Physics 661 **Homework 10** **Due Friday, April 25, 2025**

Reading assignment: sections 23.1 through 23.6 of the text.

Problem 1. Particles of mass m and incident energy E scatter from a potential

$$V(r) = \begin{cases} V_0 & (\text{for } r < a) \\ 0 & (\text{for } r > a), \end{cases}$$

where V_0 and a are constants. In the following, use $E = \hbar^2 k^2/2m$ for the incident particles moving in the positive $\hat{z}$ direction, and apply the first-order Born approximation.

(a) Find the differential scattering cross-section for small $|V_0|$. Write your answer in terms of $q = 2k\sin(\theta/2)$. Check that your answer has units of area.

(b) Show that in the limit of small ka, the differential cross-section found in part (a) is constant with respect to the scattering angle, and show that the total cross-section is $\sigma = nV_0^2 a^6$ where n is a quantity that you will find. (Check the units.)

(c) Working now to the next-to-leading order in an expansion in small ka, show that the differential cross-section has the form $d\sigma/d\Omega = b + c\cos(\theta)$, and determine the quantities b and c.

Problem 2. Consider the scattering of a spinless point particle with mass m, energy E, and charge $-e$, due to a point charge Ze fixed at the origin, which is screened at large distances by a uniform thin spherical shell of opposite charge at $r = R$, so that:

$$V(r) = \begin{cases} -Ze^2/r & (\text{for } r < R), \\ 0 & (\text{for } r > R). \end{cases}$$

Use the first-order Born approximation for scattering to:

(a) Find the differential cross section.

(b) Find the lowest energy E for which $d\sigma/d(\cos\theta)$ vanishes for backward scattering ($\theta = \pi$).

(c) Find the differential cross section and the total cross section in the low energy limit, keeping the leading and next-to-leading non-zero contributions.

Problem 3. Suppose that the differential cross-section for particles of momentum $\hbar\vec{k}$ to scatter from a potential $V_0(\vec{r})$ centered at the origin is known to be $d\sigma_0/d\Omega$ in the leading Born approximation. Now consider scattering from N such potentials centered at positions $\vec{a}_n$, so that the total potential is $V(\vec{r}) = \sum_{n=1}^{N} V_0(\vec{r} - \vec{a}_n)$. Show that in the leading Born approximation the differential cross-section is

$$\frac{d\sigma}{d\Omega} = \left| \sum_{n=1}^{N} \exp\left(i(\vec{k} - \vec{k}') \cdot \vec{a}_n\right) \right|^2 \frac{d\sigma_0}{d\Omega}.$$