Physics 686 Homework Set 9 **Due Friday, November 19, 2021**

Problem 1. Consider the process of top-quark decay:

$$t \to bW^+.$$

This is a weak interaction process, which occurs due to the following term in the interaction Lagrangian:

$$\mathcal{L} = -\frac{g}{\sqrt{2}} W_\mu^- (\bar{b}\gamma^\mu P_L t) + \text{c.c.}$$

where t, b are the Dirac spinor fields for the top quark and bottom quark. (Ignore the fact that the bottom quark appearing here is not quite a mass eigenstate; this is a very small effect.) The top quark decays very quickly, as you will discover below, so it does *not* form complicated bound states like the lighter quarks. Therefore, one can just use the simple charged-current weak-interaction Feynman rule implied by the above interaction Lagrangian.

(a) Let the 4-momentum of the top quark be p^μ, and that of the W^+ boson be k_1^μ, and that of the bottom quark be k_2^μ. Find the kinematic quantities:

$$p^2; \qquad k_1^2; \qquad k_2^2; \qquad p \cdot k_1; \qquad p \cdot k_2; \qquad k_1 \cdot k_2$$

in terms of the symbols m_t and m_W. Treat the bottom quark as massless. (This is a good approximation, since $m_t = 173.1$ GeV, $m_W = 80.4$ GeV, and $m_b \approx 5$ GeV. Note that kinematic quantities generally involve the *squares* of ratios of masses.)

(b) Draw the Feynman diagram and write down the reduced matrix element for top quark decay.

(c) Take the complex square of the reduced matrix element, and sum over the final state polarizations of the W^+ boson, using eq. (7.7.10). Then average over the t spin, and sum over the spin of the b. (The quarks have 3 colors, but the color of the final state bottom quark is constrained to be the same as that of the initial state top quark. Since one should average over the initial state quark color, the net color factor is just 1.)

(d) Compute the decay rate of the top quark. You should find a result of the form:

$$\Gamma(t \to bW^+) = \frac{g^2 m_t^3}{N_1 \pi M_W^2} \left(1 + N_2 \frac{M_W^2}{m_t^2}\right) \left(1 - \frac{M_W^2}{m_t^2}\right)^{N_3}$$

where N_1, N_2, and N_3 are positive integers that you will find.

(e) Find the numerical value of the decay width in GeV, and the corresponding top quark lifetime in seconds. Use $g = 0.65$, $m_t = 173.1$ GeV, $m_W = 80.4$ GeV. What is the dimensionless ratio $\Gamma(t \to bW^+)/m_t$?

Problem 2. Consider the process of W^- boson decay, through the interaction Lagrangian:

$$\begin{aligned}\mathcal{L} &= -\frac{g}{\sqrt{2}} W^-_\mu J^{+\mu} + \text{c.c.} \\ J^{+\mu} &= \overline{b}\gamma^\mu P_L t + \overline{s}\gamma^\mu P_L c + \overline{d}\gamma^\mu P_L u + \overline{\tau}\gamma^\mu P_L \nu_\tau + \overline{\mu}\gamma^\mu P_L \nu_\mu + \overline{e}\gamma^\mu P_L \nu_e.\end{aligned}$$

Notice that we are again making the approximation of ignoring the issue of quark mass eigenstates being not quite the same as the fields that couple to the W^-.

(a) The W^- boson cannot decay into a final state with a bottom quark, within the very good approximation just mentioned. Why? (This is a useful thing sometimes; if your experiment tags a bottom quark jet, you can say it almost certainly didn't come from a W^- decay unless it was mis-tagged.)

(b) Treating the electron as massless, compute the decay rate for $W^- \to e^- \overline{\nu}_e$, in terms of g and M_W. [Draw the Feynman diagram, write down the reduced matrix element, take its complex square, average over the *three* possible initial polarizations of the W^- boson using eq. (7.7.10), sum over all possible final state spins, and apply eq. (6.2.14) with $m_1 = 0$.]

(c) From your answer to part (b), infer the results for: $\Gamma(W^- \to \mu^- \overline{\nu}_\mu)$ and $\Gamma(W^- \to \tau^- \overline{\nu}_\tau)$ and $\Gamma(W^- \to d\overline{u})$ and $\Gamma(W^- \to s\overline{c})$, treating all of the final-state fermions as massless. Remember that each quark in the final state has 3 possible colors, which you must sum over. The antiquarks are constrained to have the opposite color, so once you have summed over the quark colors, you should not sum over the antiquark's anticolors. Because the W^- has a large mass and the decay happens quickly, you can assume that the strong interactions of the quarks and antiquarks are irrelevant until long after the decay has occurred.

(d) From the above results, predict the total decay width of the W boson in GeV, its lifetime in seconds, and its branching ratio into each of the possible final states.

(e) Extra credit: draw as many Feynman diagrams as you can think of that contribute to W boson decay at the next order in perturbation theory. Note that these will include both 1-loop diagrams and tree-level diagrams with an additional final-state particle. Don't bother with other loop diagrams whose only effect is to correct external lines.