STEVE MARTIN

SUSY = symmetry between fermions and bosons

Motivations:

6) Mathematical beauty (What's that?)

5) Unification of gauge couplings. $M_U = 2 \times 10^{16}$ GeV (major extrapolation)

4) A light Higgs (hints from precision electroweak)

3) Easy agreement with (non-flavor) precision electroweak

2) Cold dark matter particle (maybe)

1) Hierarchy Problem

A parable: electron mass

If e^- pointlike, classical electrostatic energy $= \infty$.

Model as solid sphere of radius R.

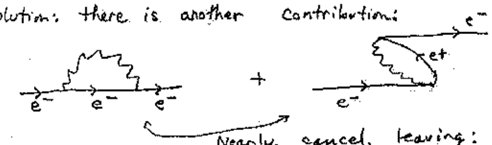
$$\Delta E_{Coulomb} = \frac{3 g_e^2}{20\pi \epsilon_0 R}$$

So $\underbrace{m_{e,phys}}_{0.511 \text{ MeV}} = m_{e,bare} + (1 \text{ MeV}/c^2)\left(\frac{0.9 \times 10^{-15} m}{R}\right)$

To avoid having to tune $M_{e,bare}$ to 1% accuracy, $R \gtrsim 10^{-17}$ meters.

But, e^- is certainly smaller!

Resolution: there is another contribution:



$$m_{e,phys} = m_{e,bare}\left[1 + \frac{3\alpha}{4\pi} \ln\left(\frac{\hbar/m_e c}{R}\right) + \ldots\right] \qquad \frac{\hbar}{m_e c} = 4 \times 10^{-13} \text{ m}$$

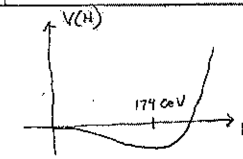
Problem solved! Even if $R \sim l_{Planck} \sim 10^{-35}$ m, only get 9% correction.

Moral: the existence of a "partner" particle for electron, the positron, eliminates dangerously huge mass contribution.

Associated symmetry: Poincaré invariance of QED.

Hierarchy Problem of SM

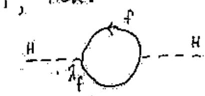
Higgs potential $V(H) = m_H^2 H^2 + \frac{\lambda}{2} H^4$ (H: neutral part of Higgs field)



$$\langle H \rangle = \sqrt{\frac{-m_H^2}{\lambda}} \approx 174 \text{ GeV}$$

If $\lambda \lesssim 1$, then: $-(\text{few } 100 \text{ GeV})^2 \lesssim m_H^2 < 0$

However:



$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2\underbrace{M_{UV}^2}_{\text{cutoff energy}} + 6 m_f^2 \ln\left(\frac{M_{UV}}{m_f}\right)\right]$$

If $M_{UV} \sim M_{Planck}$ or M_{string}, then $-\Delta m_H^2 \gg (100 \text{ GeV})^2$. Fine tuned!

Boson loops and fermion loops contribute with opposite signs.

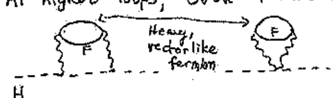
$$\Delta m_H^2 = -\frac{\lambda_f^2}{16\pi^2}(2 M_{UV}^2) + \ldots \qquad \text{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_s}{16\pi^2} M_{UV}^2 + \ldots \qquad \text{(complex scalar)}$$

Need 2 complex scalars for each Dirac fermion, and $\lambda_s = \lambda_f^2$.

A conspiracy!

At higher loops, even indirect effects are crucial:



$$\Delta m_H^2 = \# \left(\frac{g^2}{16\pi^2}\right)^2 \left[\underbrace{a}_{\text{depends on regulator}} M_{UV}^2 + 24 m_F^2 \ln\left(\frac{M_{UV}}{m_F}\right) + \ldots\right]$$

Grows with m_F^2, which can be arbitrarily large. Need to cancel this with some heavy scalar, mass $\approx m_F$.

A vast conspiracy!

SUSY makes the cancellation automatic.

SUSY takes bosons $\leftrightarrow$ fermions. An operator Q acts like:

$Q|\text{boson}\rangle = |\text{fermion}\rangle \qquad Q|\text{fermion}\rangle = |\text{boson}\rangle$

Q = anticommuting spinor. $Q^\dagger$ has same schematic action.

<u>Haag-Lopuszanski-Sohnius Theorem</u>

In a 4-d theory with chiral fermions, non-trivial scattering:

$$\{Q, Q^\dagger\} = P^\mu = (H, \vec{P}) \qquad \text{(schematic)}$$

$$\{Q, Q\} = 0 \qquad \{Q^\dagger, Q^\dagger\} = 0$$

$$[Q, P^\mu] = 0 \qquad [Q, \underbrace{T^a}_{\text{gauge generators}}] = 0$$

Single particle states fall into <u>supermultiplets</u>.

fermions bosons (superpartners)

If $|\Omega\rangle$ and $|\Omega'\rangle$ are superpartners, then $|\Omega'\rangle = f(Q, Q^\dagger)|\Omega\rangle$.

Since $[Q, P^2] = 0$ and $[Q, T^a] = 0$, superpartners must have same $(\text{mass})^2$, gauge charges.

Can also show $n_B = n_F$ within each supermultiplet.

<u>Types of supermultiplets</u>

* Chiral: 1 Weyl fermion, spin-1/2 $(n_F = 2)$
  2 real bosons, spin-0 $(n_B = 2)$
  (SM quarks, leptons, Higgs fit here).
* Vector: 1 Weyl fermion, spin-1/2 $(n_F = 2)$
  1 real boson, spin-1, massless $(n_B = 2)$
  (SM γ, W, Z, g fit here.)
* Gravity: 1 Weyl fermion, spin-3/2, massless $(n_F = 2)$
  1 real graviton, spin-2, " $(n_B = 2)$.

Each SM quark, lepton is 1 Dirac = 2 Weyl fermions

electron $\underbrace{\Psi_e}_{\text{Dirac, 4 components}} = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$ ← 2-component Weyl LH fermion / ← 2-component Weyl RH fermion

e_L and e_R each have independent spin-0 superpartners,

$\tilde{e}_L$ = LH selectron (each complex = 2 real scalars)

$\tilde{e}_R$ = RH selectron

Complete table of Minimal SUSY Standard Model (MSSM) chiral supermultiplets (all LH):

	Name	spin 0	spin 1/2	$SU(3)_c \times SU(2)_L \times U(1)_Y$
x3 families	Q	$(\tilde{u}_L \ \tilde{d}_L)$	$(u_L \ d_L)$	$(3, 2, \frac{1}{6})$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, \frac{1}{3})$
	L	$(\tilde{\nu} \ \tilde{e}_L)$	$(\nu \ e_L)$	$(1, 2, -\frac{1}{2})$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, +1)$
	H_u	$(H_u^+ \ H_u^0)$	$(\tilde{H}_u^+ \ \tilde{H}_u^0)$	$(1, 2, +\frac{1}{2})$
	H_d	$(H_d^0 \ H_d^-)$	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	$(1, 2, -\frac{1}{2})$

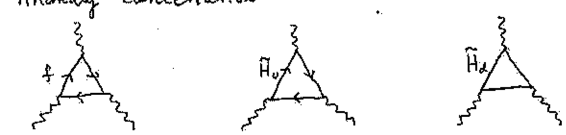
Scalar partners of quarks = "<u>squarks</u>".

" " " leptons = "<u>sleptons</u>".

Fermion partners of Higgs = "<u>higgsinos</u>".

