**Physics 751** **Homework 8** **Due Monday, April 8, 2019**

**Reading assignment: sections 8.1, 8.2, 8.3 of Carroll.**

Problem 1. Consider an observer falling on a radial path into a Schwarzschild black hole of mass $M$. The observer starts at rest from infinity, so $E = 1$. Meanwhile, an emitter at constant $r = \infty$ is shining a laser beam with photons of frequency $\omega_\infty$ along the same radial direction in towards the doomed observer. Let $\omega_{\rm obs}$ be the angular frequency of the light as seen by the observer. Find the ratio $Z = \omega_{\rm obs}/\omega_\infty$ as a function of the observer's radial coordinate $r$. Make a numerical table of $Z$ for the values $r/2GM = 1000, 100, 10, 2, 1, 0.5, 0.1, 0.01, 0.001$. [Hint: we did a somewhat similar example in class, where the roles of emitter and observer were reversed, and outgoing Eddington-Finkelstein coordinates $(u, r, \theta, \phi)$ were useful, because outgoing radial photons have $u$ =constant. In the present problem, ingoing Eddington-Finkelstein coordinates $(v, r, \theta, \phi)$ are useful, because ingoing radial photons have $v$ =constant.]

Problem 2. Considering the Reissner-Nordstrom metric eq. (6.53), find: the Christoffel symbols, the non-vanishing Riemann tensor components, the Ricci tensor, and the Einstein tensor. From this, use Einstein's equations to find the stress-energy tensor, and show that it is non-zero and equal to the result for a point charge. [Hint: you will want to make use of eqs. (5.38)-(5.41).]

Problem 3. For the case of the Reissner-Nordstrom metric eq. (6.53), compute the Komar energy integral eq. (6.38) when $\partial\Sigma$ is a 2-sphere at finite constant $r$. What happens to $E_R$ when $r \to \infty$? Interpret the remaining contribution (which is non-zero for all finite r, but vanishes for $r \to \infty$.)

Problem 4. The Kerr-Newmann black hole (see the sentence straddling pages 261 and 262 in Carroll) has mass $M$, charge $Q$, and angular momentum $J$. It is natural to ask whether the electron (or any other fundamental charged particle) could be a Kerr-Newman black hole. For this to work, there would presumably have to be a horizon $r_+$ (a solution to $g^{rr} = 0$), to avoid having a naked singularity and associated acausal behavior. Show that this constraint amounts to:

$$c^2 J^2/M^2 + G\frac{q_e^2}{4\pi\epsilon_0} < G^2 M^2,$$

where $c$ and $4\pi\epsilon_0$ have been inserted in the appropriate way for the metric system. Plug in the numbers for each of the three terms. Is this inequality satisfied for the electron?