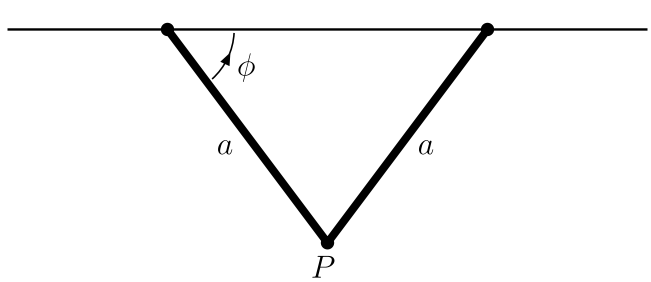
PHYS 600 **Homework 7** **Due Friday, November 7, 2025**

In order to facilitate study for the second midterm on Monday November 10, solutions for this problem set will be emailed to you very early on November 8. Therefore, it cannot be accepted late.

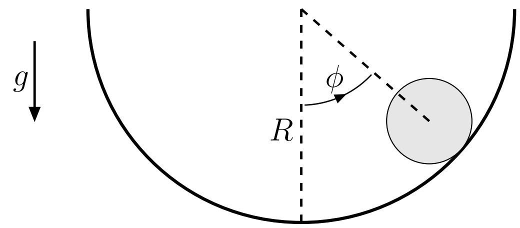
Reading assignment: pages 144-168 of Fetter and Walecka.

Problem 1. A homogeneous solid cube with mass M and sides of length a is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a tiny displacement and allowed to fall in a uniform gravitational field with acceleration g.
(a) What is the moment of inertia tensor for the cube, with respect to its principal axes through its center of mass?
(b) Find the angular velocity of the cube when one face strikes the plane, assuming that the edge cannot slide due to friction.
(c) Same question as (b), but now assuming that the edge slides without friction on the plane.
(d) For the frictionless case in part (c), what is the force exerted by the surface on the cube just before the face strikes the plane?

Problem 2. Two identical thin uniform rods, each of fixed length a and mass m, are hinged together at the point P, as shown. The other ends of the rods **both** slide freely on a horizontal rail. The system is in a uniform vertical gravitational field with acceleration g pointing down. All motion is frictionless. At time $t = 0$, the angle ϕ is held equal to ϕ_0, and the rods are released from rest.
(a) Find the Lagrangian of the system and the equation of motion in terms of the single configuration variable ϕ.
(b) For small times t, $\phi(t) = \phi_0 + \alpha t^2 + \beta t^4 + \ldots$. Find the constants α and β.
(c) Find the amount of time needed for the hinge P to reach its lowest point. You may leave your answer in terms of a single definite integral. Do not assume the time is small.
(d) Find the speed at which each of the free ends of the rods are moving when the hinge P reaches its lowest point.



Problem 3. A cylinder of mass M and radius a rolls without slipping inside another stationary cylinder with radius R. The moving cylinder is released from rest at time $t = 0$ from an angle $\phi = \phi_0$ through the center of the ball, as defined in the figure.



(a) Find the kinetic energy of the moving cylinder, and show that it is of the form

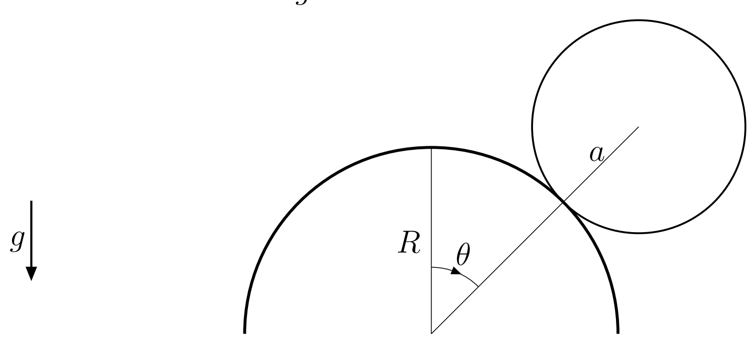
$$T = nM(R - a)^2\dot{\phi}^2,$$

where n is a certain rational number that you will discover. If you used the moment of inertia through the center, check your answer by using the moment of inertia through an axis parallel to it on the rim, or vice versa.

(b) Derive an equation of motion for the angle ϕ. Solve this equation of motion for the case of $\phi_0 \ll 1$.

(b) What is the velocity of the center of the ball when $\phi = 0$? (Do not assume $\phi_0 \ll 1$ here.)

Problem 4. A uniform **solid sphere** of radius a and mass M rolls off of a fixed **cylinder** of radius R, starting nearly from rest at the top of the cylinder, in the Earth's constant gravitational field with acceleration g.



(a) Write the Lagrangian for the system using the single configuration variable θ shown.

(b) At what angle will the sphere leave the cylinder?

(c) If the sphere had the same radius and total mass, but was a thin shell instead of a uniform solid, would the angle be greater or less than in part (b)? Explain your answer.