PHYS 600 Homework 2 Due Tuesday, September 16, 2025

Reading assignment: sections 5 (pages 18-27) and 13 (pages 49-52) of Fetter and Walecka.

Problem 1. A very small asteroid with mass m and velocity v_∞ approaches a planet of radius R and mass M, with impact parameter b.
(a) What is the condition that the asteroid will strike the planet?
(b) What is the total cross-section for the asteroid to hit the planet? [Hint: this is much easier than finding the differential cross-section for scattering.]
(c) If the asteroid just misses the planet, what will be its final angle of deflection?

Problem 2. The force of attraction between a very heavy star and a planet of mass m is:

$$F = \frac{a}{r^2} + \frac{3b\ell^2}{r^4}$$

where ℓ is the angular momentum of the planet and a, b are both positive constants. [Note: this does approximate the force of attraction between a planet and a black hole, in the non-relativistic limit, with $a = GMm$ where M is the mass of the star.]
(a) Under what conditions is a stable circular orbit possible? Give the radius of the stable circular orbit in terms of the given parameters (m, a, b, ℓ).
(b) What is the smallest radius possible for any circular orbit as a function of a and b, allowing for arbitrary ℓ? (Hint: this occurs in the limit of very large ℓ.) Is this circular orbit stable or unstable?

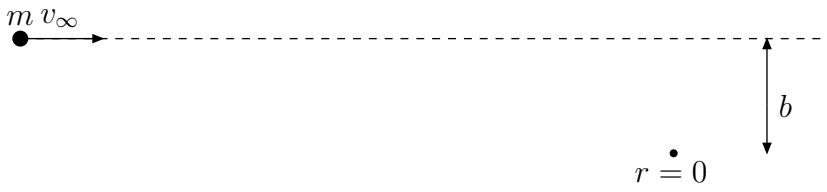
Problem 3. A very small comet with mass m is discovered approaching the Sun (with mass $M \gg m$) along an orbit that is a perfect parabola. At the present time the comet is noted to be moving with speed v_∞, and has an impact parameter distance b with respect to the Sun. For parts (b) and (c) below, you may leave your answer in terms of definite integrals, if you choose.
(a) Find the distance of closest approach of the comet to the Sun, and find its speed when it reaches closest approach.
(b) How long does it take the comet to again reach its present distance from the Sun?
(c) When the comet again reaches its present distance from the Sun, what angle (as measured from the Sun) has it swept out on its orbit?

Problem 4. Consider a point particle of mass m moving in three dimensions in a central potential

$$V(r) = -\frac{g}{r} - \frac{k}{r^2}$$

where g and k are positive constants and r is the distance from the origin. The particle

approaches from very far away with speed v and impact parameter b, as shown. (The dashed line is what the path would be if there were no potential.)



(a) Find $r_{\min}$, the distance of closest approach of the particle to $r = 0$. Show that the particle will go through the origin if $k > k_{\text{crit}}$, where k_{crit} is a critical value that you will determine in terms of the other given quantities.

In the remainder of this problem, you should assume $k < k_{\text{crit}}$, and you may leave your answers in terms of $r_{\min}$ and the other given quantities.

(b) What is the maximum speed reached by the particle on its trajectory?
(c) What is the maximum acceleration reached by the particle on its trajectory?
(d) When the particle is very far from the origin again, find the angle by which it has been scattered from its original direction. You may leave your answer in terms of a single definite integral.