Why <u>two</u> Higgs supermultiplets? Two reasons:

1) Anomaly cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \quad + 2\left(\tfrac{1}{2}\right)^3 \quad + 2\left(-\tfrac{1}{2}\right)^3 \quad = 0 \ \checkmark$$

needs <u>both</u>

2) Quark and lepton masses

Only $\langle H_u \rangle$ gives masses to $+2/3$ quarks (u, c, t)

Only $\langle H_d \rangle$ " " " $\begin{cases} -1/3 \text{ quarks}; & (d, s, b) \\ -1 \text{ leptons} & (e, \mu, \tau) \end{cases}$

<u>Vector Supermultiplets</u>

Spin 1	Spin ½	$SU(3)_c \times SU(2)_L \times U(1)_Y$
g	$\tilde{g}$ (gluino)	$(8, 1, 0)$
$W^\pm, W^0$	$\widetilde{W}^\pm, \widetilde{W}^0$ (winos)	$(1, 3, 0)$
B^0	$\widetilde{B}$ (bino)	$(1, 1, 0)$

generically, "gauginos"

Winos, bino will mix with higgsinos.

SUSY must be broken (spontaneously)

Otherwise, $m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511$ MeV,

$m_{\tilde{\nu}_L} = m_{\tilde{\nu}_R} = m_\nu$,

$m_{\tilde{g}} = m_g = 0$, etc.

Breaking must be <u>soft</u> = only masses, couplings with positive mass dim. to preserve solution to hierarchy problem. $(\lambda_s = \lambda_F^2)$

$$\mathcal{L} = \underbrace{\mathcal{L}_{SUSY}}_{\substack{\text{all gauge,} \\ \text{Yukawas}}} + \underbrace{\mathcal{L}_{soft}}_{\substack{\text{SUSY breaking} \\ \text{masses}}} \sim m_{soft}$$

Then $\Delta m_H^2 = m_{soft}^2 \left[\frac{\lambda}{16\pi^2} \ln\left(\frac{M_{UV}}{m_{soft}}\right) + \ldots \right]$.

So, expect $m_{soft} \lesssim 1$ TeV. Within reach of LHC!

<u>Notations</u>

I use the metric $\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$,

and two-component fermions. SUSY Primer hep-ph/9709356, Haber, Dreiner, SPM 0812.1594, $-+++$ version on my web page.

$\mathcal{L}_{Dirac} = i \Psi^\dagger \gamma^\mu \partial_\mu \Psi - M \overline{\Psi} \Psi$.

$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$ $\quad \sigma^0 = \bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad \sigma^i = -\bar{\sigma}^i =$ Pauli matrices.

$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$ $\quad \alpha = 1,2$, $\dot{\alpha} = 1,2$

ξ_α = LH Weyl spinor (undotted indices)

$\chi^{\dagger\dot{\alpha}}$ = RH Weyl spinor (dotted indices, always daggered)

$P_L \Psi_D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Psi_D = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}$, $\quad P_R \Psi_D = \begin{pmatrix} 0 \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$.

$\xi^\dagger_{\dot{\alpha}} \equiv (\xi_\alpha)^\dagger = (\xi^\dagger)_{\dot{\alpha}}$ and $(\xi^{\dagger\dot{\alpha}})^\dagger = \xi^\alpha$.

Heights of spinor indices are important.

$\xi_\alpha = \epsilon_{\alpha\beta} \xi^\beta$ and $\xi^\alpha = \epsilon^{\alpha\beta} \xi_\beta$,

with $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$

$\epsilon^{11} = \epsilon^{22} = \epsilon_{11} = \epsilon_{22} = 0$.

$\sigma^\mu, \bar{\sigma}^\mu$ carry indices like $(\sigma^\mu)_{\alpha\dot{\alpha}}$ and $(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}$.

Suppress spinor indices when contracted like

${}^{\alpha}{}_{\alpha}$ or ${}_{\dot{\alpha}}{}^{\dot{\alpha}}$

Example:

$$\xi\chi \equiv \xi^\alpha \chi_\alpha = \xi^\alpha \epsilon_{\alpha\beta} \chi^\beta = -\chi^\beta \epsilon_{\alpha\beta} \xi^\alpha = \chi^\beta \epsilon_{\beta\alpha} \xi^\alpha = \chi^\beta \xi_\beta = \chi\xi$$

Similarly:

$$\xi^\dagger \chi^\dagger = \chi^\dagger \xi^\dagger = (\xi\chi)^*$$

Homework:

Check $\xi^\dagger_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \chi_\alpha \equiv \xi^\dagger \bar{\sigma}^\mu \chi = -\chi \sigma^\mu \xi^\dagger = (\chi^\dagger \bar{\sigma}^\mu \xi)^* = -(\xi \sigma^\mu \chi^\dagger)^*$

$\xi \sigma^\mu \bar{\sigma}^\nu \chi = \chi \sigma^\nu \bar{\sigma}^\mu \xi = (\chi^\dagger \bar{\sigma}^\nu \sigma^\mu \xi^\dagger)^* = (\xi^\dagger \bar{\sigma}^\mu \sigma^\nu \chi^\dagger)^*$

$\chi_\alpha (\xi\psi) = -\xi_\alpha (\psi\chi) - \psi_\alpha (\chi\xi)$

Other useful identities:

$$\sigma^\mu_{\alpha\dot\alpha}\,\bar\sigma_\mu^{\dot\beta\beta} = -2\,\delta_\alpha^\beta\,\delta_{\dot\alpha}^{\dot\beta}$$

$$\left[\sigma^\mu\bar\sigma^\nu + \sigma^\nu\bar\sigma^\mu\right]_\alpha{}^\beta = -2\eta^{\mu\nu}\delta_\alpha^\beta\,.$$

Going back to Dirac Lagrangian:

$$\mathcal{L}_{\text{Dirac}} = i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi + i\chi^\dagger\bar\sigma^\mu\partial_\mu\chi - M(\xi\chi + \xi^\dagger\chi^\dagger)$$

(dropped a total derivative).

A Dirac fermion = 2 Weyl fermions connected by a mass.

By convention, LH always undaggered,
RH always daggered.

<u>Simplest SUSY Model</u>

Wess-Zumino = 1 chiral supermultiplet $= \begin{cases} 1 \text{ complex scalar } \phi \\ 1 \text{ Weyl fermion } \psi_\alpha \end{cases}$

$$\text{Action} = \int d^4x\,(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}})$$

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu\phi^*\partial_\mu\phi \qquad \mathcal{L}_{\text{fermion}} = i\psi^\dagger\bar\sigma^\mu\partial_\mu\psi$$

SUSY transformation for scalar:

$\delta\phi = \epsilon\psi \qquad \epsilon^\alpha$ = infinitesimal, constant spinor

$\delta\phi^* = \epsilon^\dagger\psi^\dagger$

Then: $\delta\mathcal{L}_{\text{scalar}} = -\epsilon\partial^\mu\psi\,\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\,\partial_\mu\phi$ ..

Need to cancel this with $\delta\mathcal{L}_{\text{fermion}}$.

What is $\delta\psi_\alpha$?

- Linear in $\epsilon^\dagger$ and ϕ
- Contains one ∂_μ

Only try: $\delta\psi_\alpha = \underbrace{-i}_{\text{lucky guess}}(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi$

Then $\delta\psi^\dagger_{\dot\alpha} = +i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^*$

Check for homework:

$$\delta\mathcal{L}_{\text{fermion}} = -\delta\mathcal{L}_{\text{scalar}} + \partial_\mu(\text{something}).$$

So, this is a SUSY theory.

