Physics 686 **Homework 1** **Due Wednesday, September 1, 2021**

Fun with relativistic kinematics!

Problem 1. Using data from the "Review of Particle Properties" and/or the lecture notes, find numerical values for the mean distance traveled before decay by: a muon, a neutral pion, a charged pion, a tau lepton, a K^+ meson, a D^+ meson, and a B^+ meson produced in high-energy physics experiments, for the following values of the particle energy: 10 GeV, 100 GeV, 1000 GeV. Present your results in a table.

Problem 2. What is the threshold energy for the initial-state proton, for the reaction $p^+n \to p^+p^+\pi^-$, assuming the neutron is initially at rest?

Problem 3. Consider a particle of mass M, which undergoes a 2-body decay to particles a, b of masses m_a and m_b. You are encouraged to set the speed of light c to be 1 (this is a choice of units). Assuming that all motion is along the z direction, find the four-momenta of the particles a, b for:
(a) the special case $m_b = m_a$.
(b) the special case $m_b = 0$.
(c) the general case. Write your answer in the simplest form. [Hint: write your answer in terms of $\sqrt{\lambda(m_a^2, m_b^2, M^2)}$, where $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is sometimes called the "triangle function".] Check that your answer agrees with the special cases in parts (a) and (b).

Problem 4. Consider the elastic scattering of a muon neutrino off of an electron: $\nu_\mu e^- \to \nu_\mu e^-$. Suppose that in the lab frame the electron is initially at rest and the neutrino has energy E, and treat the neutrino as massless.
(a) Write expressions for the 4-vectors of the particles in the center-of-momentum frame.
(b) In the lab frame, what is the maximum angle of the scattered electron with repect to the incoming neutrino?

Problem 5. Two photons, with energies E_1 and E_2, annihilate with each other in empty space, with a collision angle θ. The final state is a muon + antimuon pair. In this problem, denote the mass of the muon as M, and its mean lifetime in its rest frame as τ.
(a) Find the necessary inequality for the process to occur, in terms of the given quantities. Discuss the behavior of this requirement for the two limiting cases $\theta = 0$ and $\theta = \pi$.
(b) Now suppose that the collision occurs head-on ($\theta = 0$), and that $E_1 > E_2$. What is the maximum energy that one of the resulting muons can have?]
(c) Now suppose that $\theta = 0$, $E_1 = 9M$, and $E_2 = M$. What will be the maximum mean lifetime of the longer-lived muon?