Physics 485 **Homework 9** **Due April 15, 2003**

Problem 1. What are the solutions of

$$\begin{aligned}\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}-\frac{n(n+1)}{x^2}y &= \delta(x-x')\\ \frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}-\frac{n(n+1)}{x^2}y &= f(x)\end{aligned}$$

on the interval $0 < x < \infty$, subject to the boundary conditions $y(0) = y(\infty) = 0$? Assume n is a positive integer and $0 < x' < \infty$, and $f(x)$ is an arbitrary function.

Problem 2. The solution of

$$y'' + \omega^2 y = g(x) \qquad (0 \le x \le 2\pi),$$

with arbitrary ω, subject to the boundary conditions

$$y'(0) = 0, \qquad y(2\pi) = 0,$$

can be written in the form:

$$y(x) = \int_0^{2\pi} G(x, x', \omega) g(x') dx'.$$

Find TWO expressions for $G(x, x', \omega)$, one involving an infinite sum over eigenfunctions, and another not involving an infinite sum but using $x_< \equiv \min(x, x')$ and $x_> \equiv \max(x, x')$.

[Note that the boundary conditions involve the derivative of y at the origin, and y at 2π. Physically, this corresponds to a string with one end clamped down at 2π amd the other end allowed to slide frictionlessly along the y direction at $x = 0$.]

Problem 3. Consider the differential equation:

$$\left[(1-x^2)\frac{d^2}{dx^2} - ax\frac{d}{dx} + n(n+a-1)\right] u_n(x) = 0.$$

(a) Transform this equation into the Sturm-Liouville form, and identify the functions $p(x)$, $q(x)$, and $w(x)$ and also the eigenvalue λ_n.
(b) Show that the eigenfunctions $u_n(x)$ are orthogonal for different n. Specify the interval of integration and the weighting factor.
(If necessary, you may assume that your solutions are polynomials, but you do not need to find them explicitly.)