the final state. This is increasingly important for larger $\tan\beta$. Tau identification a crucial limiting factor for experimental SUSY?

3) Gluino Decays

The gluino can only decay through squarks, either on-shell or virtual.

If $m_{\tilde{t}_1} \ll$ other squark masses, top quarks are plentiful in these decays.

For example:



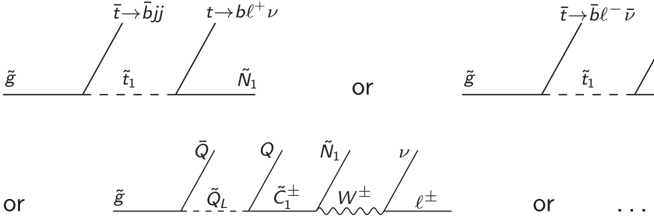
$jj + \not{E}$ or $t\bar{t} + \not{E}$

$jjjj + \not{E}$ or $jj\ell^+\ell^- + \not{E}$ or
$t\bar{t}jj + \not{E}$ or $t\bar{t}\ell^+\ell^- + \not{E}$

$jjjj + \not{E}$ or $jj\ell^\pm + \not{E}$ or
$t\bar{t}jj + \not{E}$ or $t\bar{t}\ell^\pm + \not{E}$

The possible signatures of gluinos and squarks can be numerous and complicated because of these and other **cascade decays**.

An important feature of gluino decays with one lepton, for example:



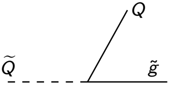
The lepton has either charge with equal probability. (The gluino does not "know" about electric charge.) So, when two gluinos are produced, probability 0.5 to have opposite-charge leptons, and probability 0.5 to have **same-charge leptons**.

$$(\text{SUSY}) \to \ell^\pm \ell'^\pm + \text{ jets } + E_T^{\text{miss}}$$

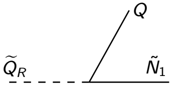
Same-charge lepton signals can be important at the LHC, because Standard Model backgrounds are much smaller. Note lepton flavors are uncorrelated. SUSY events may also have 2 or 4 taggable b jets.

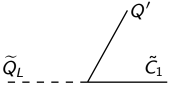
4) Squark Decays

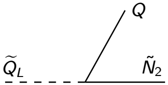
If a decay $\tilde{Q} \to Q\tilde{g}$ is kinematically allowed, it will always dominate, because the squark-quark-gluino vertex has QCD strength:



Otherwise, right-handed squarks prefer to decay directly to a bino-like LSP, while left-handed squarks prefer to decay to a wino-like $\tilde{C}_1$ or $\tilde{N}_2$:



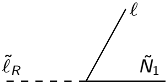




If a top squark is light, then the decays $\tilde{t}_1 \to t\tilde{g}$ and $\tilde{t}_1 \to t\tilde{N}_1$ may not be kinematically allowed, so it may decay only into charginos: $\tilde{t}_1 \to b\tilde{C}_1$. If all those decays are closed, then $\tilde{t}_1 \to bW\tilde{N}_1$. If even that is closed, it has only a suppressed flavor-changing decay $\tilde{t}_1 \to c\tilde{N}_1$ or 4-body decay $\tilde{t}_1 \to bf\bar{f}'\tilde{N}_1$.

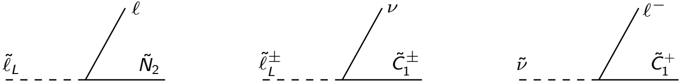
5) Slepton Decays

When $\tilde{N}_1$ is the LSP and mostly large bino, the sleptons $\tilde{e}_R$, $\tilde{\mu}_R$ (and often $\tilde{\tau}_1$ and $\tilde{\tau}_2$) prefer the direct two-body decays with strength proportional to g'^2:



(seen in detector as $\ell^\pm + \not{E}$)

However, the left-handed sleptons $\tilde{e}_L$, $\tilde{\mu}_L$, $\tilde{\nu}$ have no coupling to the bino component of $\tilde{N}_1$, so they often decay preferentially through mostly wino $\tilde{N}_2$ or $\tilde{C}_1$, with strength proportional to g^2:



with $\tilde{N}_2$ and $\tilde{C}_1$ decaying as before.

