Physics 470 **Homework 4** **Due Wednesday, September 27, 2017**

Reading assignment: the rest of Chapter 7, and section 8.1, of Griffiths.

Problem 1. A long cylindrical cable has the z-axis as its axis of symmetry. It consists of a thin cylindrical outer shell of radius b, and a solid inner cylinder of radius a. The region with $a < r < b$ can be treated as a vacuum. (Here r is the cylindrical radial coordinate.) The inner solid cylinder carries a current I in in the $+\hat{z}$ direction, distributed uniformly over its circular cross section. The current I returns along the outer surface in the opposite direction.

(a) Find the magnetic field $\vec{\mathbf{B}}$ inside and outside of the cable.

(b) Find the energy per unit length of cable that is stored in the magnetic field.

(c) Use the results above to compute the self-inductance per unit length of the cable.

Problem 2. A transmission line is constructed from two straight parallel metal ribbons. The ribbons have width w and are a very small distance h apart and have a very long length ℓ (so that $h \ll w \ll \ell$). A current I travels down one ribbon and back along the other, and in each case it is spread out uniformly over the surface of the ribbon. The potential difference between the ribbons is held to be a constant, V.

(a) Find the energy per unit length associated with the electric field between the ribbons.

(b) Find the capacitance per unit length, $\mathcal{C}$.

(c) Find the energy per unit length associated with the magnetic field between the ribbons.

(d) Find the inductance per unit length, $\mathcal{L}$.

(e) What is $1/\sqrt{\mathcal{LC}}$, numerically? [$\mathcal{C}$ and $\mathcal{L}$ vary from one kind of transmission line to another. However, $1/\sqrt{\mathcal{LC}}$ is independent of the cross-sectional shape of the two elements of the transmission line, as long as they are separated by vacuum. It turns out to be equal to the speed at which an electromagnetic pulse would propagate down the line.]

(f) Find the Poynting vector $\vec{\mathbf{S}}$ between the ribbons.

(g) Calculate the power (energy per unit time) transported down the transmission line.

Problem 3. A very long solenoid, of radius a, with n turns per unit length, carries a current I_s. Its axis of symmetry is the z axis. A circular ring of wire with radius $b \gg a$ is coaxial with the solenoid, lies in the xy plane, and has resistance R. When the current in the solenoid is slowly increased according to $I_s = I_0 + kt$, a current I_r is induced in the ring.

(a) Calculate I_r in terms of k.

(b) The power $I_r^2 R$ delivered to the ring must have come from the solenoid. Confirm this by calculating the Poynting vector for points just *outside* the solenoid. Integrate over the entire outer surface of the solenoid, and check that you recover the correct total power. (Notes: the Poynting vector is not constant over the surface of the solenoid; it is larger closer to the ring and smaller as you move away. The electric field can be found from the changing magnetic flux in the solenoid. However, the magnetic field to be used in the Poynting vector is due to the current of the *ring*, not the solenoid, since we are outside the solenoid. Since $b \gg a$, you can use the expression for the magnetic field near the axis of a circular current ring, to be found in Example 5.6 on page 227 of Griffiths.)