

Reading assignment: sections 6.4, 6.5, and then 5.1, 5.2, and 5.4 of the notes.

**In order to facilitate study for the midterm exam on Monday, October 9, solutions for this homework will be sent to you on Friday October 6 at 11:00AM. Therefore, no late solutions will be accepted after that time.**

Problem 1. A quantum mechanical particle of mass  $m$  is confined to a 1-dimensional box of length  $a$ , with the allowed region  $-a/2 < x < a/2$ . Suppose it is in the state  $|n\rangle$ , with energy  $E_n$ . (Here  $n = 1$  denotes the ground state, and you may or may not find it convenient to treat the cases of even and odd  $n$  separately.)

- Compute  $\Delta X$  and  $\Delta P$ , as functions of  $n$ .
- For each of  $n = 1, 2, 3, 4, 5, 6$ , compute  $(\Delta X)(\Delta P)$  as a decimal number (to 3 significant figures) multiplying  $\hbar$ . One of your answers should be  $5.40\hbar$ . How do your results compare to the uncertainty principle inequality?
- For general  $n$ , what is the probability that a measurement of the particle's position will find it within a distance  $a/4$  of the center of the box? For  $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ , write your answer in simplest form, which should not involve any trigonometric functions. Check that your probabilities are dimensionless numbers between 0 and 1.

Problem 2. Consider a quantum mechanical particle of mass  $m$  moving in 1 dimension in the potential:

$$\begin{aligned} V(x) &= V_0 & (x < -a), \\ &= 0 & (-a < x < 0), \\ &= \infty & (x > 0). \end{aligned}$$

- Derive a transcendental equation for the energy eigenvalues of bound states.
- Find the condition that must be satisfied by  $V_0$ ,  $m$ , and  $a$  in order for a least one bound state to exist. Sketch the wavefunction of the ground state as a function of  $x$ .

Problem 3. A quantum mechanical particle of mass  $m$  moves in 1 dimension, in the presence of an attractive delta-function potential  $V(x) = -A\delta(x)$ , where  $A$  is a positive constant.

- Show that there is always a bound state solution with energy  $E = -mA^2/2\hbar^2$ , and obtain its normalized position-space wavefunction  $\psi(x) = \langle x|\psi\rangle$  and momentum-space wavefunction  $\tilde{\psi}(p) = \langle p|\psi\rangle$ . [Hint: Solve the time-independent Schrödinger's equation for each of the two regions  $x < 0$  and  $x > 0$ , assuming  $E < 0$ . Then match the two solutions at the boundary  $x = 0$ , using the continuity of the wavefunction and the discontinuity in its first derivative, obtained as discussed in class and in the textbook surrounding eqs. (6.3.5) and (6.3.6).

- (b) Are there any other bound states?
- (c) For the ground state, sketch  $\psi(x)$ , and compute:  $\langle X \rangle$ ,  $\langle X^2 \rangle$ , and  $\Delta X$ .
- (d) For the ground state, sketch  $\tilde{\psi}(p)$ , and compute:  $\langle P \rangle$ ,  $\langle P^2 \rangle$ , and  $\Delta P$ .
- (e) What is  $(\Delta X)(\Delta P)$ ? How does it compare to the result obtained for a Gaussian wavefunction?