6) NLSP decays in Gauge-Mediated SUSY Breaking

Recall that GMSB models have the special property that the LSP is a very light Goldstino/gravitino ($\tilde{G}$). The Next-Lightest SUSY Particle (NLSP) can decay into its Standard Model partner and $\tilde{G}$.

This can completely change the SUSY signals at colliders!

In general, the NSLP can have a decay length that is microscopic, comparable to detector elements, or macroscopic:

$$\Gamma(\mathrm{NLSP} \to \mathrm{SM\,particle} + \tilde{\mathrm{G}}) \;=\; \kappa \left(\frac{\mathrm{m_{NLSP}}}{100\,\mathrm{GeV}}\right)^5 \left(\frac{\sqrt{\langle \mathrm{F} \rangle}}{100\,\mathrm{TeV}}\right)^{-4} \; 2 \times 10^{-3}\,\mathrm{eV}$$

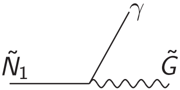
where κ is a mixing matrix factor. If the NLSP has energy E in the lab frame, its decay length will be:

$$d = \left(\frac{E^2}{m^2_{\mathrm{NLSP}}} - 1\right)^{1/2} \left(\frac{m_{\mathrm{NLSP}}}{100\,\mathrm{GeV}}\right)^{-5} \left(\frac{\sqrt{\langle F \rangle}}{100\,\mathrm{TeV}}\right)^{4} \; 9.9 \times 10^{-3}\,\mathrm{cm}\,\frac{1}{\kappa}$$

which can be anywhere from sub-micron to kilometers, depending on $\langle F \rangle$.

Neutralino NLSP in Gauge-Mediated SUSY Breaking

If the NLSP is $\tilde{N}_1$, it can have decays:



There are three general possibilities:

1) If the $\tilde{N}_1$ decays promptly, then every SUSY event has two additional **energetic, isolated photons**. There is still missing energy carried away by the $\tilde{G}$. Standard Model backgrounds are very small, so it is relatively easy to discover SUSY with the inclusive signal (X means "anything")

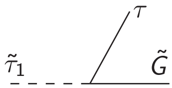
$$X + \gamma\gamma + \not{E}.$$

2) If the $\tilde{N}_1$ decays are delayed, but still occur within the detector, then one can look for photons that do not point back to the interaction vertex. This can be a striking signal, depending on the experimental environment.

3) If the $\tilde{N}_1$ decays occur outside of the detector, then the signals are the same as discussed earlier.

Stau NLSP in Gauge-Mediated SUSY Breaking

If the NLSP is the lightest stau, $\tilde{\tau}_1$, then it can have decays:

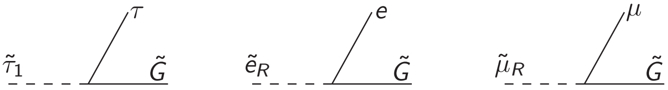


1) If the $\tilde{\tau}_1$ decays are prompt, then every SUSY event will be tagged by two **energetic, isolated τ's**.

2) If the $\tilde{\tau}_1$ decays occur outside of the detector, then one can look for slow, highly ionizing tracks as they move through the detector. These may appear to be slow "muons", or they may be missed if the timing gates do not accomodate them. They can be identified by their anomalously high ionization rate in the detector, or by their long time-of-flight.

