Physics 485 **Homework 7** **Due March 18, 2003**

<u>Problem 1.</u> (a) Show that

$$\int_{-1}^{1} x^n P_l(x)\,dx \;=\; 0 \qquad \text{for } n < l.$$

[Hint: Use Rodrigues' formula, and integrate by parts repeatedly.] Use this to show that

$$\int_{-1}^{1} P_n(x) P_l(x)\,dx \;=\; 0 \qquad \text{for } n \neq l.$$

(b) Evaluate

$$\int_{-1}^{1} x^l P_l(x)\,dx$$

in terms of l. [Hint: Use Rodrigues' formula, and integrate by parts l times. After a change of variables $u = x^2$, the part of the final integral from $x = 0$ to $x = 1$ can be written in terms of a Beta function, and the integral from $x = -1$ to $x = 0$ is the same. The resulting Beta function can then be written in terms of Gamma functions as usual, which can then be simplified.]

<u>Problem 2.</u> Use the generating function for Legendre polynomials to evaluate the following:

$$P_l(0) \qquad \text{and} \qquad \left.\frac{dP_l(x)}{dx}\right|_{x=0}.$$

[Hint: in each case, use the binomial expansion for $(1 + t^2)^n$.]