PHYS 660 **Homework 5** **Due Friday, October 4, 2024**

In order to facilitate study for the midterm exam on Wednesday, October 9, solutions for this homework will be sent to you on Monday October 7 at 10:00AM. Therefore, no late solutions will be accepted after that time.

Reading assignment: sections 6.4, 6.5, and then 5.1, 5.2, and 5.4 of the text.

Problem 1. A quantum mechanical particle of mass m is confined to a 1-dimensional box of length a, with the allowed region $-a/2 < x < a/2$. Suppose it is in the state $|n\rangle$, with energy E_n. (Here $n = 1$ denotes the ground state, and you may or may not find it convenient to treat the cases of even and odd n separately.)

(a) Compute ΔX and ΔP, as functions of n.
(b) For each of $n = 1, 2, 3, 4, 5, 6$, calculate $(\Delta X)(\Delta P)$ as a decimal number (to 3 significant figures) multiplying $\hbar$. One of your answers should be $5.40\hbar$. How do your results compare to the uncertainty principle inequality?
(c) For general n, what is the probability that a measurement of the particle's position will find it within a distance $a/4$ of the center of the box? For $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, write your answer in simplest form, which should not involve any trigonometric functions. Check that your probabilities are dimensionless numbers between 0 and 1.

Problem 2. Consider a quantum mechanical particle of mass m moving in 1 dimension in the potential:

$$\begin{aligned} V(x) &= V_0 \qquad (x < -a), \\ &= 0 \qquad (-a < x < 0), \\ &= \infty \qquad (x > 0). \end{aligned}$$

(a) What is the transcendental equation that determines the bound state energy eigenvalues?
(b) Find the condition that must be satisfied by V_0, m, and a in order for a least one bound state to exist. Sketch the wavefunction of the ground state as a function of x.

Problem 3. A quantum mechanical particle of mass m moves in 1 dimension, in the presence of an attractive delta-function potential $V(x) = -A\delta(x)$, where A is a positive constant.

(a) Show that there is always a bound state solution with energy $E = -mA^2/2\hbar^2$, and obtain its normalized position-space wavefunction $\psi(x) = \langle x|\psi\rangle$ and momentum-space wavefunction $\widetilde{\psi}(p) = \langle p|\psi\rangle$. [Hint: Solve the time-independent Schrödinger's equation for each of the two regions $x < 0$ and $x > 0$, assuming $E < 0$. Then match the two solutions at the boundary $x = 0$, using the continuity of the wavefunction and the discontinuity in its first derivative, obtained as discussed in class and the notes from Schrödinger's equation by considering $\int_{-\epsilon}^{\epsilon} \frac{d^2\Psi}{dx^2}\, dx$, where ϵ is infinitesimal, in the discussion leading to eqs. (6.3.5) and (6.3.6)]

(b) Are there any other bound states?

(c) For the ground state, sketch $\psi(x)$, and compute: $\langle X \rangle$, $\langle X^2 \rangle$, and ΔX.

(d) For the ground state, sketch $\widetilde{\psi}(p)$, and compute: $\langle P \rangle$, $\langle P^2 \rangle$, and ΔP.

(e) What is $(\Delta X)(\Delta P)$? How does it compare to the result obtained for a Gaussian wave-function?