Physics 751 Homework 11 Due 6:00 PM, Friday May 8, 2026. Not accepted late.

Problem 1. Make a **very rough** numerical estimate based on dimensional analysis [using Carroll eq. (7.192) as a guide] of the power in watts (kg·m^2/s^3) lost to gravitational radiation by:
(a) A basketball being dribbled.
(b) The Earth as it moves around the Sun.
(c) Two black holes, each of 30 solar masses, rotating around each other in approximately circular orbits with a separation distance of nR_S, where R_S is one of their Schwarzschild radii. Give this answer in terms of n.

Problem 2. Consider two stars of unequal masses M_1 and M_2 that are revolving about their center of mass in circular orbits in the xy plane according to the Newtonian physics approximation, so that their coordinates are

$$\begin{aligned}(x, y, z) &= \frac{2RM_2}{M_1 + M_2}(\cos(\omega t),\ \sin(\omega t),\ 0) \qquad \text{(for } M_1\text{)},\\ (x, y, z) &= -\frac{2RM_1}{M_1 + M_2}(\cos(\omega t),\ \sin(\omega t),\ 0) \qquad \text{(for } M_2\text{)}.\end{aligned}$$

so that the distance between them is $2R$. In this problem, everything you do should be a generalization of results given in

- the text in the example starting from the middle of page 305 to the top of page 307, and in the second half of page 314 (note that the text uses Ω instead of ω), and/or
- the handwritten lecture notes, lecture group 9 pages 8-10 and group 10 page 1 (but, watch out for sloppy mistakes on my part. There is at least one minus sign mistake, in the coordinates of one of the masses on page 8 of lecture group 9).

Therefore, you should find it helpful to check that your intermediate steps, and answers for each part below, always reduce to the results in the special case $M_1 = M_2 = M$.
(a) What is the relation between ω and M_1, M_2, R, in the Newtonian approximation?
(b) What is the reduced quadrupole moment J_{ij}, in terms of M_1, M_2, R, $\cos(2\omega t)$, and $\sin(2\omega t)$?
(c) What is the radiated power P, as a function M_1, M_2, R (with ω eliminated)? (It should be proportional to G^4/c^5.)

(d) What is the Newtonian total energy of the system, as a function of M_1, M_2, and R (with ω eliminated)?

(e) Use the results above to show that the rate of change of R is

$$\frac{dR}{dt} = -n_1 \frac{G^3}{c^5 R^3}(M_1 M_2)^{n_2}(M_1 + M_2)^{n_3},$$

where n_1, n_2, and n_3 are certain numbers that you will discover.

(f) Use the results above to show that, as claimed in class, the rate of change of the angular frequency for the (nearly) circular binary inspiral can be written as

$$\frac{d\omega}{dt} = \frac{96}{5}\left(\frac{G\mathcal{M}}{2^{1/5}c^3}\right)^{5/3} \omega^{11/3}$$

where

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}. \tag{1}$$

[The sharp dependence of $d\omega/dt$ on ω, with power 11/3, leads to the characteristic "chirp" in gravitational waves from binary mergers, but it breaks down in the final stages when R becomes comparable to the Schwarzschild radii or the physical sizes of the stars. If the orbits are elliptical rather than circular, the above results are modified significantly. However, it turns out that the orbits become more circular as the distance between the stars decreases due to gravitational radiation, so that in many cases the circular approximation is a good one before the Newtonian approximation to the orbits breaks down. You don't need to show this.]