Commutator of SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) = (\epsilon_2\sigma^\mu\epsilon_1^\dagger - \epsilon_1\sigma^\mu\epsilon_2^\dagger)\,i\underbrace{\partial_\mu}_{\substack{\text{generates infinitesimal translations,}\\ P_\mu}}\phi$$

However, not quite true for ψ_α:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) = (\epsilon_2\sigma^\mu\epsilon_1^\dagger - \epsilon_1\sigma^\mu\epsilon_2^\dagger)\,i\partial_\mu\psi_\alpha$$
$$\underbrace{+\,i\epsilon_{1\alpha}\,\epsilon_2^\dagger\bar\sigma^\mu\partial_\mu\psi - i\epsilon_{2\alpha}\,\epsilon_1^\dagger\bar\sigma^\mu\partial_\mu\psi}_{\substack{=0 \text{ by eqns of motion;}\\ \text{only on-shell.}}}$$

To fix this, introduce <u>auxiliary field</u> F. (complex scalar)

$\mathcal{L}_{\text{aux}} = F^*F \qquad$ So F has dimension $[\text{mass}]^2$, no derivatives.

Eqn. of motion: $F = F^* = 0$.

But, does transform under SUSY:

$$\begin{cases} \delta\phi = \epsilon\psi \\ \delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi + \epsilon_\alpha F \\ \delta F = -i\epsilon^\dagger\bar\sigma^\mu\partial_\mu\psi \end{cases}$$

Check for homework:

$$\mathcal{L} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{aux}}$$

has $\delta\mathcal{L} = 0$.

Also, for $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1} X) - \delta_{\epsilon_1}(\delta_{\epsilon_2} X) = (\epsilon_2 \sigma^\mu \epsilon_1^\dagger - \epsilon_1 \sigma^\mu \epsilon_2^\dagger)\, i \partial_\mu X.$$

SUSY algebra closes, gives spacetime translations.

Count degrees of freedom:

	ϕ	ψ	F	
on-shell	2	2	0	$n_B = n_F = 2$
off-shell	2	4	2	$n_B = n_F = 4$

Lagrangian linear in $\frac{\partial}{\partial t}$, momentum conjugate to ψ is $\psi^\dagger$, not independent on-shell

Noether current for SUSY:

J^μ_α = supercurrent

$$\epsilon J^\mu + \epsilon^\dagger J^{\mu\dagger} = \sum_X \delta X \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu, \quad \text{where } \delta\mathcal{L} = \partial_\mu K^\mu$$

Find $J^\mu_\alpha = (\sigma^\nu \bar{\sigma}^\mu \psi)_\alpha \partial_\nu \phi^*$

Conserved by eqns. of motion: $\partial_\mu J^\mu_\alpha = 0$

Construct conserved charges:

$$\underbrace{Q_\alpha = \sqrt{2}}_{\text{silly convention}} \int d^3\vec{x}\, J^0_\alpha \qquad Q^\dagger_{\dot\alpha} = \sqrt{2} \int d^3\vec{x}\, J^{\dagger 0}_{\dot\alpha}$$

Then $[\epsilon Q + \epsilon^\dagger Q^\dagger, X] = -i\sqrt{2}\, \delta_\epsilon X$.

Can now check:

SUSY algebra
$$\begin{cases} \{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \\ \{Q_\alpha, Q_\beta\} = 0 \qquad \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \\ [Q_\alpha, P^\mu] = 0 \qquad [Q^\dagger_{\dot\alpha}, P^\mu] = 0 \end{cases}$$

$P^\mu = (\underbrace{H}_{\text{Hamiltonian}}, \vec{P})$

What about masses, interactions?

Given $(\phi_i, \psi_{i\alpha}, F_i)$, construct superpotential:
i ↖ "flavor" index

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$$

holomorphic function of ϕ_i (not ϕ_i^*), then let

$$W^i = \frac{\partial W}{\partial \phi_i} \qquad W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

Can check that

$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$ is SUSY invariant, with

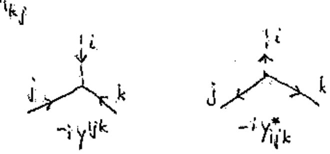
$$\mathcal{L}_{free} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i \psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i$$

$$\mathcal{L}_{int} = \left(-\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i\right) + \text{c.c.}$$

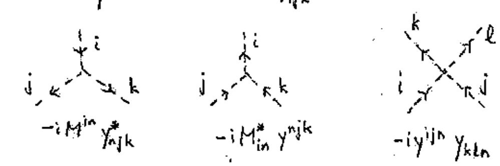
After eliminating auxiliary fields F_i, F^{*i} by eqns. of motion:

$m_\phi^2 = m_\psi^2 = M^{ik} M^*_{kj}$

Yukawa interactions:



Scalar interactions:



All masses, non-gauge couplings fixed by W.

<u>SUSY Gauge Theories</u>

$\underbrace{A^a_\mu}_{\text{vector boson}}, \quad \underbrace{\lambda^a_\alpha}_{\text{gaugino}}, \quad D^a$ ← auxiliary = <u>real</u> scalar

$$a = \begin{cases} 1, \ldots, 8 & \text{for } SU(3)_c \\ 1, 2, 3 & \text{for } SU(2)_L \\ 1 & \text{for } U(1)_Y. \end{cases}$$

Count degrees of freedom:

	A_μ	λ	D
on-shell	2	2	0
off-shell	3	4	1 (real)

(1 removed by gauge transformation.)

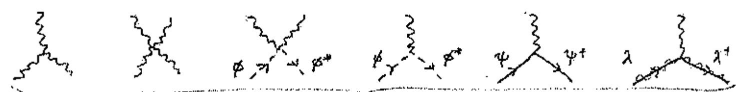
$$\mathcal{L}_{gauge} = -\tfrac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i \lambda^{\dagger} \bar{\sigma}^{\mu} \nabla_\mu \lambda^a + \tfrac{1}{2} D^a D^a$$

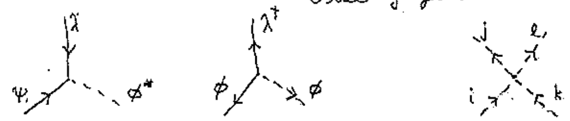
Coupled to chiral multiplets (ϕ_i, ψ_i, F_i):

$$\mathcal{L} = \mathcal{L}_{chiral} + \mathcal{L}_{gauge} - \sqrt{2} g (\phi^* T^a \psi) \lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^{\dagger} T^a \phi) + g(\phi^* T^a \phi) D^a$$

Eliminate $D^a = -\phi^* T^a \phi$.

Interactions are:





$-i\sqrt{2} g T^a$ $\quad$ $-i\sqrt{2} g T^a$ $\quad$ $-ig^2 (T_i^{aj} T_k^{a\ell} + T_i^{a\ell} T_k^{aj}) =$ "D-term"

All determined by SUSY. (No choices to make.)

<u>Soft SUSY breaking</u>

$$\mathcal{L}_{soft} = -\tfrac{1}{2}(M_a \lambda^a \lambda^a + c.c.) - (m^2)^i_j \phi^{*j} \phi_i$$
$$- (\tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + c.c.)$$

- M_a = gaugino masses $\sim m_{soft}$
- scalar (mass)^2 $(m^2)^i_j$, b^{ij} $\sim m^2_{soft}$
- (scalar)^3 a^{ijk} $\sim m_{soft}$

} To be explained by deeper model

with $m_{soft} \lesssim$ TeV

<u>How to build a SUSY theory</u>

1) Choose a gauge group
(Already done in MSSM: $SU(3)_c \times SU(2)_L \times U(1)_Y$)

2) Choose a superpotential $W(\phi_i)$, or $W(\Phi_i)$ equivalently
(ϕ_i: scalars; Φ_i: superfields)

(In MSSM, almost done; Yukawas already known.)

3) Choose $\mathcal{L}_{soft}$ (or a method for SUSY breakdown)

(In MSSM, this is where unknowns are.)

