Physics 751 **Homework 2** **Due Friday, February 8, 2019**

Reading assignment: sections 2.1 through 2.8 of Carroll.

Problem 1. Which of the following are possibly valid tensor equations? For the rest, state why they are not.

$$
\begin{array}{ll}
(a) & T^{\nu}{}_{\mu\nu\rho} = R_{\rho\mu} \\
(b) & A_{\mu}B_{\nu\rho} = X_{\mu\nu} \\
(c) & C_{\mu\nu\nu} = Y_{\mu} \\
(d) & T_{\mu\nu} + R_{\mu\rho} = Z_{\nu\rho} \\
(e) & A_{\mu}(B_{\nu} + C_{\nu}) = P_{\mu\nu} \\
(f) & A_{\mu}B_{\nu} = A_{\nu}B_{\mu}
\end{array}
$$

(I'm not telling you anything about the tensors involved, so it is impossible for you to say whether any of these equations are *really* true. I'm asking if they *could* be valid.)

Problem 2. Let A be a vector field on R^2 whose components in the standard Cartesian basis are $A^x = xy$ and $A^y = x^2 + y^2$. What are the components of A in the basis defined by polar coordinates?

Problem 3. Consider a spacetime with the line element:

$$ds^2 = -dt^2 + dx^2 + dy^2 + (1 + ax^2)dz^2 + bx(dxdz + dzdx),$$

where a and b are some constants. Assume that the coordinates (t, x, y, z) are good for the whole spacetime.

(a) Write down a list of the 16 components $g_{\mu\nu}$ of the metric.

(b) What are the 16 components $g^{\mu\nu}$ of the inverse metric?

(c) What is the determinant of the metric? Use it to find a condition on a, b that is necessary for the metric to have Lorentzian signature for all t, x, y, z.

(d) Consider a new coordinate system (t', x', y', z'), where $t' = t$ and $x' = x$ and $y' = y$ and $z' = z + kx$, for some constant k. Find the line element and the metric components in the primed coordinate system.

Problem 4. Do exercise 6 on page 91 of Carroll.