NIU Physics PhD Candidacy Exam - Spring 2018

Quantum Mechanics

YOU MAY SOLVE ALL FOUR PROBLEMS! THE **three best graded** COUNT TOWARDS YOUR TOTAL SCORE.
(40 POINTS EACH; TOTAL POSSIBLE SCORE: 120 POINTS)

I. SPINS IN A CUBIC BOX *[(22+9+9) PTS]*

Two *identical* spin-$1/2$ fermions of mass M are confined in a cubic box of side L. The sides of the box have infinite potential. The identical fermions interact according to the attractive potential:

$$V(\vec{r}_1, \vec{r}_2) = -\epsilon L^3 \delta^{(3)}(\vec{r}_1 - \vec{r}_2),$$

where ϵ is small and positive, and should be treated as a perturbation. Do not neglect the effects of spin degrees of freedom in this problem. [Hint: you may or may not find $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$ and/or $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$ to be useful in doing an integral.]

a) Find the total spin, the degeneracy, and the energy of the ground state, working to first order in ϵ.

b) What is the total spin and the degeneracy of the first excited state?

c) If the potential is instead repulsive ($\epsilon < 0$), what is the total spin and the degeneracy of the first excited state?

II. MYSTERY POTENTIAL *[(10+10+6+14) PTS]*

The wave function for a particle of mass m moving in one dimension, subject to a potential $V(x)$ is given by

$$\psi(x,t) = \Theta(x) x e^{-Bx} e^{-iCt/\hbar},$$

where $\Theta(x)$ is the Heaviside step function (0 for $x < 0$ and 1 for $x > 0$) and B & C are real constants such that $\psi(x,t)$ is a properly normalized wave function that obeys the time-dependent Schrödinger equation for potential $V(x)$.

a) Sketch this wave function at time $t = 0$. Mark any significant features.

b) Using what you know about ψ, make a qualitative sketch of the potential $V(x)$ governing this system, indicating in particular any classically forbidden regions and classical turning points.

c) Is the particle in a state corresponding to a definite energy? If so, what is the energy (in terms of any or all of B and C); if not, why not?

d) Determine the potential $V(x)$ in terms of B, C, m, and $\hbar$. Does your result agree with your sketch?

III. ISOTROPIC HARMONIC OSCILLATOR *[(8+10+10+12 PTS]*

We consider a particle with mass m confined in a three-dimensional isotropic harmonic potential

$$V_0 = \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$$

a) Give a general expression for the energy eigenvalues. Also, give explicit expressions for the lowest five energy eigenvalues including their degeneracies.

b) We add a perturbation $V_1 = k\, x$ with a real constant k. Evaluate the energy eigenvalues in lowest nonvanishing order perturbation theory. Again give the lowest five energy eigenvalues including their degeneracies.

c) Next we consider the perturbation $V_2 = \frac{k}{\sqrt{2}}\,(x+y)$. Again evaluate the energy eigenvalues in lowest nonvanishing order perturbation theory? What are now the lowest five energy eigenvalues including their degeneracies?

d) Solve the Schrödinger equation for the potential $V_0 + V_2$ exactly. What are the energy eigenvalues? Compare with part a).

IV. P-ELECTRON *[(10+10+10+10) PTS]*

An electron in a p orbital ($l = 1$) feels an additional interaction

$$\mathrm{H}' = -\frac{\omega_0}{\hbar} (\mathrm{L}_x^2 - \mathrm{L}_y^2),$$

where ω_0 is a parameter and

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad L_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

a) Write H' in matrix form.

b) Determine the eigenvalues and eigenvectors of H'.

c) If the electron is initially in the state with $m = 1$, how does it oscillate between the different states as a function of time?

d) Calculate the probability of finding $m = 1$ as a function of time.