PHYS 660 **Homework 1** **Due Wednesday, September 4, 2024**

Reading assignment: Chapter 1 and sections 2.1 and 2.2 of the class text.

Problem 1. Estimate the time needed for a Hydrogen atom to reach infinite binding energy, in a mythical world with no quantum mechanics, assuming that the electron starts in a circular orbit of radius R about the (very heavy, pointlike) proton and radiates away electromagnetic energy adiabatically, so that the orbit stays nearly circular, as discussed in class and the notes. First, use the results in the class notes to write the answer for the lifetime in terms of r_0, and the mass and charge of the electron, and any other constants that you need. Then plug in the numbers to estimate the time numerically in seconds, if $r_0 = 5 \times 10^{-11}$ meters.

Problem 2. The cosmic background radiation is nearly blackbody radiation with a present temperature of 2.73 Kelvin. Compute the numerical total energy per cubic meter in this radiation:
(a) within the visible frequency range (defined here as $4 \times 10^{14}\,\mathrm{s}^{-1} < \nu < 8 \times 10^{14}\,\mathrm{s}^{-1}$), if it were governed by the classical Rayleigh-Jeans formula. (For comparison, the energy density in starlight is roughly $10^{-15} J/m^3$.)
(b) for all frequencies, as given by the quantum Planck formula. (Most of this is in the microwave; convince yourself that the visible range contribution is ridiculously tiny.)

Problem 3. Consider a 3-dimensional complex vector space with a basis $|1\rangle$, $|2\rangle$, $|3\rangle$, with inner product given by

$$\langle j|k\rangle = \begin{cases} 3 & \text{for } j = k, \\ 1 & \text{for } j \neq k. \end{cases}$$

Construct an orthonormal basis $|1'\rangle$, $|2'\rangle$, $|3'\rangle$ for the space, in terms of the original basis.

Problem 4. Consider a 2-dimensional complex vector space with a basis $|1\rangle$, $|2\rangle$, with an inner product such that $\langle 1|1\rangle = 3$, and $\langle 2|2\rangle = 2$, and $\langle 1|2\rangle = i$. Construct an orthonormal basis $|1'\rangle$, $|2'\rangle$ for the space, in terms of the original basis.

Problem 5. The Schwarz inequality says that for any two kets $|V\rangle$ and $|W\rangle$ in a complex inner product space,

$$|\langle V|W\rangle| \leq |V||W|,$$

where the norm is defined by $|V| = \sqrt{\langle V|V\rangle}$. The proof is given in the class notes. Use this to prove the triangle inequality for the norm of $|V + W\rangle \equiv |V\rangle + |W\rangle$:

$$|V + W| \leq |V| + |W|$$

and show that equality holds if and only if $|W\rangle = a|V\rangle$ where a is a non-negative real number. [Hint: start with $|V + W|^2$, and use the property of complex numbers $\mathrm{Re}(z) \leq |z|$.]