Slepton co-NLSP in Gauge-Mediated SUSY Breaking

In GMSB models, it can happen that $\tilde{\tau}_1$, $\tilde{e}_R$, $\tilde{\mu}_R$ are degenerate to within less than $m_\tau = 1.8$ GeV. In that case, SUSY particles will decay to final states involving one of them, and they each act as the NLSP, with decays:

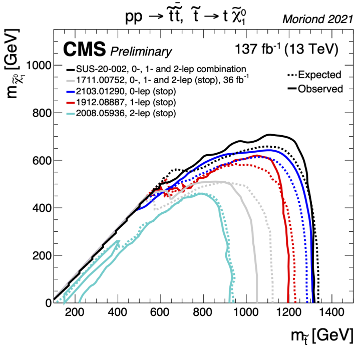


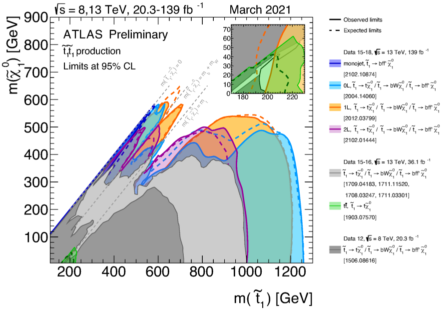
1) If the NLSP decays are prompt, then every SUSY event will be tagged by two **energetic, isolated leptons** (e, μ, τ) with uncorrelated flavors, and often uncorrelated charges..

2) If the NLSP decays occur outside of the detector, then one can look for slow, highly ionizing tracks, just as for the stau NLSP case.

The LHC vs. Supersymmetric Models

Constraints on SUSY are often colloquially overstated, perhaps due to temptation to make grand statements.

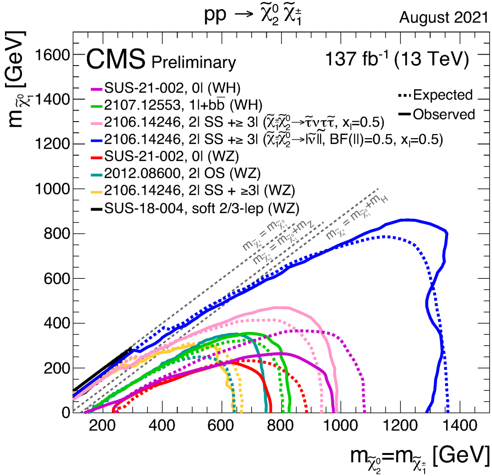




Pessimist says: Exclusion of top-squark masses now up to 1300 GeV!

Optimist says: No constraints at all on direct top-squark pair production, if LSP mass exceeds 700 GeV. (Includes compressed, dark-matter favored models.)

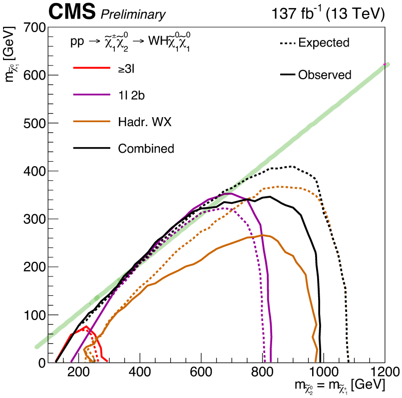
Constraints on wino-like charginos and neutralinos that decay through sleptons: $pp \to \tilde{C}_1 \tilde{N}_2 \to$ leptons $+ \not{E}_T$



Pessimist: Exclusion of electroweakinos above 1300 GeV!

Optimist: Models with decays through staus have much weaker constraints: no exclusion for $m_{\tilde{C}_1} > 1000$ GeV or LSP mass > 450 GeV. Furthermore. . .

Constraints on wino-like charginos and neutralinos decaying through W, h:



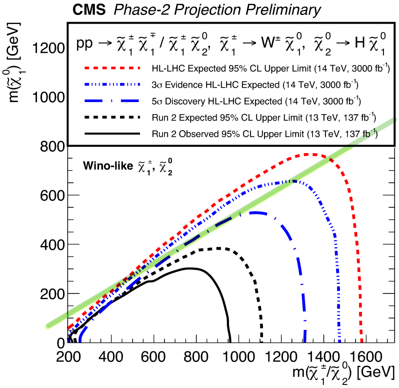
The decay $\tilde{N}_2 \to h\tilde{N}_1$ dominates in SUSY models. In older papers this was known as the "spoiler mode", but after improvements in the last few years, it now gives the best reach!

