Physics 661 **Homework 11** **Due Friday, May 2, 2025**

Note: solutions for this homework set will be made available after midnight on May 2, so it will not be accepted late after that time.

Reading assignment: sections 23.6 through 23.9, and 3.5, of the text.

Problem 1. Consider an experiment in which slow neutrons of momentum $\hbar k\hat{z}$ are scattered by a diatomic molecule; suppose that the molecule is aligned along the y axis, with one atom at $y = b$ and the other at $y = -b$. The beam of neutrons is directed in the $\hat{z}$ direction. Assume the atoms to be infinitely heavy so that they remain fixed throughout the experiment. The potential due to the atoms as seen by the neutrons can be represented by a delta function, so:

$$V(\vec{r}) = a[\delta(x)\delta(y-b)\delta(z) + \delta(x)\delta(y+b)\delta(z)]$$

(a) Calculate the scattering amplitude and the differential cross section $d\sigma/d\Omega$, as a function in simplest form of θ, ϕ, and the parameter a, b, k, and m, in the first-order Born approximation. (Note that this potential is not spherically symmetric!)
(b) Suppose that there is a small detector that is located far from the target, along the y axis. For what values of the deBroglie wavelength of the incident neutrons is the observed flux of scattered particles minimized? (This is a very non-classical phenomenon.)

Problem 2. A graduate student is assigned to measure the differential cross-section of a beam of spinless particle of mass m with known kinetic energy E (which is assumed to be low) on an unknown target of spherically symmetric atoms. She finds that the differential cross-section is not isotropic, and can be fit to the form:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\hbar^2}{mE}[c_0 + c_1\cos(\theta) + c_2\cos^2(\theta)].$$

where c_0, c_1, and c_2 are measured quantities that depend on E but not on θ. Assume that the target is very small compared to the experimental apparatus, and that multiple scatterings within the target are not an issue. Also assume that the results should be interpreted as a combination of S-wave and P-wave scattering.
(a) What are the dimensions (units) of c_0, c_1, and c_2 ?
(b) Find real equations relating the phase shifts δ_0 and δ_1 to the measured quantities c_0, c_1, and c_2.
(c) Use partial wave unitarity to find theoretical upper bounds on c_0 and c_2.
(d) What are the theoretical upper and lower bounds on c_1? [Hint: both upper and lower bounds are realized by phase shifts satisfying $\delta_0 = \pi n_0/6$ and $\delta_1 = \pi n_1/6$ where n_0 and n_1 are certain positive integers less than 6. You may use this fact without proving it.]