Problem 1. Consider the scalar $\lambda \phi^4$ theory discussed in section 4.4. Draw all Feynman diagrams corresponding to $\phi \phi \to \phi \phi \phi \phi$ scattering, to order $\lambda^2$. You should find 10 distinct diagrams, which can be organized into two distinct classes. Clearly label the particle 4-momenta for each external particle and internal propagator line in your diagram. Use Feynman rules to write down an expression for the reduced matrix element for the process. (You do not need to find the cross-section.)

Problem 2. Consider the process of antimumon scattering off of an electron:

$$\mu^+ e^- \to \mu^+ e^-$$

You may assume $m_e$ is negligible, but keep $m_\mu$. Work in the center-of-momentum frame. Assign the incoming electron and antimumon 4-momenta $p_a$ and $p_b$, and call the magnitude of their 3-momenta $P$. Assign the final-state electron and antimumon 4-momenta $k_1$ and $k_2$, and note that the magnitude of their 3-momenta is also $P$. (In working this problem, do not use crossing symmetry. For the purposes of this homework problem, that is considered cheating. If you don’t know what crossing symmetry is yet, that’s fine.)

(a) Draw the Feynman diagram(s) which contribute to the reduced matrix element for this process at order $e^2$.

(b) Define $\theta$ to be angle between the initial-state $e^-$ and the final state $e^-$ directions in the center-of-momentum frame. Work out all of the following quantities in terms of $m_\mu$, $P$, and $\cos \theta$:

\[
\begin{align*}
p_a^2, & \quad p_b^2, & \quad k_1^2, & \quad k_2^2, \\
(p_a \cdot p_b), & \quad (p_a \cdot k_1), & \quad (p_a \cdot k_2), \\
(p_b \cdot k_1), & \quad (p_b \cdot k_2), & \quad (k_1 \cdot k_2), \\
s = (p_a + p_b)^2, & \quad t = (p_a - k_1)^2, & \quad u = (p_a - k_2)^2.
\end{align*}
\]

(c) Use the Feynman rules of QED to obtain the reduced matrix element $\mathcal{M}$. 
(d) Take the complex square of the reduced matrix element you found. Sum over final state spins, and average over initial state spins, and simplify. Write the result in terms of Mandelstam variables $s, t, u$. Then, rewrite it in terms of $P$ and the scattering angle $\theta$.

(e) Find the differential cross section. Simplify your answer as much as possible.

(f) Now take $m_\mu \to 0$. What is the differential cross section? You should note something interesting for a particular value of $\cos \theta$.

(g) Consider the 16 possible helicity processes:

\[
\begin{align*}
\mu_L^+ e_L^- & \to \mu_L^+ e_L^-; \quad \mu_L^+ e_L^- \to \mu_L^+ e_L^-; \\
\mu_L^- e_L^- & \to \mu_L^- e_L^-; \quad \mu_L^- e_L^- \to \mu_L^- e_L^-; \\
\mu_L^+ e_R^- & \to \mu_L^+ e_L^-; \quad \mu_L^+ e_R^- \to \mu_L^+ e_L^-; \\
\mu_L^- e_R^- & \to \mu_L^- e_L^-; \quad \mu_L^- e_R^- \to \mu_L^- e_L^-; \\
\mu_R^+ e_L^- & \to \mu_R^+ e_L^-; \quad \mu_R^+ e_L^- \to \mu_R^+ e_L^-; \\
\mu_R^- e_L^- & \to \mu_R^+ e_L^-; \quad \mu_R^- e_L^- \to \mu_R^+ e_L^-; \\
\mu_R^+ e_R^- & \to \mu_R^+ e_L^-; \quad \mu_R^+ e_R^- \to \mu_R^+ e_L^-; \\
\mu_R^- e_R^- & \to \mu_R^+ e_L^-; \quad \mu_R^- e_R^- \to \mu_R^+ e_L^-.
\end{align*}
\]

Do not compute them. Instead, figure out which ones vanish by helicity conservation in the limit $m_e, m_\mu \ll \sqrt{s}$. 