Physics 485 **Homework 5** **Due February 20, 2003**

Problem 1. The motion of a falling body in a resisting medium may be described by:

$$m\frac{dv}{dt} = mg - bv,$$

when the retarding force is proportional to the velocity, v. Find the velocity as a function of time. Evaluate the constant of integration by demanding that $v(0) = 0$.

Problem 2. The rate of evaporation from a spherical drop of liquid (constant density) is proportional to its surface area. Assuming this to be the only mechanism of mass loss, write down a differential equation for the radius of the drop as a function of time, and solve it.

Problem 3. Find the general solutions for:

(a) $\displaystyle \frac{dy}{dx} = \frac{2xy}{y - x^2}$ (b) $\displaystyle \frac{dy}{dx} = \frac{y}{e^y - x}$

[Hint: for (b), you may find it easiest to solve for x as a function of y.]

Problem 4. As discussed in class, the differential equation governing the density (number of particles per unit volume) of annihilating dark matter particles in an expanding universe is:

$$\frac{dn}{dt} + 3H(t)n = -kn^2 + S(t)$$

where $H(t) = \dot{a}/a$ is known as the "Hubble constant" (although it is not really constant). Assume that the source term $S(t)$ is negligible (this is not a completely realistic assumption). The result is a special case of Bernoulli's equation.

(a) Suppose that the scale factor $a(t)$ is given by $a_0(t/t_0)^{2/3}$. (This is the case for a universe in which the expansion rate is said to be "matter dominated".) Convert the above equation into a linear equation, find an integrating function, and find the general solution. How does the solution behave for very large t? In that limit, which is more important in determining the density $n(t)$, the expansion of the universe, or the annihilation? Answer the same questions for very small t.

(b) Repeat part (a) if the scale factor is $a(t) = a_0(t/t_0)^{1/2}$ and again if $a(t) = a_0e^{H_0t}$. (These two cases correspond to universes in which the expansion rate is said to be "radiation dominated" and "vacuum energy dominated", respectively.)