Physics 470 **Homework 9** **Due Friday, November 16, 2017**

Reading assignment: Griffiths sections 9.5 and 10.1.1.

Problem 1. Light in the visible range in water has an index of refraction which approximately obeys Cauchy's law for dispersion:

$$n(\omega) = 1 + n_0 + n_1\omega^2$$

where n_0 and n_1 are positive constants.

(a) Find expressions for the phase velocity v and the group velocity v_g, in terms of n_0, n_1, and ω. Prove that $v_g < v < c$.

(b) The color of light, whether in vacuum, air or water, is determined by its frequency. Let us define our standard "red" light to have wavelength $\lambda = 7.6 \times 10^{-7}$ meters in vacuum, and our standard "blue" light to have $\lambda = 4.3 \times 10^{-7}$ meters in vacuum. What are the angular frequencies of these two colors?

(c) Suppose the index of refraction in water is measured to be 1.329 for red light and 1.340 for blue light. Use this data to find the constants n_0 and n_1. Use these in turn to predict the index of refraction for green light with wavelength in vacuum of $\lambda = 5.3 \times 10^{-7}$ meters.

(d) What are the wavelengths in water of red, green, and blue light as defined above?

(e) What are the numerical values of the phase velocity and the group velocity for red light? What is the numerical difference between them?

(f) A beam of white light (containing red and blue components, among others) is incident at an angle of 30 degrees on a perfectly flat water surface. What is the angular separation between the red and blue refracted components under the water?

Problem 2. Researchers at the University of East Sionilli are measuring the properties of the extremely rare and nearly transparent gas meta-Kryptonite. For light in the visible range, they have found that its index of refraction is approximately

$$n(\omega) = 1.0011 \; + \; (2.0 \times 10^{-36}\, s^2)\, \omega^2.$$

Assume that each meta-Kryptonite atom contains two free electrons with the same natural angular frequency ω_0, with negligible damping in the visible range. Find ω_0, and the number of meta-Kryptonite atoms per unit volume in the sample used for the above measurement.

Problem 3. Consider a rectangular wave guide with sides a and b along the x and y directions respectively. The z-components of the (real) electric and magnetic fields in the wave guide are

$$\begin{aligned} E_z &= 0\,, \\ B_z &= B_0 \cos(2\pi x/a)\, \cos(2\pi y/b)\, \cos(kz - \omega t)\,. \end{aligned}$$

(a) Find k in terms of the other quantities in the problem.
(b) Find all other components of the real electric and magnetic fields within the waveguide.

Problem 4. Repeat Problem 3, but now with:

$$\begin{aligned} E_z &= 0\,, \\ B_z &= B_0 \cos(\pi x/a)\, \sin(kz - \omega t)\,. \end{aligned}$$