Superpotential for MSSM:

$$W_{MSSM} = \bar{u}\, y_u\, Q H_u - \bar{d}\, y_d\, Q H_d - \bar{e}\, y_e\, L H_d + \mu H_u H_d$$

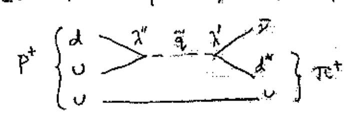
y_u, y_d, y_e: 3x3 Yukawa matrices

μ: SUSY Higgs, higgsino mass. 1 new parameter

Actually, could have:

$$W_{\Delta L=1} = \tfrac{1}{2} \lambda L L \bar{e} + \lambda' L Q \bar{d} + \mu' L H_u$$
$$W_{\Delta B=1} = \tfrac{1}{2} \lambda'' \bar{u} \bar{d} \bar{d}$$

If both are present, rapid proton decay!



So $p^+ \to \pi^+ \bar{\nu}$ or $K^+ \bar{\nu}$ or $\pi^0 \ell^+$ or...

$$\Gamma_{p^+} \sim \frac{m_p^5}{m_{\tilde{q}}^4} |\lambda'' \lambda'|^2 \ll 1 \text{ second if } \lambda'', \lambda' \sim 1.$$

To avoid this, assume <u>matter parity</u> conserved.

$$P_M = (-1)^{3(B-L)}$$

B = baryon number
L = lepton number

$$P_M = \begin{cases} -1 & \text{quarks, leptons, squark, sleptons} \\ +1 & \text{everything else.} \end{cases}$$

Forbids $W_{\Delta L=1}$, $W_{\Delta B=1}$. ✓

<u>R-parity</u> $\quad P_R = (-1)^{3(B-L)+2\underbrace{J}_{\text{spin}}}$ exactly <u>equivalent</u> to matter parity

$$P_R = \begin{cases} +1 & \text{all SM particles (quarks, leptons, } \gamma, W, Z, g, \text{Higgs)} \\ -1 & \text{all superpartners} \end{cases}$$

Consequences:
* Lightest SUSY Particle (LSP) is <u>stable</u>. $(P_R = -1)$.
  - Dark matter - Missing energy at colliders.
* Superpartners produced in <u>pairs</u>.
* Heavier supepartners eventually decay to LSP.

$$\begin{aligned}\mathcal{L}^{MSSM}_{soft} = & -\tfrac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B}\right) + c.c. && \leftarrow \text{gaugino masses} \\ & -\tilde{Q}^+ m_Q^2 \tilde{Q} - \tilde{L}^+ m_L^2 \tilde{L} - \tilde{\bar{u}}^+ m_{\bar{u}}^2 \tilde{\bar{u}} - \tilde{\bar{d}}^+ m_{\bar{d}}^2 \tilde{\bar{d}} - \tilde{\bar{e}}^+ m_{\bar{e}}^2 \tilde{\bar{e}} && \leftarrow 3\times 3 \text{ squark slepton (mass)}^2 \\ & - m_{H_u}^2 H_u^+ H_u - m_{H_d}^2 H_d^+ H_d - (b H_u H_d + c.c.) && \leftarrow \text{Higgs soft (mass)}^2 \\ & -\left(\tilde{\bar{u}}\, a_u \tilde{Q} H_u + \tilde{\bar{d}}\, a_d \tilde{Q} H_d - \tilde{\bar{e}}\, a_e \tilde{L} H_d + c.c.\right) && \leftarrow (\text{scalar})^3 \end{aligned}$$

Careful count: ~100 new parameters!

Let's work out MSSM mass spectrum.

Higgs sector:

$$\begin{aligned} V = & \left(|\mu|^2 + m_{H_u}^2\right)|H_u^0|^2 + \left(|\mu|^2 + m_{H_d}^2\right)|H_d^0|^2 - (b H_u^0 H_d^0 + c.c.) && \leftarrow \phi^2 \\ & + \tfrac{1}{8}(g^2+g'^2)\left(|H_u^0|^2 - |H_d^0|^2\right)^2 && \leftarrow \phi^4 \end{aligned}$$

The ϕ^4 coupling is fixed by SUSY to be small.

$\frac{1}{8}(g^2+g'^2) \simeq 0.07 \quad \longrightarrow$ Light Higgs!

Two VEVs at minimum: $\quad v_u = \langle H_u^0 \rangle \qquad v_d = \langle H_d^0 \rangle$.

$v_u^2 + v_d^2 = (174 \text{ GeV})^2$ (known).

$\frac{v_u}{v_d} = \tan\beta$ (unknown, but likely between 2 and 60 in MSSM)

Minimizing potential gives:

$$m_Z^2 = \frac{g^2+g'^2}{2}(v_u^2+v_d^2) = -2\underbrace{(m_{H_u}^2 + |\mu|^2)}_{} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right)$$

These need to nearly cancel: $m_{H_u}^2 \simeq -|\mu|^2$, requires % fine tuning?

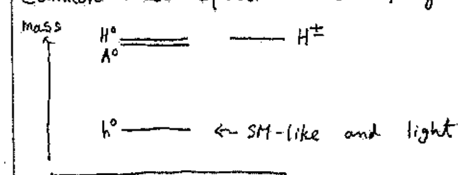
Find mass eigenstate Higgs bosons: $h^0, H^0, A^0, \underbrace{G^0, H^\pm, G^\pm}_{\text{would-be Goldstones; eaten by } W, Z}$

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}}\begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix}\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

h^0, H^0 = scalars $\qquad A^0$ = pseudoscalar

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix}\begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

Common mass spectrum: decoupling limit $m_{A^0}^2 \gg m_Z^2$



$$m_{h^0}^2 = \underbrace{m_Z^2 \cos^2(2\beta)}_{\text{maximum for large } \beta} + \frac{3}{4\pi^2}\sin^2\beta\, y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$

important, messy

$\tilde{t}_1, \tilde{t}_2$ = top squarks

Need $\tilde{t}_1, \tilde{t}_2$ not too light, <u>or</u> highly mixed from $\tilde{t}_L, \tilde{t}_R$, <u>or</u> non-minimal model.

<u>Neutralinos</u> mixtures of $\widetilde{B}$, $\widetilde{W}^0$ gauginos, $\widetilde{H}_u^0$ $\widetilde{H}_d^0$ higgsinos

I call them $\widetilde{N}_i$ $(i=1,2,3,4)$. Others: $\widetilde{\chi}_i^0$

$$M_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ " & " & 0 & -\mu \\ " & " & -\mu & 0 \end{pmatrix} \quad \text{(symmetric)}$$

Many models assume $M_1 \ll M_2, |\mu|$, so

$LSP = \widetilde{N}_1 =$ bino-like

<u>Charginos</u> mixtures of $\widetilde{W}^\pm$ gauginos, $\widetilde{H}_u^+, \widetilde{H}_d^-$ higgsinos

I call them $\widetilde{C}_i^\pm$ $(i=1,2)$. Others: $\widetilde{\chi}_i^\pm$

Mass matrix: $\begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix}$.

Typical mass spectrum:

$\uparrow$ mass

$\widetilde{N}_4$, $\widetilde{N}_3$	$\widetilde{C}_2^\pm$	← higgsino-like	$\sim \mu$
$\widetilde{N}_2$	$\widetilde{C}_1^\pm$	← wino-like	$\sim M_2$
$\widetilde{N}_1 = LSP$		← bino-like	$\sim M_1$

<u>Gluino</u> Color octet fermion; can't mix with anything.

A popular assumption: $M_1 = M_2 = M_3$ at GUT scale

then run down to TeV scale:

$M_3^{phys} : M_2^{phys} : M_1^{phys} \simeq 6 : 3 : 1$ ← <u>Totally</u> assumption! Don't believe the hype!

