Physics 661 **Homework 9** **Due Tuesday, April 23, 2024**

Reading assignment: sections 5.6, 6.6, and 20.1 of the text.

Problem 1. Consider an electron trapped in a (three-dimensional) harmonic oscillator potential with angular frequency Ω. The electron is initially in the first excited state with $l = 1$, $m = 0$, and spontaneously emits a photon to decay to the ground state.
(a) Consider the angular frequency of the emitted photon, ω. How is it related to the angular frequency Ω of the harmonic oscillator potential?
(b) Compute the lifetime of the first excited state.

Problem 2. Consider a particle of mass m moving in one dimension in a potential

$$V(x) = -aV_0[\delta(x-a) + \delta(x+a)],$$

where $a > 0$ and $V_0 > 0$. (Note that V_0 has units of energy.) Recall that in the presence of a delta function potential, the wavefunction is continuous, but the derivative of the wavefunction has a discontinuity where the delta function's argument vanishes; see the discussion surrounding eqs. (6.3.1) through (6.3.6) on pages 126-127 of the text. In this problem, we will be interested in bound states, which have $E < 0$. The given potential is invariant under $x \to -x$, so we should be able to choose stationary states that are also eigenstates of parity.
(a) What is the form of the wave function for a bound stationary state with even parity? Choose your answer so that $\psi(x) = e^{-\kappa x}$ for large positive x. (You do not need to worry about the overall normalization of your wavefunction.)
(b) Find an equation that determines the bound state energies for even parity states, and determine graphically how many even parity bound states there are. [Hint: you should be able to write your equation in the form $\kappa a =$ (polynomial in $e^{-\kappa a}$). Graph the qualitative shape of the right-hand side as a function of κa.]
(c) Repeat parts (a) and (b) for odd parity. For what values of V_0 are there no odd-parity bound stationary states? [Hint: you should again be able to write your equation in the form $\kappa a =$ (polynomial in $e^{-\kappa a}$). Graph the qualitative shape of the right-hand side, and consider its slope at $\kappa a = 0$.]
(d) Solve for the even parity bound state energy analytically in the limit of $V_0 \ll \hbar^2/ma^2$.
(e) Find the even and odd parity state binding energies in the limit $V_0 \gg \hbar^2/ma^2$. (They are nearly equal to each other in that limit.)

Problem 3. For the potential in problem 2, suppose particles are incident from the left with energy $E > 0$.

(a) Look for a stationary state with $E = \hbar^2 k^2/2m > 0$ and wavefunction of the form:

$$\psi(x) = \begin{cases} e^{ikx} + Be^{-ikx} & (x \leq -a) \\ De^{ikx} + Fe^{-ikx} & (-a \leq x \leq a) \\ Ce^{ikx} & (x \geq a) \end{cases}$$

(Note that in 1-d scattering problems, we are not interested in eigenstates of parity.) Derive four equations that relate the coefficients B, C, D, and F. You are strongly encouraged to simplify the notation for your work below by writing things in terms of the dimensionless quantity

$$n = \frac{maV_0}{\hbar^2 k}.$$

Also, if you want to check the four equations that you got, before you proceed to the next part, I will be happy to confirm or deny them by email.

(b) Solve for C. From it, get the transmission coefficient T. Some partial results:

$$C = \frac{1}{1 - iN_1 n + n^2(e^{iN_2 ka} - N_3)},$$

where N_1, N_2, and N_3 are certain positive integers that you will discover, and you should write the transmission coefficient in the form:

$$T = \frac{1}{P_1 + P_2 \cos(N_2 ka) + P_3 \sin(N_2 ka)}$$

where P_1, P_2, and P_3 are certain polynomials in n with integer coefficients.

(c) Check that your transmission coefficient has the expected behavior when E becomes very large. How does it behave when E is very small?