Physics 751 **Homework 5** **Due Monday, March 4, 2019**

Reading assignment: sections 3.8, 3.9, 4.1, 4.2, and 4.3 of Carroll.

Problem 1. Do exercise 8 on pages 148-149 of Carroll. On part (c), you need only check the following special cases:
(i) $\rho = \mu = \theta$, $\nu = \sigma = \phi$,
(ii) $\rho = \mu = \theta$, $\nu = \sigma = \psi$,
(iii) $\rho = \mu = \phi$, $\nu = \sigma = \theta$,
(iv) $\rho = \mu = \theta$, $\nu = \phi$, $\sigma = \psi$.

Problem 2. Consider the three dimensional cylinder, with independent coordinates (z, θ, ϕ), and metric:

$$ds^2 = dz^2 + A^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where A is a constant. Find the Riemann tensor, the Ricci tensor, and the Ricci scalar. Show that there is no constant K for which $R_{\rho\sigma\mu\nu} = K g_{\rho[\mu} g_{\nu]\sigma}$. Hence, this is *not* a maximally symmetric space; compare to equation (3.191) and the surrounding discussion in Carroll. This is despite the fact that the Ricci scalar curvature is constant everywhere. (In fact, it is possible to show the even stronger statement that the Riemann tensor is covariantly constant, $\nabla_\kappa R_{\rho\sigma\mu\nu} = 0$, but you don't need to do that.) How many independent Killing vector fields does this metric have? You don't need to find them from scratch, but you ought to be able to write them down from other results that we've seen. In words, what isometries do they correspond to?

Problem 3. Do exercise 13 on page 149 of Carroll. Hints: the Christoffel symbols can be written in terms of a, b, a', and b', where the prime indicates a derivative with respect to u. You are encouraged to derive them using the method that Carroll illustrates at the beginning of section 3.5, in the example that runs from eq. (3.72)-(3.83). Personally, I found it easiest to solve for the Killing vector components with *raised* indices. In the end, some of the Killing vector components can be written in terms of the indefinite integral quantities $\int du/[a(u)]^2$ and $\int du/[b(u)]^2$. Since you aren't told what the functions a and b are, you can and should just leave them in that form. Note that each indefinite integral comes with a constant of integration.