Green line = prediction from gaugino mass unification

No exclusion for $m_{\tilde{C}_1} > 1000$ GeV or $m_{\tilde{N}_1} > 350$ GeV.

Bounds will be weaker if you take into account $\mathrm{BR}(\tilde{N}_2 \to h\tilde{N}_1) < 1$ and t-channel u, d squark exchange, even if squark masses are ≥ 2 TeV.

Projections for HL-LHC with $\sqrt{s} = 14$ TeV and 3000 fb^{-1}, for wino-like charginos and neutralinos decaying through W, h:

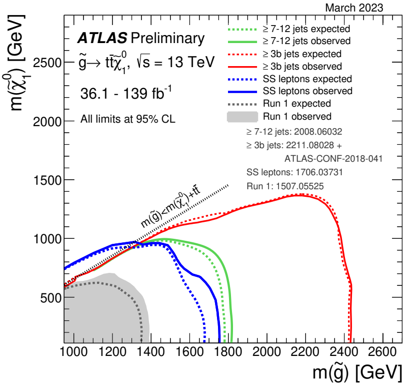


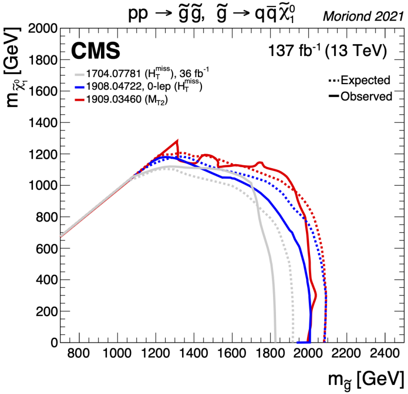
From *Snowmass White Paper Contribution: Physics with the Phase-2 ATLAS and CMS Detectors*

At least according to this projection, the expected 5σ exclusion almost coincides with the gaugino mass unification line. Can one do better?

For 95% exclusion, can go to equivalent of perhaps 4500-5000 GeV in gluino mass, for models with gaugino mass unification. This is far beyond what can be done for direct gluino pair searches at HL-LHC.

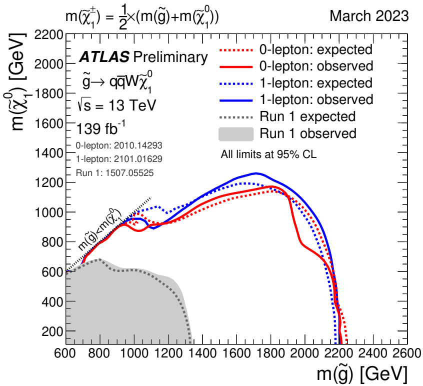
Constraints on gluino pair production tend to have the highest reach:

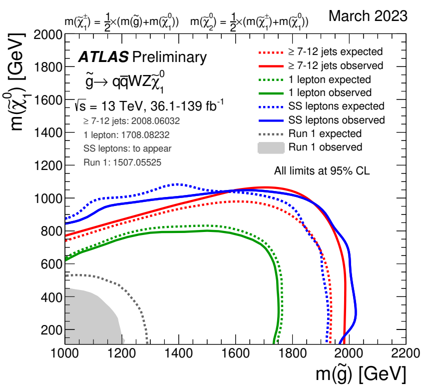




But again, there is no exclusion if the gluino-LSP mass difference is not large. For example, $M_{\tilde{g}} = 1500$ GeV, $M_{\tilde{N}_1} = 1200$ GeV is alive, at least for now. "Compressed SUSY" models with small mass differences are more difficult because visible energy in each event is smaller.

