

Reading assignment: sections 13.1, 13.2, 13.6 of the class text.

Problem 1. Consider a harmonic oscillator supplemented by a quartic potential term, so that the Hamiltonian is

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 + \lambda X^4$$

Treat the last term as a small perturbation.

- (a) Compute the correction to the energy of the state $|n\rangle$ at first order in λ . For fixed n , your answer is good for sufficiently small λ . However, you should find that it grows quadratically with n , so if λ is fixed, no matter how small it is, for sufficiently large n the perturbation expansion will break down. (To think about: what feature of the harmonic oscillator wavefunction at level n causes this breakdown in perturbation theory?)
- (b) Find the ground state eigenket in terms of the unperturbed energy eigenkets, at first order in λ .

Problem 2. The Hamiltonian of a rigid rotator is given by

$$H = aL^2 + bL_z + cL_x,$$

where a , b , and c are positive constants, and \vec{L} is the angular momentum operator.

- (a) Write down the exact energy eigenvalues. (This should be quick. Start by writing $bL_z + cL_x = b'\hat{n} \cdot \vec{L}$, where \hat{n} is a unit vector; what is b' ?)
- (b) Now treat the last term as a perturbation, so $c \ll a, b$. Find the unperturbed energy eigenvalues with $c = 0$ for the eigenkets $|l, m\rangle$ of L^2 and L_z , and use them to obtain the energy eigenvalues at second order in the perturbation. Compare them to the exact answer from part (a). [Hint: you will want to use the matrix elements $\langle l', m' | L_x | l, m \rangle$. They can be obtained from eq. (8.4.4) of the notes.]

Problem 3. A particle of mass m is confined to a 1-dimensional box of length L with a bump of width a in the middle:

$$V(x) = \begin{cases} \infty & (|x| > L/2), \\ V_0 & (|x| < a/2), \\ 0 & (a/2 < |x| < L/2) \end{cases}$$

- (a) Treat V_0 as a perturbation, and calculate the energy and the position-space wavefunction for the first excited ($n = 2$, odd parity) energy eigenstate, to first order in V_0 . Your result for the wavefunction will involve an infinite sum. Check that your answer is exact when $a = L$. (Hint: we did the ground state in class and in the notes.)

Problem 4. Consider a particle of mass m confined to a 3-dimensional cubic box with sides L . At the center of the box is a potential $V = \lambda\delta^{(3)}(\vec{r})$, which you should treat as a perturbation.

- (a) What are the metric system units of λ ?
- (b) Write down the unperturbed ground state wavefunction (normalized to 1) and energy eigenvalue.
- (c) Find the energy eigenvalue of the ground state to first order in the perturbation.