Physics 470 **Homework 11** **Due December 8, 2017**

The solutions for this homework will be provided at 10:00 AM on Friday, December 8. Therefore it cannot be turned in for credit after that time.

Problem 1. A particle of mass m and charge q is attached to a spring with force constant k, hanging from the ceiling as shown in figure 11.18 on page 497 of the textbook. The particle's equilibrium position is a distance h above the floor. It is pulled down a distance d below equilibrium and released at time $t = 0$. Therefore, there is electric dipole radiation, since the electric dipole moment (with respect to the equilibrium position) oscillates with time.

(a) Under the usual assumptions that $d \ll \lambda \ll h$, show that the intensity of the radiation hitting the floor, as a function of the distance R from the point directly below q, is

$$\langle \vec{\mathbf{S}} \rangle \cdot \hat{\mathbf{n}} = K \frac{\mu_0 q^2 d^2 k^2 R^2 h}{c m^2 (R^2 + h^2)^{5/2}}$$

where K is a constant that you will compute. [Note: the intensity here is the average power per unit area of *floor*, and $\hat{n}$ is the unit normal to the floor, downwards.]

(b) Assume the floor is of infinite extent, and calculate the average energy per unit time striking the floor. [Hint: you may find the integral

$$\int_0^\infty \frac{x^3}{(x^2 + h^2)^{5/2}} dx = \frac{2}{3h}$$

to be useful.] Explain how your answer compares to the total power radiated by the charge.

Problem 2. The **radiation resistance** of a wire is defined to be the resistance that would give the same average power loss (to heat) as is actually radiated away in the form of electromagnetic waves.

(a) In the case of electric dipole radiation in the model discussed in class and in the text, show that the radiation resistance of the wire is approximately $R_{\text{rad}} = 790\,(d/\lambda)^2\,\Omega$, where d is the length of the wire and λ is the wavelength of the radiation, and Ω is the symbol for the metric system unit Ohms.

(b) In the case of magnetic dipole radiation from a circular wire of radius a, show that the radiation resistance is approximately $R_{\text{rad}} = 3 \times 10^5 (a/\lambda)^4\,\Omega$.

Problem 3. One can show (but you don't need to do it) that if an arbitrary slowly time-varying current $I(t)$ flows around a circular ring with radius a in the xy plane centered at the origin, then the retarded vector potential for large r is

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) \approx \frac{\mu_0 a^2}{4c} \left[\dot{I}(t - r/c)\right] \frac{\sin\theta}{r} \widehat{\phi}.$$

(a) Find expressions for the electric and magnetic fields in spherical coordinates, using the radiation zone approximation. (This means keep only terms proportional to $1/r$.)

(b) Find the Poynting vector and the radiated power. Write your answers in terms of $\ddot{m}(t - r/c)$, the second time derivative of the magnetic moment evaluated at the retarded time. (Note that $\ddot{m} = \pi a^2 \ddot{I}$.)

Problem 4. Suppose an electron is decelerated from some initial velocity v_0 to rest in a time T, with a constant acceleration. What fraction of the initial kinetic energy is lost to radiation? (The rest is absorbed by whatever mechanism causes the acceleration.) Assume $v_0 \ll c$ so that the Larmor formula can be used. Write your answer as a function of the charge of the electron q_e, the mass of the electron m_e, v_0, and T. (One of these quantities will drop out of the final expression.)