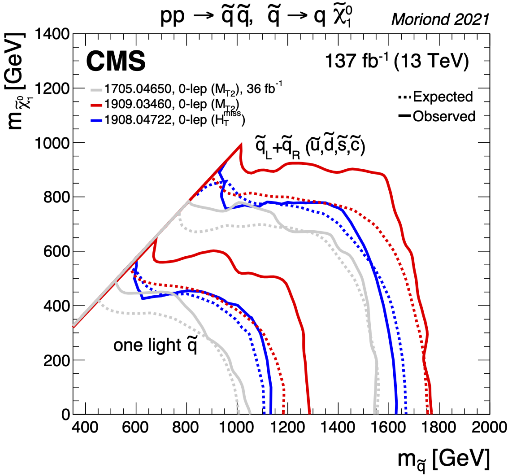
Cascade decays: gluino exclusions can be somewhat weaker if more steps.





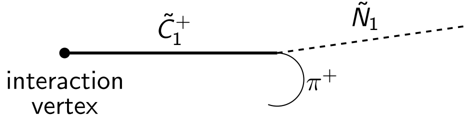
A general lesson: "Simplified SUSY models" are not actually SUSY!
In real SUSY models, things are more complicated.

Squark limits are weaker than for gluino:



Note: these are simplified SUSY models. In real SUSY models, squarks don't always decay as assumed here! (Especially true for $\tilde{q}_L$, which couple to winos.) Exclusions could be stronger or weaker.

Disappearing track signal for nearly degenerate higgsinos or winos



The distance traveled by the chargino is macroscopic, perhaps of order millimeters or centimeters. The pion is very soft, so curls up in the magnetic field and is not detected.

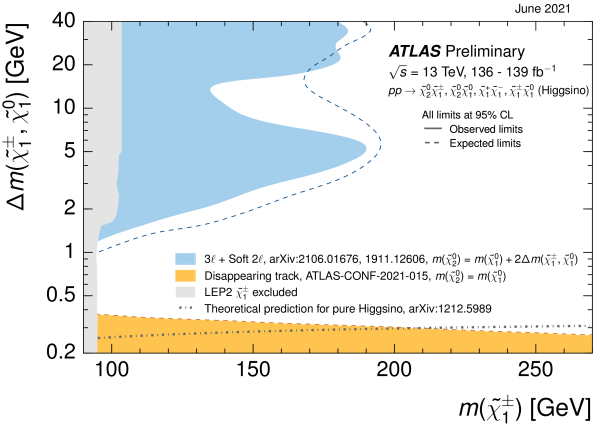
The mass reach is much lower for higgsinos than for winos because:

- track is shorter,
- production cross-section is much smaller.

Longer $\tilde{C}_1$ tracks can also be detected by measuring the anomalous energy deposited per distance traveled in the detector (dE/dx).

Nearly degenerate higgsino-like $\tilde{C}_1^{\pm}$, $\tilde{N}_2$, and $\tilde{N}_1$

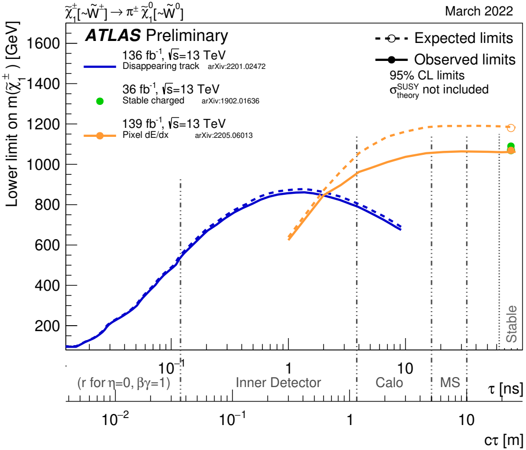
Look for soft leptons if mass difference is large enough, otherwise disappearing tracks if the mass difference is smaller than few hundred MeV. This includes the limit of pure Higgsino.



Exclusion presently limited to 210 GeV. Recall higgsino-like thermal dark matter has $m_{\tilde{N}_1} \approx 1.1$ TeV.