<u>Squarks + Sleptons</u>

In general:
* 6×6 $(mass)^2$ matrix for $\tilde{u}_L\ \tilde{c}_L\ \tilde{t}_L\ \tilde{u}_R\ \tilde{c}_R\ \tilde{t}_R$
* 6×6 $(mass)^2$ matrix for $\tilde{d}_L\ \tilde{s}_L\ \tilde{b}_L\ \tilde{d}_R\ \tilde{s}_R\ \tilde{b}_R$
* 6×6 $(mass)^2$ matrix for $\tilde{e}_L\ \tilde{\mu}_L\ \tilde{\tau}_L\ \tilde{e}_R\ \tilde{\mu}_R\ \tilde{\tau}_R$
* 3×3 $(mass)^2$ matrix for $\tilde{\nu}_e\ \tilde{\nu}_\mu\ \tilde{\nu}_\tau$

Usual assumption: small flavor mixing for 1st, 2nd families.

Evidence:

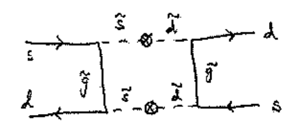
1) <u>$\mu \to e\gamma$</u>

$$\Gamma(\mu^- \to e^-\gamma) = 5\times10^{-15}\,eV \left(\frac{m^2_{\tilde{\mu}_L \tilde{e}_L}}{\widetilde{M}^2}\right)^2 \left(\frac{100\ GeV}{\widetilde{M}}\right)^4$$

Experiment: $\Gamma(\mu^- \to e^-\gamma) < 3.6\times10^{-21}\,eV$

So, $\left(\frac{m^2_{\tilde{\mu}_L \tilde{e}_L}}{\widetilde{M}^2}\right) < 0.02 \left(\frac{\widetilde{M}}{500\ GeV}\right)^2$.

2) <u>Meson mixing</u> $K^0 \leftrightarrow \overline{K^0}$



Constraint: $\frac{\left[m^2_{\tilde{s}_R \tilde{d}_R}\ m^2_{\tilde{s}_L \tilde{d}_L}\right]^{1/2}}{m^2_{\tilde{q}}} < 0.002 \left(\frac{m_{\tilde{q}}}{1\ TeV}\right)$

So, assume SUSY breaking is <u>flavor-blind</u>.

1st and 2nd family sfermions come in 7 nearly degenerate, unmixed lines:

$(\tilde{e}_R, \tilde{\mu}_R)$ $(\tilde{e}_L, \tilde{\mu}_L)$ $(\tilde{\nu}_e, \tilde{\nu}_\mu)$ $(\tilde{u}_R, \tilde{c}_R)$ $(\tilde{d}_R, \tilde{s}_R)$ $(\tilde{u}_L, \tilde{c}_L)$ $(\tilde{d}_L, \tilde{s}_L)$.

The 3rd family quarks and leptons are different, because of large RG effects from Yukawas, and $(scalar)^3$ couplings.

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<u>mSUGRA</u> (aka "CMSSM") = "minimal supergravity"

Assume at RG scale $Q = 2 \times 10^{16}$ GeV:

* $M_1 = M_2 = M_3 \equiv m_{1/2}$ common gaugino mass
* $m^2_{\phi_i} = m_0^2$ common scalar $(\text{mass})^2$
* $a_f = A_0 y_f$ common $(\text{scalar})^3$

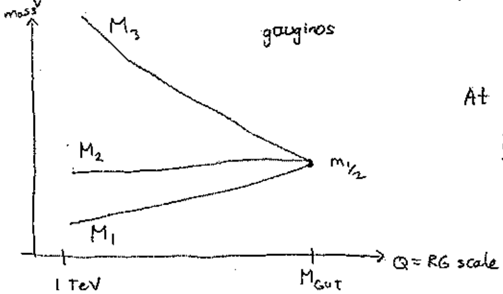
Other inputs: $\tan\beta$, $\text{Arg}(\mu)$ usually assumed real.

Crucial Fact about mSUGRA: it's <u>wrong</u>.

But, it is a convenient benchmark.

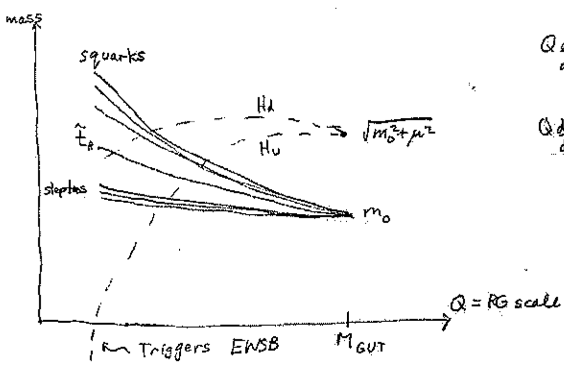
Use RG equations to find physical masses.

Programs that do this: SoftSUSY, SuSpect, ISASUSY, Spheno.



At 1-loop:

$$\frac{M_i}{m_{1/2}} = \frac{\alpha_i}{\alpha_{GUT}}$$



$$Q\frac{d}{dQ} m^2_{\text{squark}} = -\frac{8\alpha_s}{3\pi} M_3^2 + \ldots$$

$$Q\frac{d}{dQ} m^2_{H_u} = \frac{3y_t^2}{8\pi^2}\left(m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R}\right)$$

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More general than mSUGRA = Flavor-Preserving MSSM.

* $M_1\ M_2\ M_3$ (3 real gaugino masses)
* $m^2_{\tilde{Q}}, m^2_{\tilde{u}}, m^2_{\tilde{d}}, m^2_{\tilde{L}}, m^2_{\tilde{e}}$ (5 sfermion $[\text{mass}]^2$)
* A_{u0}, A_{d0}, A_{e0} (3 real $[\text{scalar}]^3$ couplings)
* $m^2_{H_u}, m^2_{H_d}, b, \mu$ (4 real Higgs masses)

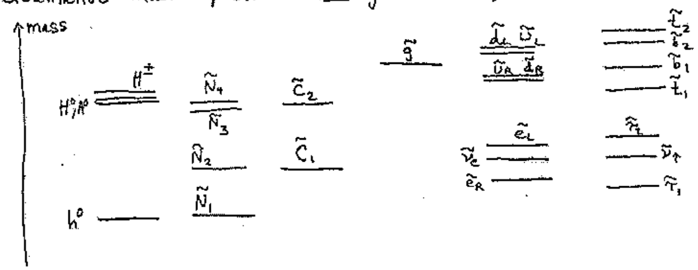
} 15 real parameters

Trade μ and b for $\sqrt{v_u^2+v_d^2} = 174$ GeV and $\frac{v_u}{v_d} = \tan\beta$ and $\text{Arg}(\mu)$.

Also, need an input scale Q_0 for RG equations.

$Q_0 = M_{GUT}$? M_{Planck}? something else?

Qualitative mass spectrum (<u>not</u> guaranteed)



Columns may slide up/down, get compressed, change order, etc.

Top-squark mixing:

$$\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}}\, e^{i\delta} \\ -\sin\theta_{\tilde{t}}\, e^{-i\delta} & \cos\theta_{\tilde{t}} \end{pmatrix} \underbrace{\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}}_{\text{mass eigenstates}}$$

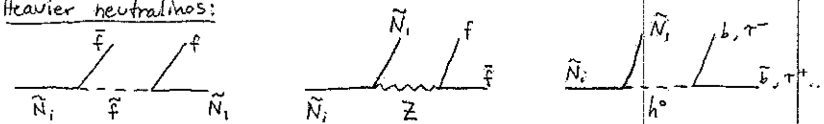
$$(\text{mass})^2 \text{ matrix} = \begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + \Delta_L & a_t^* v_u - \mu y_t v_d \\ " \quad " \nearrow & m^2_{\tilde{u}_3} + m_t^2 + \Delta_R \end{pmatrix}$$

Large mixing if $a_t H_u \tilde{t}_L^* \tilde{t}_R$ coupling is big.

