Physics 751 **Homework 7** **Due Wednesday, March 27, 2019**

Reading assignment: sections 5.8, 6.1, 6.2, 6.4, 6.5, 6.6, 6.7 and Appendix H.

Problem 1. Consider a **three-dimensional** Riemannian manifold with spherical symmetry, so that the metric is given by

$$ds^2 = f(r)dr^2 + r^2 d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ as usual. Show that a new "isotropic radial coordinate" $\bar{r}$ can be defined so that the metric in the new coordinates takes the form

$$ds^2 = g(\bar{r})[d\bar{r}^2 + \bar{r}^2 d\Omega^2].$$

For the case of four-dimensional Schwarzschild, show that the metric in the isotropic radial coordinate system is

$$ds^2 = -\left(\frac{1 - GM/2\bar{r}}{1 + GM/2\bar{r}}\right)^{N_1} dt^2 + (1 + GM/2\bar{r})^{N_2}[d\bar{r}^2 + \bar{r}^2 d\Omega^2],$$

where N_1 and N_2 are certain integers that you will find.

Problem 2. A particle is released at rest at $r = R$ outside of a Schwarzschild black hole. You should assume that $R > 2GM$.

(a) Find the time τ measured by the particle's clock as a function of r as it falls. [Hint: the result will probably involve an inverse trig function.]

(b) As a special case, check that if the point of initial release from rest is $R = 4GM$, then the time needed to pass the horizon is $\tau = \sqrt{2}GM(\pi + 2)$. How much longer does it take for the particle to reach the singularity? Check that your answer is consistent with the constraint found in problem 1 of Homework Set 6 ($\Delta\tau < \pi GM$).

(c) How long in seconds does it take for a particle to fall to the horizon, and from there to the singularity, if M is the mass of the Sun, and R is the radius of the Earth's orbit?

Problem 3. Consider a metric with the form of Schwarzschild written in outgoing Eddington-Finkelstein coordinates:

$$ds^2 = -(1 - 2GM/r)du^2 - (du\, dr\, + dr\, du) + r^2 d\Omega^2.$$

However, now assume that M is *not* a constant, but is instead a function $M(u)$ of the coordinate u. Show that this is not a solution of the vacuum Einstein equations, but is a solution of the Einstein equations with a simple matter energy-momentum tensor $T_{\mu\nu}$. Express $T_{\mu\nu}$ in terms of $M'(u)$. [Hint: it may not be obvious, but should be plausible, that the resulting energy-momentum tensor corresponds to outgoing radiation.]