Nearly degenerate wino-like $\tilde{C}_1^{\pm}$ and $\tilde{N}_1$

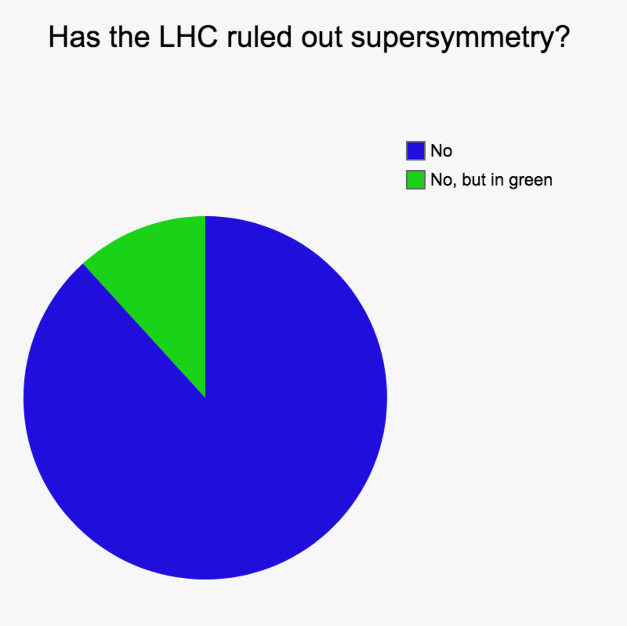
Can use both disappearing track, and anomalous energy deposition dE/dx for longer tracks.



Recall wino-like thermal dark matter has $m_{\tilde{N}_1} \approx 2.8$ TeV.

Summarizing important things to keep in mind when looking at LHC limits

- Compressed mass spectra (small mass splittings) give weaker limits, or no limit
- "Simplified SUSY models" are not SUSY
- Cascade decays can give weaker limits
- Decays through τ leptons give weaker limits
- Mass reach for squarks and gluinos will increase slowly with more LHC data in Run 3. Cross-section falls quickly with large masses because of parton distribution functions.
- Mass reach for charginos, neutralinos, sleptons still have some room to grow at LHC
- Dark matter motivates nearly degenerate and nearly pure higgsinos or winos, but with masses well beyond the reach of LHC


Has the LHC ruled out supersymmetry?
No
No, but in green

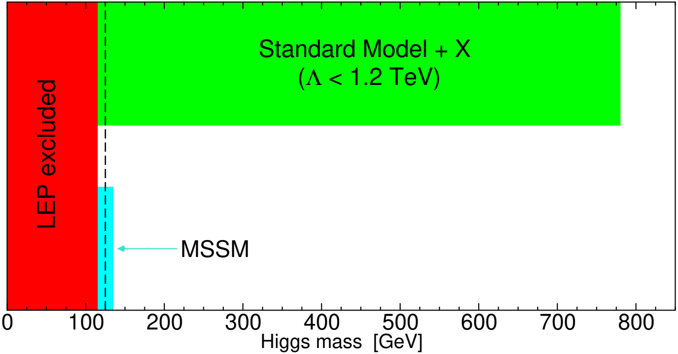
Question: **Can** the LHC rule out supersymmetry?

Answer: **No.** Supersymmetry is an example (one of many!) of a decoupling theory; the more you raise the masses of the new particles, the better it agrees with the Standard Model.

Actually, there is one exception to decoupling in SUSY: it was predicted in the 1990's that the Higgs boson had to be light ($M_h \lesssim 135$ GeV) in SUSY. When we discovered the Higgs with mass 125 GeV in 2012, we lost the last chance to rule out SUSY.

The LHC had a real chance to rule out SUSY, but it failed to do so!

The success of the Higgs scalar boson mass in SUSY, illustrated:



Once we fix the weak scale, a generic theory beyond the Standard Model could have had a Higgs scalar boson mass anywhere up to nearly 800 GeV, consistent with unitarity. It turned out to be in the range predicted by SUSY.

Despite strong limits, my personal bias is that the case for SUSY as the solution to the Big Hierarchy problem:

Why is $M_W^2 \ll M^2_{\rm Planck},\ M^2_{\rm GUT},\ M^2_{\rm seesaw},\ f^2_{\rm axion},\ \ldots$?

is about as strong as ever.

