PHYS 660 **Homework 6** **Due Friday, October 18, 2024**

Reading assignment: sections 7.1, 7.2, 7.3 of the class text.

Problem 1. (This is a redo of the first four parts of problem 2 on the midterm.) A quantum system has an orthonormal basis consisting of two states $|1\rangle$ and $|2\rangle$. The Hamiltonian of the system obeys:

$$\begin{aligned} H\,|1\rangle &= \hbar\omega\,(4\,|1\rangle + i\,|2\rangle)\,, \\ H\,|2\rangle &= \hbar\omega\,(c\,|1\rangle + 4\,|2\rangle)\,, \end{aligned}$$

where ω is a positive real number and c is a certain number.

(a) What is the number c, and how do you know?

For the remaining parts of the problem, suppose that the system is in the state $|1\rangle$ at time $t = 0$.

(b) If the energy is measured at $t = 0$, what are the possible results, and their probabilities?

(c) Find the expectation value and uncertainty of the energy.

(d) If the energy is measured at $t = 0$, and the result is the smallest possible one, find the normalized state ket of the system immediately after the measurement.

Problem 2. (This is a redo of the last part of problem 2 on the midterm, with more added.) Consider the same situation as in Problem 1 above, but now **instead** of measuring the energy at $t = 0$, the state $|1\rangle$ at time $t = 0$ is allowed to evolve.

(a) What is the state at all later times t? [Hint: start by writing the state $|1\rangle$ as a linear combination of energy eigenstates.]

(b) I can now reveal that the orthobasis kets are eigenstates of an operator A, so that $A\,|1\rangle = |1\rangle$ and $A\,|2\rangle = 2\,|2\rangle$. What is the probability of measuring A and getting the result 2, as a function of time?

(c) What are the expectation value and uncertainty of A, as functions of time?

(d) What are the expectation value and uncertainty of H, as functions of time?

Problem 3. Consider a quantum mechanical particle of mass m moving in 1 dimension in the potential:

$$V(x) \;=\; \begin{cases} \frac{1}{2}m\omega^2 x^2 & \text{(for } x > 0\text{)}, \\ \infty & \text{(for } x < 0\text{)}. \end{cases}$$

What boundary conditions should be applied to the position-representation wavefunctions? Find all of the energy eigenvalues and the corresponding unit-normalized position-representation wavefunctions. [Hint: you should use logic and reasoning and calculations already done in class and the text, rather than new calculations. But there is one subtlety involving the normalization...]

Problem 4. For the 1-dimensional harmonic oscillator with mass m and angular frequency ω:
(a) write down the expressions for the operators X and P in terms of a and $a^\dagger$.
(b) Calculate the general expressions for the following matrix elements of the energy eigenstates $|n\rangle, |k\rangle$ with n, k non-negative integers. (Your answers should make use of the Kronecker delta symbol.)

$$\begin{array}{rll} \langle k|X|n\rangle, & \langle k|P|n\rangle, & \\ \langle k|X^2|n\rangle, & \langle k|P^2|n\rangle, & \\ \langle k|PX|n\rangle, & \langle k|XP|n\rangle, & \\ \langle k|a|n\rangle, & \langle k|a^\dagger|n\rangle, & \langle k|H|n\rangle. \end{array}$$

(c) Use the results of part (b) to calculate the expectation values of the kinetic energy and the potential energy in the state $|n\rangle$, and show that they are equal.
(d) Use the results of part (b) to calculate $(\Delta X)(\Delta P)$ for the state $|n\rangle$.
(e) Calculate the following expectation values in the state $|n\rangle$:

$$\begin{array}{rl} \langle n|X^3|n\rangle, & \langle n|P^3|n\rangle, \\ \langle n|X^4|n\rangle, & \langle n|P^4|n\rangle, \\ \langle n|X^5|n\rangle, & \langle n|P^5|n\rangle. \end{array}$$

Make note of how the parity selection rules apply here.