Physics 661 **Homework 7** **Due Tuesday, April 2, 2024**

Reading assignment: sections 18.1-18.3 of the text.

Problem 1. In the Bizarro Universe, long ago and very far away, scientists have determined that everything is just like in our universe except that electrons are fermions with spin 3/2 instead of spin 1/2. Consider atomic states for Bizarro electrons.
(a) How many Bizarro electrons can fit into each s ($l = 0$) orbital? How many Bizarro electrons can fit into each p ($l = 1$) orbital?
(b) What are the atomic numbers of the lightest two Bizarro noble gases (the lightest has a filled $1s$ orbital, the next has a filled $2p$ orbital)?
(c) Consider Bizarro neon, by which we mean a neutral atom with $Z = 10$. What is the electron configuration, instead of $(1s)^2(2s)^2(2p)^6$ as in our universe? Will it behave chemically like a noble gas? What are all of the ${}^{2S+1}L_J$ spectroscopic terms for the electron configuration you found, and which of them is the ground state?

Problem 2. In class and in the notes, we evaluated the probability that a ^{3}H (tritium) atom in its ground state would end up in the ground state of ^{3}He, after beta decay, and found $P = 512/729$. In the same way, evaluate the probability that the atom ends up in each of the $n = 2$ or $n = 3$ levels instead. Check that the total probability for ending up in the $n = 1$, $n = 2$, and $n = 3$ levels does not exceed 1.

Problem 3. A particle of mass m moves freely in a 1-dimensional box of length a, with the left edge at $x = 0$ and the right edge at $x = a$. At time $t = 0$, the particle is in the ground state, and the box is suddenly lengthened by expanding the right edge out to $x = b$.
(a) Find the probability that the particle ends up in *each* of the energy eigenstates of the final box, labeled by positive integers n. (Make sure that your probabilities are dimensionless.)
(b) Use your results from part (a), and your knowledge of the laws of probability, to evaluate the following mathematical infinite sum over n:

$$S(a, b) \quad = \quad \sum_{n=1}^{\infty} \frac{\sin^2(n\pi a/b)}{(n^2a^2 - b^2)^2}.$$

Quantum mechanics knows how to do non-trivial sums!
(c) As a check, for the special case $b = 2a$, evaluate the individual probabilities numerically for each integer up to $n = 10$, and their total with 6 significant digits. (Be careful for $n = 2$; the answer is non-zero and finite.)