None of the competitor theories to explain the Big Hierarchy problem are being discovered at LHC either! And many are in worse shape than SUSY is, or are now completely dead (technicolor, top-quark condensate models, chiral quarks and leptons. . .).

If you don't like the fact that SUSY cannot be ruled out by the LHC, remember that **all** theories of physics beyond the Standard Model that remain alive after the LHC are **also** decoupling theories.

Why did we think superpartners should be light?

Minimizing the Higgs potential, we find:

$$M_Z^2 = -2(|\mu|^2 + m_{H_u}^2) + \mathcal{O}(1/\tan^2\beta) + \text{loop corrections}$$

So avoiding fine-tuning suggests that **Higgsinos should be light:** $\mu^2 \sim -m_{H_u}^2 \sim M_Z^2$.

Other superpartners should be light only if their masses are correlated with, or feed into, $m_{H_u}^2$.

Corrections from loop diagrams give:

$$\Delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2}(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\ln(\Lambda/\text{TeV}) - \frac{\alpha_S y_t^2}{\pi^3} M_{\tilde{g}}^2 \ln^2(\Lambda/\text{TeV}) + \text{small}.$$

So the **top squarks and gluino should also not be too heavy**.

But, "not too heavy" is a notoriously fuzzy statement.

My view on fine-tuning and naturalness:

- There is no way of objectively defining, let alone measuring, "fine-tuning", or "naturalness".
- Naturalness is personal and subjective, rather than scientific.
- **However**, naturalness is useful, and even crucial, for scientists. We are constantly making practical decisions about which research directions to pursue, given finite time and money.

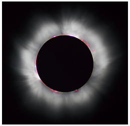
How bad is the problem, really? (opinion, not science!)

If all superpartners have masses 3-10 TeV, then they can easily explain $M_h = 125$ GeV and decouple from flavor violating effects.

We then need $m^2_{H_u}$ fine-tuned to be $-|\mu|^2$, in order to get M_Z^2 correct. Subjectively, tuning is of order 1 part in 10^2 to 10^4.

Small numbers sometimes do happen in Nature for no obvious reason!

- Electron Yukawa coupling is 3×10^{-6}. Why?
- Eclipses happen. Why?



$$\frac{R_{\rm sun}}{D_{\rm sun}} = \frac{6.955 \times 10^8 \text{ m}}{(1.496 \pm 0.025) \times 10^{11} \text{ m}} = 0.00465 \pm 0.00008$$

$$\frac{R_{\rm moon}}{D_{\rm moon}} = \frac{1.738 \times 10^6 \text{ m}}{(3.844 \pm 0.214) \times 10^8 \text{ m}} = 0.00452 \pm 0.00028$$

So maybe SUSY particles exist, and solve the big hierarchy problem, but will be somewhat beyond the LHC reach.

Another personal bias: SUSY is likely to be non-minimal; not just the MSSM. What follows is my own personal Top 7 list.

(Not including the obvious, a SUSY breaking sector.)

The first three involve adding a singlet chiral superfield in different ways. . .

1) **The NMSSM (Next-to-Minimal SSM)**: Add a singlet chiral superfield S

$$W = \lambda S H_u H_d + \dots$$

The scalar component of S gets a VEV of order $m_{\rm soft}$, and then:

$$\mu = \lambda \langle S \rangle \sim m_{\rm soft}.$$

Get an extra singlino fermion (could be dark matter, hard to see in direct detection experiments!) and singlet scalars mix with the Higgs.

2) **Kim-Nilles mechanism:** Add a singlet S with a non-renormalizable coupling

$$W = \frac{\lambda}{M_P} S^2 H_u H_d + \dots$$

Now, $\langle S \rangle \sim \sqrt{m_{\rm soft} M_P} \sim 10^{11}$ GeV, and

$$\mu = \frac{\lambda \langle S \rangle^2}{M_P} \sim m_{\rm soft}$$

Still get a TeV-scale singlino. Bonus: if S carries a Peccei-Quinn charge, get an invisible axion, solving the strong CP problem, and providing dark matter.

