PHYS 600 **Homework 4** **Due Friday, October 3, 2025**

To facilitate study for the first midterm on Monday October 6, solutions for this homework set will be distributed early in the morning of October 4. Therefore, it will not be accepted late after that time.

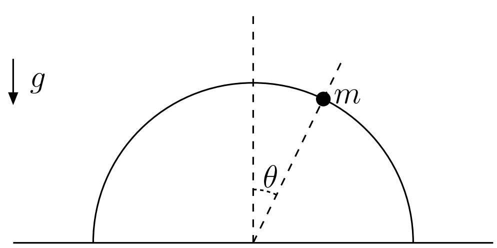
Reading assignment: pages 86-98 of Fetter and Walecka.

Problem 1. Consider a Lagrangian with a single generalized coordinate x,

$$L(x,\dot{x}) = \frac{1}{2}\dot{x}^2 F(x) + \dot{x}G(x) - V(x).$$

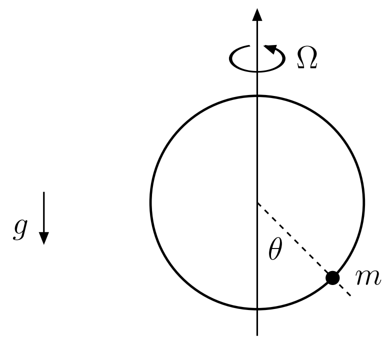
(a) What is the generalized momentum p_x, in terms of $\dot{x}$ and x ?
(b) Find the conserved energy E, written in terms of x and $\dot{x}$ (with no explicit appearance of p_x).
(c) Using your expression for E found in the previous part, compute dE/dt in terms of x, $\dot{x}$, and $\ddot{x}$, without using the equation of motion.
(d) Derive the equation of motion, and write it as an expression for $\ddot{x}$ in terms of x and $\dot{x}$. Use it to show that your answer for part (c) vanishes.
(e) Rewrite the conserved energy E in terms of x and p_x (with no explicit appearance of $\dot{x}$).

Problem 2. A particle of mass m slides frictionlessly on a hemisphere of radius $r = R$, in the presence of a uniform gravitational field with acceleration g pointed downward, as shown. The particle is initially at rest at an angle $\theta = \theta_0$. You should not need to do any integrals or solve any differential equations in this problem.



(a) Use conservation of energy to find $\dot{\theta}$ as a function of θ.
(b) Write the Lagrangian in terms of generalized coordinates r and θ and a Lagrange multiplier λ for the constraint $r = R$. Find the three equations of motion.
(c) Express the force exerted by the hemisphere on the particle as a function of m, g, R, θ, and θ_0, by evaluating the Lagrange multiplier with $\dot{\theta}$ and r eliminated.
(d) By solving for the condition that the force of constraint changes sign, determine the angle θ at which the particle leaves the surface. What is it numerically in degrees, if $\theta_0 \approx 0$? What if $\theta_0 = 45°$? What if $\theta_0 = 89°$?

Problem 4. A point mass m slides without friction on a vertical circular wire of radius a. The center of the circular wire is fixed, and it rotates with constant angular velocity Ω about the vertical diameter. There is a uniform gravitational field with acceleration g pointing downward.



(a) Construct the Lagrangian for the mass, using the angle θ (measured from the downward vertical direction as shown) as the generalized coordinate.

(b) Derive the equation of motion for θ.

(c) Show that the condition for a circular orbit at angle $\theta = \theta_0$ is $\cos(\theta_0) = g/a\Omega^2$.

(d) Investigate the stability of the orbit found in part (c) by writing $\theta(t) = \theta_0 + \epsilon(t)$, where $\epsilon(t)$ is small, and deriving the equation of motion for ϵ. Find the angular frequency ω of small oscillations about the equilibrium point.

(e) What happens if $\Omega^2 < g/a$?