Physics 661 **Homework 9** **Due Tuesday, April 15, 2025**

Reading assignment: sections 22.5, 22.6, 5.6, and 6.6 of the text.

Problem 1. In this problem you will find the lifetime of the $|n, l, m\rangle = |3, 2, 2\rangle$ state of an isolated hydrogen atom, due to spontaneous emission of a single photon. Ignore fine, hyperfine, and Lamb shift splitting effects, and use the electric dipole approximation.
(a) According to the electric dipole selection rules, what state or states could be the final state? What are the angular frequency ω, the wavelength λ, and the color of the emitted light? Note that $\lambda \gg a_0$, so that the electric dipole approximation is indeed valid.
(b) Use eq. (22.4.14) to evaluate the emission rate, and compute $\tau = 1/\Gamma$ to find the lifetime, both symbolically in terms of quantities like α, c, $\hbar$, a_0, and ω, and then numerically in seconds.

Problem 2. Consider an electron trapped in a 3-dimensional isotropic harmonic oscillator potential with angular frequency Ω. The electron is initially in the first excited state with $l = 1$, $m = 0$, and spontaneously emits a photon to decay to the ground state.
(a) How is the angular frequency ω of the emitted photon related to the angular frequency Ω of the harmonic oscillator potential?
(b) Compute the lifetime of the first excited state.

Problem 3. Consider scattering in one dimension of particles with mass m and energy E from a potential $V(x) = V_0\delta(x/a)$, where a is a length scale and V_0 is a constant potential energy.
(a) Find the transmission and reflection coefficients T and R by matching stationary-state wavefunctions at $x = 0$.
(b) Check your result for T by taking the $a \to 0$ limit of eq. (6.6.37).