Superpartner Decays (assume R-parity)

$\tilde{N}_1$ = LSP can't decay.

Heavier neutralinos:



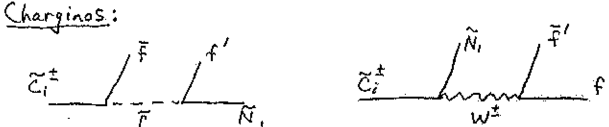
In each case, might be 2-body or 3-body.

$\tilde{N}_i \to jj + \not{E}_T$ as seen in detector.

$\tilde{N}_i \to \ell^+\ell^- + \not{E}_T$ ← more likely if $m_{\tilde{\ell}} < m_{\tilde{N}_2}$.

If allowed, $\tilde{N}_2 \to h^0 \tilde{N}_1$ can dominate, BR ~ 90%.
Often called "spoiler mode", because no leptons.
Simultaneous discovery of $h^0 \to b\bar{b}$ and SUSY?

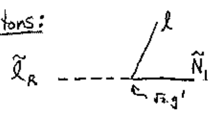
Charginos:



So $\tilde{C}_i \to jj + \not{E}_T$ or $\tilde{C}_i^\pm \to \ell^\pm + \not{E}_T$ ← more likely for light sleptons.

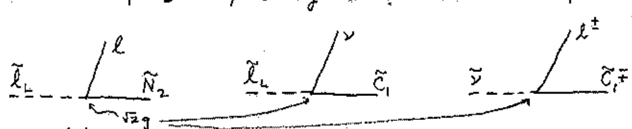
If staus are light, $\ell^\pm$ may be mostly $\tau^\pm$.

Sleptons:



$\tilde{\ell}_R \to \ell + \not{E}_T$

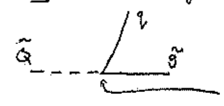
But LH sleptons may decay to wino-like states preferentially:



in addition.

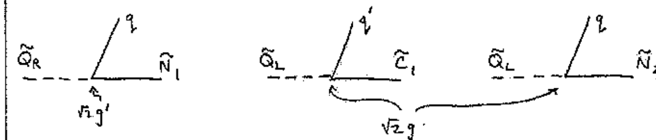
3-body decays not needed, $m_{\tilde{\ell}} > m_\ell + m_{LSP}$.

Squarks If kinematically allowed,



dominates. $\tilde{Q} \to q\tilde{g}$ for $m_{\tilde{Q}} > m_{\tilde{g}} + m_q$

Otherwise,



Top squark: $\tilde{t}_1 \to t\tilde{N}_1$ may be kinematically closed.

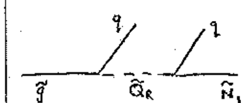
maybe $\tilde{t}_1 \to b\tilde{C}_1$. If that's also closed,

maybe $\tilde{t}_1 \to bW\tilde{N}_i$ If that's also closed, ...

maybe $\tilde{t}_1 \to c\tilde{N}_1$ (violates flavor) or $\tilde{t}_1 \to bf\bar{f}'\tilde{N}_1$ (4-body)

Stoponium? ($\tilde{t}_1^* \tilde{t}_1$) bound state, rare decay to $\gamma\gamma$.

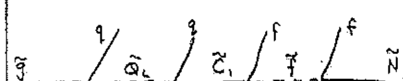
Gluino can only decay through squarks (on-shell or virtual)



$jj + \not{E}_T$ or $t\bar{t} + \not{E}_T$

Cascade decays:

$jjjj + \not{E}_T$ or $jj\,t\bar{t} + \not{E}_T$
or $jj\,\ell^+\ell^- + \not{E}_T$



$jjjj + \not{E}_T$ or $jj\,t\bar{t} + \not{E}_T$
or $jj\,\ell^\pm + \not{E}_T$

If $m_{\tilde{t}_1} \ll m_{\tilde{Q}}$, top quarks may appear.

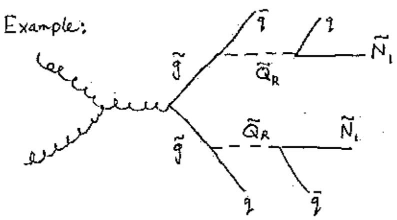
Hadron Collider Searches

Largest cross-sections are typically:

$PP \to \tilde{g}\tilde{g}$, $\tilde{g}\tilde{Q}$, $\tilde{Q}\tilde{Q}$

Get jets + $\not{E}_T$, maybe leptons from decays through $\tilde{C}$, $\tilde{N}$.

Example:



$PP \to \tilde{g}\tilde{g} \to jjjj + \not{E}_T$

Veto events with isolated energetic leptons to kill $W \to \ell\nu$ backgrounds.

Typical cuts: P_T of leading jet > 120 GeV or more

1 or more other jets $P_T > 40$ GeV (n=2,3 ?)

Large $\not{E}_T$ > 100 GeV or more

$m_{eff} \equiv \not{E}_T + \sum_n P_T(j_n)$ > 500 GeV or more ← heavy $\tilde{g}$ or $\tilde{Q}$

$\Delta\phi(\not{E}_T, P_T(j))$ > 0.2 ← avoids backgrounds from mismeasured jets.

Biggest backgrounds:

$PP \to W + \text{jets} \to \underset{\text{miss}}{\ell}\underset{\not{E}_T}{\nu} + \text{jets}$

$PP \to Z + \text{jets} \to \underset{\not{E}_T}{\nu\bar{\nu}} + \text{jets}$

} roughly comparable.

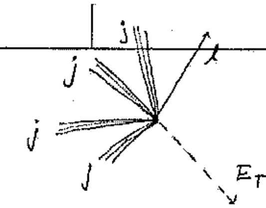
Can also look for signal

$PP \to \text{jets} + \ell^{\pm} + \not{E}_T$.

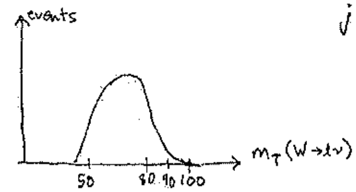
Despite the W+jets background, reach may be better for models with lots of decays through sleptons, charginos.

Require transverse mass $m_T > 100$ GeV (usually smaller for W's)

$$m_T = \sqrt{2 p_T^{\ell} \not{E}_T (1 - \cos\Delta\phi)}$$



For W's



Same sign dileptons: recall gluino can decay like

$\tilde{g} \to q\bar{q}\,\tilde{C}_1^{\pm} \to q\bar{q}\,\ell^{\pm}\nu\not{E}_T$

The gluino is Majorana, so ℓ^+ and ℓ^- each 50%, uncorrelated with rest of event.

So $PP \to \tilde{g}\tilde{g}$ (or $\tilde{g}\tilde{Q}$ or $\tilde{Q}\tilde{Q}$, if $\tilde{Q} \to q\tilde{g}$) gives

jets + $\ell^+\ell^+ + \not{E}_T$ (25%)
jets + $\ell^-\ell^- + \not{E}_T$ (25%) } <u>same sign</u>, uncorrelated flavors very low background
jets + $\ell^+\ell^- + \not{E}_T$ (50%)

Other search elements:

* Light $\tilde{t}_1$, $\tilde{b}_1$ ⟹ high b-tag rate $\tilde{t}_1 \to bW\tilde{N}_1$
* Light $\tilde{\tau}_1$ ⟹ high τ rate $\tilde{\tau}_1 \to \tau\tilde{N}_1$

* Gauge-Mediated SUSY Breaking

  $\tilde{N}_1 \to \gamma\tilde{G}$ $\tilde{G}$ = gravitino/goldstino : very light, invisible.

  $\tilde{\ell} \to \ell\tilde{G}$

  These could be stable on detector size scale; quasi-stable $\tilde{\ell}$ with anomalous ionization, time-of-flight (very heavy)

* Very non-mSUGRA-like mass spectrum.
* R-parity violation LSP decays to visible states.

