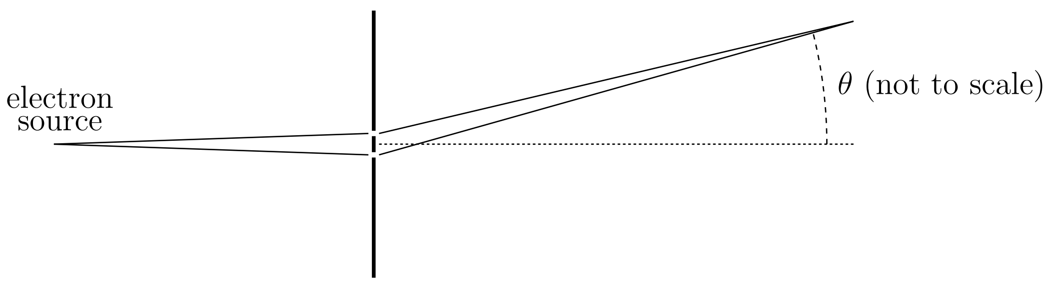
PHYS 660 **Homework 1** **Due Wednesday, September 6, 2023**

Reading assignment: Chapter 1 and sections 2.1 and 2.2 of the class notes.

Problem 1. Estimate the time needed for a Hydrogen atom to reach infinite binding energy, in a mythical world with no quantum mechanics. Assume that the electron starts in a circular orbit of radius r_0 about the (very heavy, pointlike) proton and radiates away electromagnetic energy adiabatically, so that the orbit stays nearly circular, as discussed in class and the notes. First, use the results in the class notes to write the answer for the lifetime in terms of r_0, and the mass and charge of the electron, and any other constants that you need. Then plug in the numbers to estimate the time numerically in seconds, if $r_0 = 5 \times 10^{-11}$ meters.

Problem 2. Electrons are accelerated from rest by a potential difference of one volt, and pass through a screen with two very narrow long parallel slits. How far apart must the slits be in order for the first minimum of the interference pattern at a distant detector to be at an angle $\theta = 0.2^\circ$ away from the central maximum?



Problem 3. Consider a 3-dimensional complex vector space with a basis $|1\rangle$, $|2\rangle$, $|3\rangle$, with inner product given by

$$\langle j|k\rangle = \begin{cases} 2 & \text{for } j = k, \\ 1 & \text{for } j \neq k. \end{cases}$$

Construct an orthonormal basis $|1'\rangle$, $|2'\rangle$, $|3'\rangle$ for the space, in terms of the original basis.

Problem 4. Consider a 2-dimensional complex vector space with a basis $|1\rangle$, $|2\rangle$, with an inner product such that $\langle 1|1\rangle = 1$, and $\langle 2|2\rangle = 2$, and $\langle 1|2\rangle = i$. Construct an orthonormal basis $|1'\rangle$, $|2'\rangle$ for the space, in terms of the original basis.

Problem 5. The Schwarz inequality says that for any two kets $|V\rangle$ and $|W\rangle$ in a complex inner product space,

$$|\langle V|W\rangle| \leq |V||W|,$$

where the norm is defined by $|V| = \sqrt{\langle V|V\rangle}$. The proof is given in the class notes. Use this to prove the triangle inequality for the norm of $|V+W\rangle \equiv |V\rangle + |W\rangle$:

$$|V+W| \leq |V| + |W|$$

and show that equality holds if and only if $|W\rangle = a|V\rangle$ where a is a non-negative real number. [Hint: start with $|V+W|^2$, and use the property of complex numbers $\mathrm{Re}(z) \leq |z|$.]