Physics 751 **Homework 10** **Due Friday, April 26, 2019**

Reading assignment: sections 7.1-7.4 of Carroll.

Problem 1. The deceleration parameter in a Robertson-Walker cosmology is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}.$$

As the name suggests, this is a (dimensionless) measure of how much the expansion of the universe is decelerating. When it was originally defined, it was thought (for reasons that were not very good, with the wisdom granted by hindsight) that this parameter was positive; now it is believed to be negative. In other words, the expansion of the universe is accelerating.

(a) Assuming the standard cosmological model, find a formula for the value of this parameter today, of the form (after simplification)

$$q_0 = n_1\Omega_{d,0} + n_2\Omega_{\Lambda,0} + n_3\Omega_{r,0}$$

where n_1, n_2, and n_3 are fixed non-zero rational numbers. Using the numerical values $\Omega_{d,0} = 0.30495$, $\Omega_{\Lambda,0} = 0.695$, $\Omega_{r,0} = 0.00005$, what is the prediction for the numerical value of q_0?

(b) Using eqs. (8.120) and (8.123), and assuming that there is no spatial curvature, and that we are co-moving observers, show that the luminosity distance to co-moving galaxies, *expanded in small* z, can be approximated as

$$d_L(z) = \frac{1}{H_0}\left(z + \frac{1}{2}(1-q_0)z^2 + \ldots\right).$$

Thus, a measurement of the deviation from Hubble's Law in the luminosity distances of "standard candles" as a function of their redshifts provides information about the deceleration parameter.

Problem 2. Assuming that the spatial curvature is 0, and that radiation energy density can be neglected, integrate the Friedmann equation to find an analytical formula for the age of the universe today in terms of H_0, $\Omega_{d,0}$, and/or $\Omega_{\Lambda,0}$. Evaluate your formula in terms of H_0 for the specific numerical values $(\Omega_{d,0}, \Omega_{\Lambda,0}) = (1.0, 0)$ and $(0, 1.0)$ and $(0.5, 0.5)$ and $(0.305, 0.695)$. Using $H_0 = 67.8$ km/(sec Mpc), estimate the age of the Universe today. (Note that the age of the universe in terms of the coordinate time t is also the proper time measured by any co-moving observer since the singularity at $a = 0$. The correction due to non-zero radiation energy density is numerically small.)

[Hint: you will find the following indefinite integral to be useful

$$\int \frac{dx}{\sqrt{B/x + x^2(1-B)}} = \frac{2}{3\sqrt{1-B}} \ln\left(2\sqrt{x^3(1-B)} + 2\sqrt{x^3(1-B)+B}\right)$$

where B is a constant.]

<u>Problem 3.</u> Consider the Lagrangian density in eq. (7.9). Working in terms of $h_{\mu\nu}$, derive the linearized equations of motion for empty space, and show that they imply that the linearized Einstein tensor in eq. (7.8) vanishes.