<u>LSP as dark matter</u>

$\Omega_{DM} h^2 = 0.11$ (WMAP) h = Hubble "constant" in $100 \frac{km}{sec \cdot Mpc} \approx 0.7$

fraction of critical density.

$\Rightarrow$ Thermal averaged $\langle \sigma v \rangle_{annihilation} \sim 1$ pb

If weakly interacting,

$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \left(\frac{150 \text{ GeV}}{M}\right)^2$ Very rough, but coincides with hierarchy problem. estimate.

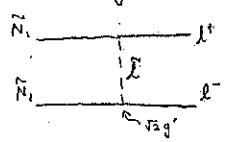
Could $\tilde{N}_1$ = LSP = dark matter?

If $\tilde{N}_1$ = mostly wino or higgsino, $\langle \sigma v \rangle$ is too big.

If $\tilde{N}_1$ = mostly bino, $\langle \sigma v \rangle$ generically too small (too much dark matter)

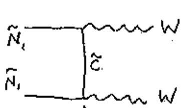
Need a mechanism for efficient annihilation.

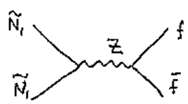
1) "Bulk region" = t-channel annihilation through sleptons



Needs light sleptons
(mSUGRA version dead from Tevatron, LHC)

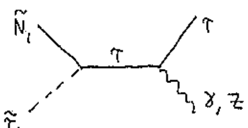
2) "Focus point / small μ" = bino/higgsino mixing

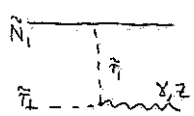




(in mSUGRA, occurs for very heavy squarks)

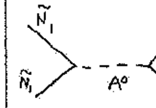
3) "Stau (or sfermion) coannihilation"
coexists with $\tilde{N}_1$ in thermal equilibrium in early universe

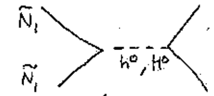




Needs $m_{\tilde{\tau}_1} - m_{\tilde{N}_1}$ small.

4) "Higgs funnel" $m_{A^0} \approx 2 m_{\tilde{N}_1}$, so near resonance:





<u>What if your model predicts too much dark matter?</u>

OK! Introduce another P_R-odd singlet fermion $\tilde{S}$. NMSSM = MSSM + singlet

$\tilde{N}_1 \to \tilde{S}$ + something in early universe,

$\Omega_{\tilde{S}} = \Omega_{\tilde{N}_1}^{predicted} \left(\frac{m_{\tilde{S}}}{m_{\tilde{N}_1}}\right)$ reduces density.

<u>What if your model predicts too little dark matter?</u>

OK! could be something else (axion?)

Direct detection of $\tilde{N}_1$ dark matter.

$\rho_{DM}^{here} \simeq 0.3 \text{ GeV/cm}^3 = 3000 \frac{\text{LSP's}}{m^3} \left(\frac{100 \text{ GeV}}{m_{\tilde{N}_1}}\right)$

elastic scattering $\Rightarrow$ 10's of keV recoil

rate = (flux) $n_{target}\, \sigma_{LSP\text{-}nucleon}\, A^2$ heavier is better

For $m_{\tilde{N}_1} = 100$ GeV, $\sigma_{LSP\text{-}nucleon} \lesssim 10^{-44}$ cm^2 XENON100

Starting to rule out SUSY models.

Caution: limit uncertain by factor of ~2, due to nuclear matrix elements.

Origins of SUSY breaking

$\mathcal{L}$ invariant under SUSY, $|0\rangle$ = vacuum state isn't.

So $Q_\alpha|0\rangle \neq 0$ and/or $Q^\dagger_{\dot\alpha}|0\rangle \neq 0$.

From SUSY algebra:

$$H = P^0 = \tfrac{1}{4}\left(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2\right)$$

So, if SUSY weren't broken, $H|0\rangle = 0$.

If SUSY is broken, then

$$\langle 0|H|0\rangle = \tfrac{1}{4}\left(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\right) > 0$$

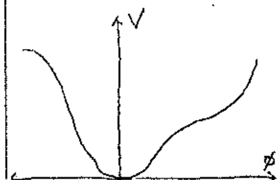
Need a theory with $|0\rangle$ having energy > 0.

From SUSY Lagrangian:

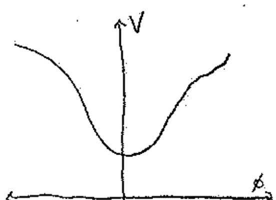
$$V = \sum_i |F_i|^2 + \tfrac{1}{2}\sum_a D_a^2 = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \tfrac{1}{2}\sum_a g_a^2 (\phi^\dagger T^a \phi)^2$$

This is ≥ 0.

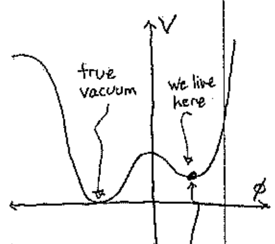
Possibilities:



SUSY not broken; not us.



SUSY broken.
Models before 2006.
No SUSY preserving vacuum.



Metastable SUSY breaking. Our vacuum will eventually decay, but lifetime $\gg$ age of universe
Intriligator, Seiberg, Shih, 0602239
"Simple" models!

Two types of SUSY-breaking models:

- $\langle F\rangle \neq 0$ O'Raifeartaigh ← most realistic known models
- $\langle D\rangle \neq 0$ Fayet-Iliopoulos

F-I Model: $U(1)$ gauge theory, supermultiplets (ϕ_i, ψ_i) charges q_i.

Add $\mathcal{L}_{FI} = -\underset{\substack{\uparrow\\ \text{parameter}}}{\kappa}\, D$ ← auxiliary field, transforms into ∂_μ(something)

$$V = \kappa D - \tfrac{1}{2}D^2 - gD\sum_i q_i|\phi_i|^2 + \sum_i \underbrace{M_i^2}_{\text{SUSY masses}}|\phi_i|^2$$

Equation of motion: $D = \kappa - g\sum_i |\phi_i|^2$

$$V = \tfrac{1}{2}\left(\kappa - g\sum_i q_i|\phi_i|^2\right)^2 + \sum_i M_i^2|\phi_i|^2$$

Can't vanish, so SUSY broken.

Global minimum of V for $\phi_i = 0$ (if $M_i^2 > gq_i\kappa$).

Scalar mass²: $M_i^2 - gq_i\kappa$
Fermion mass²: M_i^2
✓ SUSY broken

O'R Model: 3 chiral supermultiplets Φ_1, Φ_2, Φ_3.

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \tfrac{y}{2}\Phi_1\Phi_3^2$$

So $F_1^* = -\dfrac{\partial W}{\partial \Phi_1} = k - \tfrac{y}{2}\phi_3^2$

$F_2^* = -\dfrac{\partial W}{\partial \Phi_2} = -m\phi_3$

can't both be 0 ⟹ SUSY broken

$F_3^* = -\dfrac{\partial W}{\partial \Phi_3} = -m\phi_2 - y\phi_1\phi_3$

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2$$

Expand to quadratic order to find...

Scalar $(\text{mass})^2$: $0, 0, m^2, m^2, m^2+yk, m^2-yk$

Fermion $(\text{mass})^2$: $0 \quad m^2 \quad \underbrace{m^2}_{\text{SUSY broken ✓}}$

Why massless particles?

$\phi_1 \leftrightarrow$ flat direction in V $\qquad V(\phi_1, \phi_2=0, \phi_3=0) = 0$ for all ϕ_1.

This is an accidental feature of tree-level potential.

At 1-loop order, $m^2_{\phi_1} = \frac{y^4 k^2}{48\pi^2 m^2}$.

