PHYS 600 **Homework 1** **Due Thursday, September 4, 2025**

Reading assignment: sections 1-4 (pages 1-18) of Fetter and Walecka.

Problem 1. The escape velocity of a small particle at the surface of a spherical planet of mass M is defined to be the minimum velocity that the particle must have in order to escape to infinity against the gravitational field. Ignore the friction of the atmosphere, so that the force exerted on the particle of mass m is a conservative one, with gravitational potential $V(r) = -GMm/r$, where G is Newton's constant. Using conservation of energy, find the formula for the escape velocity in terms of the mass M of the planet and its radius R, ignoring the rotation of the planet. Use your result to estimate the escape velocity numerically, in km/sec, for the Earth, the Moon, and Mars.

Problem 2. Consider the gravitational potential near a planet of mass M, as in the previous problem, and consider a small mass m dropped from rest at time $t = 0$ at a distance r_0 from the center of the planet. Use conservation of energy to find a formula relating the distance r and the elapsed time t, of the form $t = f(r)$. Simplify it as much as possible. You may find the following indefinite integral useful:

$$\int \frac{du}{\sqrt{1/u - 1}} \quad = \quad -u\sqrt{1/u - 1} - \arccos(\sqrt{u}).$$

Problem 3. Consider a rocket with initial mass m_0. It gains speed by emitting propellant with constant speed v_{ex} (relative to the rocket) at a constant rate m_0/T, where T is a constant with units of time. This means that the rocket's mass is not constant, but rather is a decreasing function of time, and approaches 0 as $t \to T$. The rocket starts from rest on the earth's surface and rises vertically. Treating the gravitational field of the earth as uniform with acceleration of gravity g, find the height and velocity of the rocket as a function of time t. Sketch the height as a function of t. Discuss the behavior for small times ($t \ll T$) and for $t \to T$.

Problem 4. Consider a single particle with fixed mass m. Use the equation of motion to show that the kinetic energy T obeys the differential equation

$$\frac{dT}{dt} \quad = \quad \vec{F} \cdot \vec{v}.$$

Problem 5. A system consists of N particles with mass m_i and positions $\vec{r}_i$, with $i = 1, \ldots, N$, with total mass $M = \sum_i m_i$. Show that the magnitude of the center-of-mass position vector $\vec{R}$ obeys the equation

$$M^2 R^2 = aM \sum_i m_i r_i^2 + b \sum_i \sum_j m_i m_j |\vec{r}_i - \vec{r}_j|^2,$$

where a and b are certain fixed rational numbers that you will discover.