Physics 686 Homework Set 8 **Due Monday November 8, 2021**

Problem 1. In certain extensions of the Standard Model, including the Minimal Supersymmetric Standard Model (MSSM), it is predicted that there is an electrically neutral spin-0 particle A^0 called the "pseudo-scalar Higgs boson". (This is in addition the the usual "Standard-Model-like" Higgs boson called h^0, and another neutral scalar Higgs boson H^0, and a charged Higgs boson pair $H^\pm$.) The interaction Lagrangian of this particle with each Standard Model Dirac fermion field f has the form:

$$\mathcal{L}_{\text{int}} = y'_f A^0 \overline{f} \gamma_5 f$$

where y'_f is a coupling constant.

(a) Compute the partial decay rate for A^0 into a fermion anti-fermion pair, as a function of y'_f, the mass of the pseudo-scalar M_{A^0}, and the mass of the fermion m_f. You should find a result of the form:

$$\Gamma = N n_f y'^2_f M_{A^0} \left(1 - \frac{4m_f^2}{M_{A^0}^2}\right)^p,$$

where $n_f = 3$ for quarks and 1 for leptons, and N and p are numbers that you will compute. (Hint: the calculation is quite similar to the case of ordinary Higgs decay, given in the notes. However, in the present case, you should find $p \neq 3/2$.)

(b) In the MSSM, it has been calculated that the ratio of the couplings of A^0 to bottom quarks and to top quarks is:

$$\frac{y'_b}{y'_t} = \frac{\overline{m}_b}{\overline{m}_t} \tan^2\beta$$

where $\tan\beta$ is a parameter of the model which is usually believed to be in the range:

$$2 \lesssim \tan\beta \lesssim 55.$$

Here, $\overline{m}_t$ and $\overline{m}_b$ differ somewhat from the actual masses, because of higher-order corrections. Taking $\overline{m}_t = 165$ GeV and $\overline{m}_b = 3$ GeV and $m_t = 173$ GeV and $m_b = 5$ GeV and $m_{A^0} = 1000$ GeV, make a plot of

$$\frac{\text{BR}(A^0 \to b\overline{b})}{\text{BR}(A^0 \to t\overline{t})}$$

as a function of $\tan\beta$, using at least representative points $\tan\beta = 2, 5, 10, 20, 30, 40, 50$. Use a log scale for the vertical axis. For what values of $\tan\beta$ is the branching fraction of A^0 into $b\overline{b}$ greater than for $t\overline{t}$?

Problem 2. Professor Yxxz Xxzxy and Professor Zyzzy Yxxxxy, the most brilliant scientists on the far-off planet Xany (where particle accelerators have not yet been invented and data on nuclear decays is very scant), have proposed competing theories for muon decays. The scientists on Xany have data on the total muon decay rate and the final-state electron energy distributions. (Unfortunately for them, they are not as smart as us, so they haven't thought to consider the $V - A$ theory that we know to be correct.) Xxzxy's proposal corresponds to what we Earthlings would call a P theory of the weak interaction current:

$$\mathcal{L}_{\text{int}}^{P} = -2\sqrt{2}\, G_X\, (\overline{\nu}_\mu \gamma_5 \mu)(\overline{e}\gamma_5 \nu_e) + \text{c.c.}$$

where G_X is known as Xxzxy's Constant. In contrast, Yxxxxy's theory is an $S - P$ theory of the weak interaction current:

$$\mathcal{L}_{\text{int}}^{S-P} = -2\sqrt{2}\, G_Y\, (\overline{\nu}_\mu P_L \mu)(\overline{e} P_L \nu_e) + \text{c.c.}$$

where G_Y is known as Yxxxxy's Constant.

(a) For each of these two competing theories, calculate

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2$$

for $\mu^- \to e^- \nu_\mu \overline{\nu}_e$ decay. Neglect the electron and neutrino masses.

(b) Find the resulting distribution for the electron energy produced in muon decay according to each theory,

$$\frac{d\Gamma}{dE_e}.$$

For each theory, integrate to find Γ.

(c) What would the inhabitants of Xany conclude are the numerical values of the constants G_X and G_Y, in order for each theory to agree with the known value of the muon lifetime? What can you say about whose theory would agree better with the shape of the electron energy distribution for muon decay, Professor Xxzxy or Professor Yxxxxy?

Note that all of this takes place on a different planet, not a different universe. Muons decay the same way on Xany as they do here on Earth. Only the theories are different.

[Back here on Earth, the correctness of the $V - A$ theory was established mostly by analyzing a variety of nuclear decay experiments. Muon decay played a minor role historically.]