Ψ_1 = goldstino. Spontaneously broken SUSY $\leftrightarrow$ goldstino = massless fermion.

Remains exactly massless including loop effects.

In supergravity = local supersymmetry, goldstino is eaten by gravitino, giving it mass.

Analogy:

$\underline{SU(2)_L \times U(1)}$	$\underline{\text{Supergravity}}$
$\langle H^0 \rangle \neq 0$	$\langle F \rangle \neq 0$
Would-be Goldstones ($J=0$)	goldstino ($J=\frac{1}{2}$)
W, Z ($J=1$)	gravitino ($J=\frac{3}{2}$)

<u>SUSY breaking must be separate from MSSM</u>

1) Masses for gauginos?

$$\mathcal{L} = -\frac{F}{2M}\lambda\lambda + \text{c.c.} \quad \Rightarrow m_\lambda = \frac{\langle F \rangle}{M}.$$

But there is no renormalizable term $F\lambda\lambda$ in SUSY.

2) Sum rule:

Even after SUSY breaking, at tree level,

$$STr(m^2) = Tr(m_S^2) - 2\,Tr(m_F^2) + 3\,Tr(m_V^2) = 0.$$

In MSSM, $m^2_{\tilde{e}_1} + m^2_{\tilde{e}_2} = 2m_e^2 \quad$ X.

$\boxed{\text{SUSY breaking } \langle F \rangle \neq 0}$ - - $\underline{\text{flavor-blind}}$ "messengers" - - $\boxed{\text{MSSM } m_{soft}}$

Dimensional analysis: $m_{soft} = \frac{\langle F \rangle}{M_{mess}}$ $\qquad M_{mess}$ = mass scale of messenger physics

<u>Planck (or Gravity) Mediated</u>: $M_{mess} = M_{Planck}$

Since $m_{soft} \sim 10^3$ GeV, $M_{Planck} \sim 2.4 \times 10^{18}$ GeV, need $\sqrt{\langle F \rangle} \sim 10^{10}$ or 10^{11} GeV

Write an effective theory that couples to MSSM $\begin{cases} \text{gauginos } \lambda^a \\ \text{scalars } \phi_i \end{cases}$

$$\mathcal{L}_{PMSB} = \left(-\frac{f^a}{2M_p} F \lambda^a \lambda^a + \text{c.c.}\right) - \frac{k_i^j}{M_p^2} F F^* \phi_i \phi^{*j}$$
$$- \left(\frac{\alpha^{ijk}}{6M_p} F \phi_i \phi_j \phi_k + \frac{\beta^{ij}}{2M_p} F \phi_i \phi_j + \text{c.c.}\right)$$

Now when $\langle F \rangle \neq 0$:

- Gaugino masses $\quad M_a = \frac{f^a \langle F \rangle}{M_p} \sim m_{soft}$
- Scalar $(\text{mass})^2 \quad (m^2)_i{}^j = k_i^j \frac{|\langle F \rangle|^2}{M_p^2} \sim m^2_{soft}$

$$b^{ij} = \beta^{ij} \frac{\langle F \rangle}{M_p} \sim m^2_{soft}$$

- $(\text{scalar})^3$ couplings $\quad a^{ijk} = \alpha^{ijk} \frac{\langle F \rangle}{M_p} \sim m_{soft}$

Not obvious why flavor blind. Does <u>not</u> follow from Equivalence Principle.

For mSUGRA, need $f^1 = f^2 = f^3$ and $k_i^j = k\,\delta_i^j$ and $\alpha^{ijk} = \alpha\, y^{ijk}$.

<u>Might</u> follow from deeper theory, but not obvious.

<u>Gauge-Mediated SUSY Breaking</u>

Usual $SU(3)_c \times SU(2)_L \times U(1)_Y$ communicates the breaking.

Minimal model:

New, vectorlike, chiral supermultiplets, <u>messengers</u>

$q \sim (3, 1, -\frac{1}{3}) \quad \bar{q} \sim (\bar{3}, 1, \frac{1}{3}) \quad \ell \sim (1, 2, \frac{1}{2}) \quad \bar{\ell} \sim (1, 2, -\frac{1}{2})$

Superpotential:

$$W_{mess} = y_3 S q \bar{q} + y_2 S \ell \bar{\ell}$$

Assume S is part of SUSY-breaking, so that

$\langle S \rangle \neq 0$ and $\langle F_S \rangle \neq 0$.

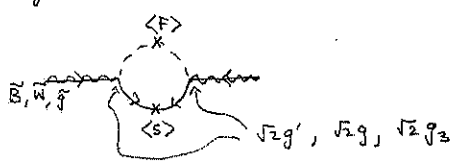
This splits the messenger scalars, fermions

$q, \bar{q}$: $\quad m^2_{fermions} = |y_3 \langle S \rangle|^2 \quad m^2_{scalars} = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F \rangle|^2$

$\ell, \bar{\ell}$: $\quad m^2_{fermions} = |y_2 \langle S \rangle|^2 \quad m^2_{scalars} = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F \rangle|^2$

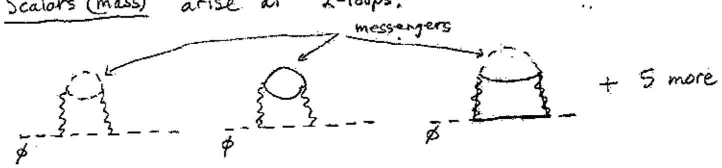
This SUSY breaking is communicated to MSSM.

<u>Gaugino masses:</u>



Result: $M_a = \frac{\alpha_a}{4\pi} \Lambda$ with $\Lambda = \frac{\langle F \rangle}{\langle S \rangle}$

<u>Scalars (mass)^2</u> arise at 2-loops:



After some calculation...

$$m^2_{\phi \text{ MSSM scalar}} = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3^\phi + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2^\phi + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1^\phi \right], \text{ where}$$

$C_3^\phi = \begin{cases} 4/3 & \text{for squarks} \\ 0 & \text{for sleptons} \end{cases}$

$C_2^\phi = \begin{cases} 3/4 & \text{for } \phi = \tilde{Q}, \tilde{L}, H_u, H_d \quad \text{(doublets)} \\ 0 & \text{for } \phi = \tilde{u}, \tilde{d}, \tilde{e} \quad \text{(singlets)} \end{cases}$

$C_1^\phi = \frac{3}{5} Y_\phi^2 \qquad Y_\phi = $ weak hypercharge

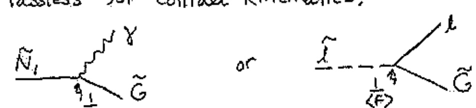
Successes:
- Flavor blind ✓
- Scalar $(\text{mass})^2 > 0$ ✓ (could have been negative!)

Gauginos, sfermions $\quad M_{soft} \sim \alpha_a \frac{\langle F \rangle}{M_{mess}}$ (robust for generalized models)

If $\langle F \rangle \sim M^2_{mess}$, then need $\sqrt{\langle F \rangle} \sim M_{mess} \gtrsim \frac{1\,TeV}{\alpha_3} \approx 10\,TeV$

Gravitino mass $\quad M_{3/2} \sim \frac{\langle F \rangle}{M_{Planck}}$ could be as low as 0.1 eV.

Massless for collider kinematics,



$$\Gamma(\text{NLSP} \to \text{SM partner} + \tilde{G}) = |(\text{mixing factor})|^2 \left(\frac{M_{NLSP}}{100\,GeV}\right)^5 \left(\frac{\sqrt{\langle F \rangle}}{100\,TeV}\right)^{-4} 2 \times 10^{-3}\,eV$$

The NLSP could be long-lived on detector scales.

The LHC is confronting SUSY <u>now!</u>

If it is the solution to the hierarchy problem, I expect at least hints from the current run, with few fb^{-1}!