Physics 485 **Homework 11** **Due May 1, 2003**

<u>Problem 1.</u> (a) What linear homogeneous second-order differential equation has

$$x^A J_{\pm m}(Bx^C)$$

as its solutions? Show that it can be written in the form:

$$y'' + \frac{k_1}{x}y' + \left[k_2 x^{k_3} + \frac{k_4}{x^2}\right] y = 0,$$

where k_1, k_2, k_3, and k_4 are constants that you will evaluate in terms of A, B, C, and m.

(b) Use your answer to part (a) to find the general solutions to each of:

$$\begin{aligned} y'' + 4x^2 y &= 0, \\ y'' - \frac{1}{x}y + (4 + \frac{1}{x^2})y &= 0, \\ 4xy'' + 2y' + y &= 0. \end{aligned}$$

<u>Problem 2.</u> A cylindrical metal cavity has a radius a and a height L. The ends, $z = 0, L$, are at zero electrical potential. The cylindrical walls, $r = a$, are at a potential given by $V(\phi, z)$, which you may assume obeys $V(\phi, z) = -V(-\phi, z)$. Inside the cylinder is a vacuum.

(a) Show that the electrical potential everywhere inside the cylinder can be written in the form

$$V(r, \phi, z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} I_m(k_n r) \sin(k_n z) \sin(m\phi),$$

where the a_{mn} are coefficients, and $k_n = n\pi/L$, and $I_m(x)$ are the modified Bessel functions.

(b) Use the boundary conditions, and the orthogonality of the sine functions, to show that the coefficients a_{mn} are given by:

$$a_{mn} = \frac{2}{\pi L I_m(k_n a)} \int_0^{2\pi} d\phi \int_0^L dz\, V(\phi, z) \sin(k_n z) \sin(m\phi)$$