Physics 470 **Homework 10** **Due Friday, December 1, 2017**

Reading assignment: Griffiths chapter 10 (except section 10.2.2) and section 11.1.1.

Problem 1. Consider a rectangular wave guide with sides 1.8 cm and 1.0 cm. Which TE modes will propagate in this wave guide, if the driving frequency (not angular frequency) is $\nu = 2.6 \times 10^{10}$ Hz? For each of these modes, what is the wave number k and what is the group velocity v_g? Suppose you wanted to have one and *only* one TE mode propagate; what range of frequencies could be used?

Problem 2. Consider the potential and vector potential:

$$V(\vec{\mathbf{r}}, t) = 0, \qquad \vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{Q}{4\pi\epsilon_0} \frac{t}{r^2} \hat{r}$$

in spherical coordinates.

(a) Find the electric and magnetic fields.

(b) Use a gauge transformation function with $\lambda(\vec{\mathbf{r}}, t) = Xt/r$, where X is left as an arbitrary constant, to transform the potential and vector potential.

(c) What value of X should you choose in order to make $\vec{\mathbf{A}} = 0$? What is V in that case?

Problem 3. Suppose that in some region of vacuum (this means $\rho = 0$ and $\vec{\mathbf{J}} = 0$): the potential and vector potential are given by:

$$\begin{aligned} V(\vec{\mathbf{r}}, t) &= V_0 \sin(\omega t - az) \\ \vec{\mathbf{A}}(\vec{\mathbf{r}}, t) &= \frac{V_0}{c} \sin(\omega t - az)\, (\hat{y} + \hat{z}) \end{aligned}$$

(a) Find expressions for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$. By requiring that they satisfy Maxwell's equations, derive a relation between the quantities ω and a.

(b) Taking into account the results of (a), do the given $V(\vec{\mathbf{r}}, t)$ and $\vec{\mathbf{A}}(\vec{\mathbf{r}}, t)$ satisfy the Coulomb gauge condition, Lorenz gauge condition, both, or neither?

(c) Perform a gauge transformation so as to make $V(\vec{\mathbf{r}}, t) = 0$. What is $\vec{\mathbf{A}}(\vec{\mathbf{r}}, t)$ in the new gauge? Do the new $V(\vec{\mathbf{r}}, t)$ and $\vec{\mathbf{A}}(\vec{\mathbf{r}}, t)$ satisfy the Coulomb gauge condition, Lorenz gauge condition, both, or neither?

Problem 4. An infinitely long straight wire carries no current for $t < 0$. At time $t = 0$, the current everywhere in the wire starts to increase linearly with time:

$$I(t) = \begin{cases} 0 & (t \leq 0) \\ kt & (t \geq 0) \end{cases}$$

where k is a constant.

(a) Find the potentials $\vec{\mathbf{A}}$ and V in Lorenz gauge as a function of r (the distance to the wire) and t.

(b) Find the electric and magnetic fields as a function of r and t.

HINT: A similar problem was done in class.

Problem 5. A particle of charge Q is forced to move in a circle of radius a at a constant speed v. Assume that the circle lies in the xy plane, centered at the origin, so that its position at time t is:

$$\vec{\mathbf{r}}' = a[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}].$$

(a) Find the Liénard-Wiechert potentials for points on the z-axis. [Hint: note that the *magnitude* of the retarded position vector is constant in this case. This makes it easier than usual to solve for the retarded time.]
(b) Find the z-component of the electric field for points on the z-axis. [Hint: note that

$$E_z = -\frac{\partial V}{\partial z} - \frac{\partial A_z}{\partial t}$$

follows from equation (10.3) on page 437 of the text.]