3) **Giudice-Masiero mechanism:** couple the singlet to the Higgs through the kinetic term (Kahler potential):

$$\mathcal{L} = \int d^2\theta d^2\theta^\dagger \left[H_u^* H_u + H_d^* H_d + \frac{\lambda}{M_P} S^* H_u H_d + \ldots \right]$$

This time,

$$\mu = \frac{\lambda}{M_P} \langle F_S \rangle \sim m_{\rm soft}$$

provided that

$$\langle F_S \rangle \sim m_{\rm soft} M_P \sim 10^{11}\,\mathrm{GeV}.$$

This is the same order of magnitude for the F-term VEV as is needed to explain the usual MSSM soft terms.

4) **Dirac gauginos:** chiral supermultiplets in adjoint representation.
For example, the usual gluino $\tilde{g}$ can mix with a color octet fermion $\tilde{g}'$:

$$\mathcal{L} = -\begin{pmatrix} \tilde{g} & \tilde{g}' \end{pmatrix} \begin{pmatrix} M_3 & M_D \\ M_D & \mathcal{M} \end{pmatrix} \begin{pmatrix} \tilde{g} \\ \tilde{g}' \end{pmatrix} + \text{c.c.}$$

- If $M_3 = \mathcal{M} = 0$, the gauginos are pure Dirac.
- If $M_D = 0$, the gauginos are pure Majorana.
- Otherwise, mixed Dirac/Majorana.

Rich phenomenology, including suppression of production cross-sections.
Can be motivated theoretically in different ways, including "supersoft symmetry breaking".

5) **Vectorlike quarks and leptons:** chiral supermultiplets in real ("vectorlike") representation of gauge group.

$$\begin{aligned} \mathcal{Q} + \overline{\mathcal{Q}} &= (\mathbf{3}, \mathbf{2}, 1/6) + (\overline{\mathbf{3}}, \mathbf{2}, 1/6), \\ \mathcal{U} + \overline{\mathcal{U}} &= (\mathbf{3}, \mathbf{1}, 2/3) + (\overline{\mathbf{3}}, \mathbf{1}, -2/3), \\ & \ldots \end{aligned}$$

As their masses are raised, decouple from low-energy physics, <u>except</u> for M_h.

$$W = \lambda H_u \mathcal{Q}\overline{\mathcal{U}} + M_{\mathcal{Q}} \overline{\mathcal{Q}}\mathcal{Q} + M_{\mathcal{U}} \overline{\mathcal{U}}\mathcal{U}$$

gives a correction to the lightest Higgs boson mass:

$$\Delta M_h^2 = \frac{3}{4\pi^2} \lambda^4 v^2 \left[\ln(M_S^2/M_F^2) - 5/6 + M_F^2/M_S^2 \right],$$

where M_S and M_F are vectorlike squark, quark masses.

Vectorlike quarks are easy to search for, vectorlike leptons are much more of a challenge, especially if they are $SU(2)_L$ singlets.

6) **A fixed point for the weak scale?**

Define: $m^2 \equiv |\mu|^2 + m^2_{H_u}$, determines the Higgs VEV.

Can we drive it towards 0, as a power law, due to some strong conformal scaling dynamics?

$$\beta_{m^2} = Q \frac{\partial}{\partial Q} m^2 = K m^2$$

where Q is the renormalization scale and K is a not-too-small constant.

This implies:

$$m^2(Q) \, = \, \left(\frac{Q}{Q_0} \right)^K m^2(Q_0) \, \rightarrow \, 0$$

Then we could have a natural explanation for $m_W \ll m_{\rm soft}$.

Not clear to me if this can really work, but enticing. Some literature:
Roy, Schmaltz 0708.3593, Murayama, Nomura, Poland 0709.0775
Perez, Roy, Schmaltz 0811.3206, Knapen, Shih 1311.7107,
SPM, 1712.05806.

7) **Something that nobody has thought of yet...**

This is the most exciting, and most likely, possibility!

Thank you for your attention.