PHYS 600 **Homework 3** **Due Friday, September 26, 2025**

Reading assignment: pages 53-74 of Fetter and Walecka.

Problem 1. Consider a particle of mass m moving in a repulsive central potential $V(r)$.
(a) Show that the scattering angle θ and the impact parameter b are related by

$$\theta = \pi - 2b \int_0^{u_{\max}} du \frac{1}{\sqrt{1 - V/E - b^2u^2}},$$

where $u = 1/r$ and $u_{\max}$ is the classical turning point, corresponding to the closest approach $r_{\min}$.
(b) What is the corresponding formula for an attractive potential $V(r)$?

Problem 2. A uniform beam of particles with energy E is scattered by a repulsive central potential $V(r) = k/r^2$, where k is a positive constant.
(a) Use the results of the previous problem to show that the impact parameter b and the scattering angle θ are related by

$$b^2 = \frac{k}{E} f(\theta/\pi),$$

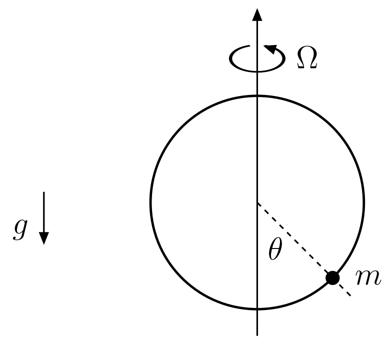
where $f(x)$ is a certain rational function that you will discover. [Hint: you should find that the numerator and denominator of $f(x)$ are both quadratic polynomials with integer coefficients.]
(b) Find the differential cross-section $d\sigma/d\Omega$ in simplest form.
(c) Expand the differential cross-section near the forward and backward directions, by expanding to the lowest non-trivial order about $\theta = 0$ and $\theta = \pi$, respectively. You should find that the differential cross-section for backward scattering approaches a non-zero constant.
(d) Is the total cross-section finite (as for hard-sphere scattering) or infinite (as for Rutherford scattering)?

Problem 3. A physical system has the Lagrangian

$$L(x, \dot{x}) = 3m^2\dot{x}^4 + 6mk\dot{x}^2x^2 - k^2x^4,$$

where m and k are constants, and x is the generalized coordinate.
(a) Derive the equations of motion, and put them into simplest form.
(b) Using your answer to part (a), show that this Lagrangian secretly describes a simple harmonic oscillator, and find its angular frequency of oscillations.
[This illustrates, again, the fact that the Lagrangian describing a physical system is not unique, even for the same choice of configuration variables.]

Problem 4. A point mass m slides without friction on a vertical circular wire of radius a. The center of the circular wire is fixed, and it rotates with constant angular velocity Ω about the vertical diameter. There is a uniform gravitational field with acceleration g pointing downward.



(a) Construct the Lagrangian for the mass, using the angle θ (measured from the downward vertical direction as shown) as the generalized coordinate.

(b) Derive the equation of motion for θ.

(c) Show that the condition for a circular orbit at angle $\theta = \theta_0$ is $\cos(\theta_0) = g/a\Omega^2$.

(d) Investigate the stability of the orbit found in part (c) by writing $\theta(t) = \theta_0 + \epsilon(t)$, where $\epsilon(t)$ is small, and deriving the equation of motion for ϵ. Find the angular frequency ω of small oscillations about the equilibrium point.

(e) What happens if $\Omega^2 < g/a$?