PHYS 660 **Homework 9** **Due Wednesday, November 22, 2023**

Reading assignment: sections 8.3-8.7 and 9.1-9.3 of the course text.

Problem 1. Consider a spinless particle in a state with spherical-coordinate wavefunction

$$\psi(r, \theta, \phi) = ae^{-br^2} \cos^2 \theta,$$

where a and b are constants.

(a) Solve for the constant a, by assuming that the wavefunction has unit normalization.

(b) If L^2 and L_z are simultaneously measured, what are the possible pairs of results, and their probabilities? Check that your probabilities are dimensionless and non-negative and that they add up to 1.

(c) Find the expectation value and uncertainty of the radial coordinate r. You should not let the symbol a appear in your answer. Check that your answers have the correct units.

Problem 2. Consider a particle with spin 1/2 that starts in an eigenstate of S_z with eigenvalue $+\hbar/2$. Suppose now that the spin along the direction $\hat{n} = \hat{x} \sin \beta + \hat{z} \cos \beta$ is to be measured.

(a) Construct the matrix representations of the operators S_x, S_y, S_z, S^2, and $\hat{n} \cdot \vec{S}$, in the basis of eigenstates of S_z, denoted $|\uparrow\rangle$ and $|\downarrow\rangle$. What are the eigenvalues and eigenvectors of each, written in that basis?

(b) What is the probability that the measurement of $\hat{n} \cdot \vec{S}$ yields the result $+\hbar/2$?

(c) Suppose that the measurement in part (b) has been carried out and the result was indeed $+\hbar/2$. Immediately afterwards, S_z is measured. What is the probability that the measurement yields $-\hbar/2$?

(d) Check that your results for parts (b) and (c) make sense when $\beta = 0$ and π. What happens when $\beta = \pi/2$?

Problem 3. Consider a spinless particle in a state with L^2 eigenvalue $2\hbar^2$ and L_z eigenvalue $\hbar$. As in the previous problem, let $\hat{n} = \hat{x} \sin \beta + \hat{z} \cos \beta$.

(a) Suppose that the angular momentum along the direction $\hat{n}$ is measured. What are the possible results, and their probabilities?

(b) For each of the possible results in part (a), suppose that L_z is then measured. What are the possible results, and their probabilities?

(c) Check that your results make sense in the special cases $\beta = 0$ and $\beta = \pi$ and $\beta = \pi/2$.

Problem 4. Consider a quantum system consisting of two independent spin-1/2 operators, $\vec{S}_1$ and $\vec{S}_2$, so that the state space is spanned by orthonormal S_{1z} and S_{2z} eigenstates $|\uparrow,\uparrow\rangle$ and $|\uparrow,\downarrow\rangle$ and $|\downarrow,\uparrow\rangle$ and $|\downarrow,\downarrow\rangle$. Here the first entry of each ket labels the S_{1z} eigenvalue as $\pm\hbar/2$, and the second labels the S_{2z} eigenvalue $\pm\hbar/2$. At time $t=0$, the system happens to be in the state:

$$|\psi(0)\rangle = c\Big(4\,|\uparrow,\uparrow\rangle + 3\,|\uparrow,\downarrow\rangle + 2\,|\downarrow,\uparrow\rangle + |\downarrow,\downarrow\rangle\Big),$$

where c is a constant that you should solve for and eliminate from your answers.

(a) At time $t=0$, we measure the z-component of spin 1, S_{1z}. What is the probability of finding $+\hbar/2$?

(b) Suppose the measurement mentioned in the previous part has happened, and the result was indeed $+\hbar/2$. What is the normalized state ket immediately after this measurement? If we then immediately measure the x-component of spin 2, S_{2x}, what results can be found, and with what probabilities?

(c) Instead of performing the above measurements, we let the system evolve under the influence of the Hamiltonian $H = \vec{B}\cdot\vec{S}_1$, where $\vec{B} = B_0\hat{z}$ and B_0 is a constant. Find the state of the system at time t. Use this to find the expectation value of the x-component of spin 1, $\langle S_{1x}\rangle$, as a function of time.