Physics 470/570 Homework 1 Due Wednesday, September 6, 2017

Reading assignment: review chapters 1-6 of Griffiths.

Problem 1. A solid sphere with radius R is made of a linear dielectric material with electric susceptibility χ_e. The sphere has a free charge density kr, where k is a constant. Outside of the sphere ($r > R$ in spherical coordinates) is a vacuum.

(a) Find, in whatever order is most convenient, the electric field $\vec{\mathbf{E}}$, the electric displacement $\vec{\mathbf{D}}$, and the polarization $\vec{\mathbf{P}}$ for all values of r. You will need to treat $r < R$ and $r > R$ separately.

(b) Find the locations and amounts of all bound charges.

(c) Determine the electric potential $V(r)$ everywhere, by integrating the electric field. Use a point infinitely far away from the sphere as your reference point. Compute the gradient of the $V(r)$ that you found, and show that it satisfies $\vec{\mathbf{E}} = -\vec{\nabla}V$, for both $r > R$ and $r < R$.

Problem 2. A long straight wire is a solid cylinder of radius R made out of a linear material (say, copper) with magnetic susceptibility χ_m. The wire carries a free current I, distributed so that the magnitude of the free current density $\vec{\mathbf{J}}_f$ in the wire is proportional to r^2. Here r is the distance from the symmetry axis, or equivalently the radial cylindrical coordinate. (Write your answers below in terms of I, χ_m, R, and r.)

(a) Use the integral form of Ampere's Law to find $\vec{\mathbf{H}}$ everywhere inside and outside of the wire.

(b) Find the magnetic field $\vec{\mathbf{B}}$, the magnetization $\vec{\mathbf{M}}$, and the locations and amounts of all bound currents.

(c) Check your answers for $\vec{\mathbf{B}}$ and $\vec{\mathbf{H}}$ found above by direct substitutions into the derivative forms of Ampere's Law.

(d) Compute $\vec{\nabla} \cdot \vec{\mathbf{B}}$ and $\vec{\nabla} \cdot \vec{\mathbf{H}}$.

(HINTS: You may use the fact that in cylindrical coordinates, $\vec{\mathbf{H}}$ and $\vec{\mathbf{B}}$ point in the $\hat{\phi}$ direction due to the symmetry of the problem. Also, remember that there is an Ampere's Law for $\vec{\mathbf{B}}$ involving the total current, and an Ampere's Law for $\vec{\mathbf{H}}$ involving the free current. Each of these comes in an integral form and a derivative form. You might want to start by finding the free current density $\vec{\mathbf{J}}_f$ in terms of the given quantities.)

Problem 3. Label each of the following statements as true or false. For the statements that are false, explain with a brief specific reason or provide a counterexample to the statement. You do not need to explain the true statements.

(a) For any vector field $\vec{\mathbf{C}}$, $\quad \vec{\nabla} \times (\vec{\nabla} \times \vec{\mathbf{C}}) = 0$.

(b) For any vector field $\vec{\mathbf{C}}$, $\quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{C}}) = 0$.

(c) For any vector field $\vec{\mathbf{C}}$, $\quad \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{C}}) = 0$.

(d) For any scalar function f, $\quad \vec{\nabla} \times (\vec{\nabla} f) = 0$.

(e) For any scalar function f, $\quad \vec{\nabla} \cdot (\vec{\nabla} f) = 0$.

(f) Magnetic monopoles do not exist.

(g) Electric monopoles do not exist.

(h) In electrostatics, $\vec{\nabla} \cdot \vec{\mathbf{D}}$ vanishes if there are no bound charges.

(i) A piece of ferromagnetic material can contain large steady-state bound currents.

(j) The magnetic field of a magnetic dipole falls off like $1/r^2$ for large r.

(k) The vector potential of a magnetic dipole falls off like $1/r^2$ for large r.