PHYS 661 **Homework Set 2** **Due Thursday, January 30, 2025**

Reading assignment: sections 15.1, 15.2, 15.6 of the class text.

Problem 1. Consider a harmonic oscillator supplemented by a quartic potential term, so that the Hamiltonian is

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 + \lambda X^4$$

Treat the last term as a small perturbation.
(a) Compute the correction to the energy of the state $|n\rangle$ at first order in λ. For fixed n, your answer is good for sufficiently small λ. However, you should find that it grows quadratically with n, so if λ is fixed, no matter how small it is, for sufficiently large n the perturbation expansion will break down. (To think about: what feature of the harmonic oscillator wavefunction at level n causes this breakdown in perturbation theory?)
(b) Find the ground state eigenket in terms of the unperturbed energy eigenkets, at first order in λ.

Problem 2. A rigid rotator has Hamiltonian given by

$$H = aL^2 + bL_z + cL_x,$$

where a, b, and c are positive constants, and $\vec{L}$ is the angular momentum operator.
(a) Write down the exact energy eigenvalues. (This should be quick. Start by writing $bL_z + cL_x = b'\hat{n} \cdot \vec{L}$, where $\hat{n}$ is a unit vector; what is b'?)
(b) Now treat the last term as a perturbation, so $c \ll a, b$. Find the unperturbed energy eigenvalues with $c = 0$ for the eigenkets $|l, m\rangle$ of L^2 and L_z, and use them to obtain the energy eigenvalues at second order in the perturbation. Compare them to the exact answer from part (a). [Hint: you will want to use the matrix elements $\langle l', m'|L_x|l, m\rangle$. They can be obtained from eq. (8.4.4) of the text.]

Problem 3. A particle of mass m is confined to a 1-dimensional box of length L with a bump of width a in the middle:

$$V(x) = \begin{cases} \infty & (|x| > L/2), \\ V_0 & (|x| < a/2), \\ 0 & (a/2 < |x| < L/2) \end{cases}$$

(a) Treat V_0 as a perturbation, and calculate the energy and the position-space wavefunction for the first excited ($n = 2$, odd parity) energy eigenstate, to first order in V_0. Your result for the wavefunction will involve an infinite sum. Check that your answer is exact when $a = L$. (Hint: we did the ground state in class and in the text.)

Problem 4. Consider a particle of mass m confined to a 3-dimensional cubic box with sides L. At the center of the box is a potential $V = \lambda\delta^{(3)}(\vec{r})$, which you should treat as a perturbation.

(a) What are the metric system units of λ?

(b) Write down the unperturbed ground state wavefunction (normalized to 1) and energy eigenvalue.

(c) Find the energy eigenvalue of the ground state to first order in the perturbation.