Physics 470 **Homework 5** **Due Wednesday, October 4, 2017**

Reading assignment: Griffiths sections 8.2 and 9.1

Problem 1. A fat wire, radius a, carries a constant current I, uniformly distributed over its cross-section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Figure 7.45 on page 336 of the book. Throughout this problem you may ignore the complications of fringing effects, which modify the fields near the edge of the capacitor.

(a) Find the electric and magnetic fields in the gap, as functions of the distance r from the axis and time t. (Assume the charge on the capacitor is 0 at time $t = 0$.)

(b) Find the electromagnetic energy density u_{EM} and the Poynting vector $\vec{\mathbf{S}}$ in the gap. Check that your answers satisfy equation (8.12) on page 359 of Griffiths.

(c) Compute the total electromagnetic energy U_{EM} within the gap. Calculate the total power P flowing into the gap, by integrating the Poynting vector over an appropriate surface. Check these results by showing that the power input is equal to the rate of increase of energy in the gap.

Problem 2. Two very long cylinders with radii a and b (with $a < b$) are coaxial with each other, and both move with speed v in the $\hat{z}$ direction along their common axis. The inner cylinder carries charge per unit area σ/a, and the outer cylinder carries charge per unit area $-\sigma/b$. (This means that any given length of the cylinders together is neutral.) Find, in terms of the given quantities:

(a) The electric and magnetic fields everywhere.

(b) The energy per unit length stored in the electromagnetic fields.

(c) The energy per unit time (power) transported by the fields through a cross-sectional annular area perpendicular to the cylinders.

(d) The momentum per unit length stored in the electromagnetic fields.

Problem 3. A very long solenoid of radius a has its axis of symmetry on the z axis and produces a constant magnetic field $\vec{\mathbf{B}} = B_0\hat{z}$ in its interior. There is also a very long wire carrying no current, but with a constant charge per unit length λ, parallel to and outside of the solenoid, at $x = d$, $y = 0$. Find the momentum per unit length stored in the electromagnetic fields. What will happen to the wire if the solenoid is turned off? (HINT: a similar example was done in class.)

Problem 4. A very long solenoid, of radius a, with n turns per unit length, carries a slowly time-varying current I_s, so that $dI_s/dt \neq 0$. Its axis of symmetry is the z axis.

(a) Use the integral form of Faraday's Law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}},$$

to find the electric field both inside and outside of the solenoid. (We used the *derivative* form of Faraday's law to do this in class for $r < a$; I want to see you use the integral version. Also, note that $\vec{\mathbf{E}}$ outside the solenoid is *not* 0, even though $\vec{\mathbf{B}}$ is.)

(b) Find the Poynting vector everywhere, in terms of n, a, I_s, and dI_s/dt.

(c) Consider an imaginary cylinder, coaxial with the solenoid, and with length d and radius r with $r < a$. Find the rate at which energy is flowing into this cylinder from the outside, by integrating the Poynting vector over the surface.

(d) Repeat part (c), but for $r > a$.

(e) The answer you got in part (c) is $\frac{d}{dt}$(something). Find a formula for the "something", and a specific interpretation for it in words in the case $r \to a$.

(f) Extra credit: Why do you think I specified that the current was *slowly* varying? [Hint: Think about the energy density in electric fields, the energy density in magnetic fields, and how each depends on dI_s/dt.]