Physics 661 **Homework 5** **Due Thursday, February 20, 2024**

Reading assignment: sections 17.3 and 18.1 of the text.

Problem 1. Consider the second excited level of atomic hydrogen (the $n = 3$ level). Make a diagram of the relative energy splittings, similar to Figure 17.2.1 on page 347 for the $n = 2$ level. Include numerical values in μeV for the fine structure, hyperfine structure, and Lamb shift splittings. For the Lamb shift, you may use the approximate formula

$$\Delta E_{\rm Lamb} = 6.5\delta_{\ell,0}\frac{\alpha^3}{n^3}\left(\frac{e^2}{2a_0}\right).$$

Don't try to draw the energy splittings exactly to scale; instead just indicate their qualitative relative sizes and label them with the numerical values. Use spectroscopic notation to label the states before the hyperfine effect, and label the hyperfine-split states by their f (grand total angular momentum) quantum numbers.

Problem 2. An electron is moving in a spherically symmetric but otherwise generic (no other special properties) potential $V(r)$, with a perturbation Hamiltonian of the spin-orbit type:

$$W = a\vec{L}\cdot\vec{S},$$

where a is a positive constant and $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin operator. Consider a set of unperturbed states that are degenerate in energy (before including W), and have L^2 eigenvalue $12\hbar^2$.

(a) Work in the total angular momentum basis $|j, m_j\rangle$ to answer the following questions. What are the energy corrections due to W? What are the remaining degeneracies of the corresponding energy eigenstates?

(b) Repeat part (a), but this time do your work in terms of the matrix elements of W in the product angular momentum basis $|m_l, m_s\rangle$. [Hint: you may want to use eq. (17.3.23).]

(c) Repeat part (a), but this time pretend that you live in a universe where the electron has spin 3/2 (instead of spin 1/2, as in our universe).

Problem 3. Deuterium, also known as heavy hydrogen, is an atom with one electron and a nucleus consisting of a deuteron (a proton-neutron bound state) with spin 1 (note not spin 1/2). The magnetic moment operator of the deuteron is

$$\vec{\mu}_d = \frac{g_d e}{2m_d c}\vec{S}_d, \tag{1}$$

where $\vec{S}_d$ is the spin-1 operator, $m_d = 1875.6$ MeV$/c^2$ is the mass, and $g_d = 1.713$ is the gyromagnetic ratio. This is the counterpart to eq. (17.2.1) in the text. Work out the hyperfine structure of the $n = 1$ state of deuterium. What are the energy, frequency, and wavelength of the photon corresponding to the hyperfine transition of the ground state?