NIU Physics PhD Candidacy Exam – Spring 2026 – Electricity and Magnetism

You may solve all four problems, if you choose. Only the three best problem scores count towards your total score. Total points on each problem = 40. Total possible score = 120.

Problem 1. Consider a magnetized cylinder with the center drilled out, carrying uniform magnetization $\vec{M} = M_0\hat{z}$. The inner and outer radii of the cylinder are a and b respectively, and it has length L. Do not assume that $L \gg a, b$. Determine the:
(a) effective (also known as bound) volume current and surface currents. [15 points]
(b) the magnetic field everywhere along the z axis, as a function of z. Take the center of the cylinder to be at $z = 0$. [25 points]

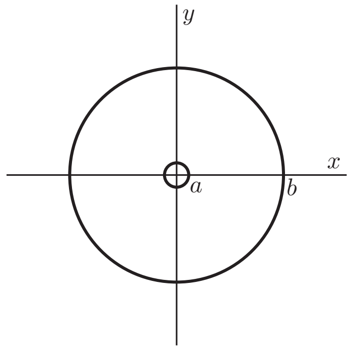
Problem 2. A spherical conductor of radius a is cut along the equator ($z = 0$). The upper hemisphere is held at electrostatic potential V_1, and insulated from the lower hemisphere which is held at electrostatic potential V_2. The potential very far away from the hemispheres is 0.
(a) Find the potential in spherical coordinates (r, θ, ϕ) in an expansion for large r, including all contributions that fall off like $1/r^4$ or slower. [25 points]
(b) Find the electric field vector in spherical coordinates at large r, corresponding to your answer in part (a). [15 points]

Problem 3. An electromagnetic wave propagates in a long coaxial cable consisting of an inner cylinder of radius a and an outer cylinder of radius b. Suppose that the magnetic field for $a < r < b$ is given in cylindrical coordinates (r, ϕ, z) by

$$\vec{B} = B_0\,(r/a)^n \cos(kz - \omega t)\hat{\phi} \tag{1}$$

where B_0, ω, and k are positive constants and n is a rational number. The region $a < r < b$ is vacuum.
(a) Solve for the constant n. [8 points]
(b) Find the electric field in the region $a < r < b$. [8 points]
(c) Find the relation between k and ω. [6 points]
(d) Find the charge density and the current density on the inner cylinder. [12 points]
(e) Find the total time-averaged power transmitted by the cable. [6 points]

Problem 4. Two circular wire loops have very different radii $a \ll b$. Both loops have resistance per unit length K. Their centers are at the origin, and at time $t = 0$ they both lie in the horizontal xy plane as shown.



(a) Suppose that the large loop carries constant current I. At time $t = 0$, it is allowed to fall under the influence of gravity with acceleration g along $-\hat{z}$, maintaining its center on the negative z axis and remaining parallel to the xy plane. Meanwhile, the small loop stays fixed. Find the mutual inductance between the two loops as a function of time, and use it to compute the current induced in the small loop. [25 points]

(b) Suppose instead that the large loop stays fixed, but the small circular loop carries constant current I, and at time $t = 0$ begins spinning slowly about the x-axis at constant angular frequency ω. Find the current induced in the large circular loop as a function of t. [15 points]

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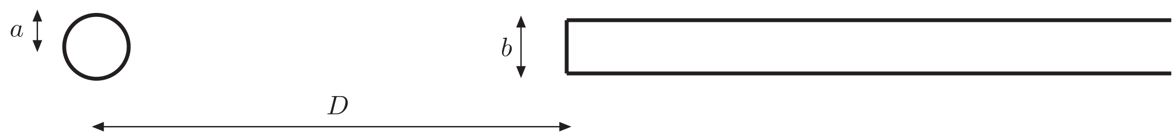
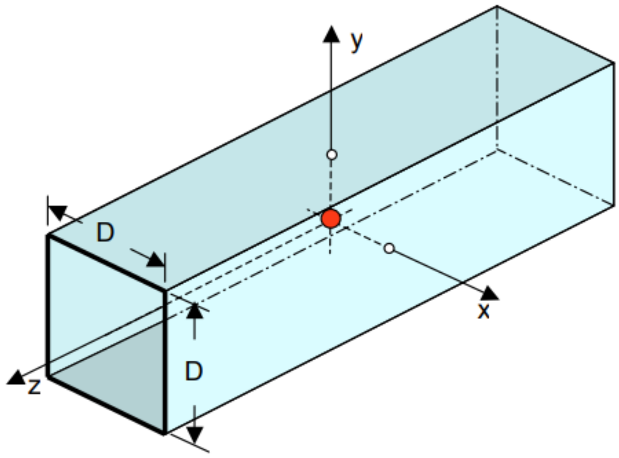
Do not just quote a result. Show your work clearly step by step.

1. [40 points] Charges $+q$ and $-q$ lie at $(x,y,z) = (a, 0, a)$ and $(-a, 0, a)$ respectively, above a grounded conducting plane positioned at $z = 0$. Find:
   a) The total force on $+q$. [14 points]
   b) The work done against electrostatic forces in assembling this system of charges. [13 points]
   c) The surface charge density at $(a, 0, 0)$. [13 points]
2. [40 points] A spherical shell, of radius R, carrying a uniform surface charge density σ, is set spinning about z axis at angular velocity ω.
   a) Find the vector potential it produces at point $\boldsymbol{r}$ (both $r \leq R$ and $r \geq R$). [20 points]
   b) Find the magnetic field inside this spherical shell. [10 points]
   c) Find the magnetic field outside this spherical shell. [10 points]
3. [40 points]A small circular wire loop is in the same plane as a very thin rectangular wire loop which extends very far off to the right (where it is closed off somewhere beyond the range shown). The radius of the small circular loop is a, the distance between the parallel wires of the rectangular loop is b, and the distance from the rectangular loop to the center of the circular loop is D. The circular loop has resistance R_1 and the rectangular loop has resistance R_2. You should assume that a and b are both very small compared to D.



   a) Suppose that a current $I(t) = I_0\cos(\omega t)$ flows counterclockwise in the circular loop, where I_0 and ω are constants. Find the current induced in the rectangular loop as a function of time. Be sure to unambiguously specify the direction. [20 points]
   b) Now suppose instead that a current $I(t) = I_0 e^{-kt}$ flows counterclockwise in the rectangular loop, where I_0 and k are constants. Find the current induced in the circular loop as a function of time. Be sure to unambiguously specify the direction. [20 points]

4. [40 points] Consider a very long, metallic and grounded waveguide with a square shaped inner cross section of side D. A point charge Q is suspended on the central axis of this guide at a location far from either end.



a) Find the electrostatic potential everywhere inside the guide. [25 points]
b) What is the asymptotic form of the potential at locations far from the point charge? [10 points]
c) Make a sketch of the electric field lines in a region far from the point charge. [5 points]

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Do not just quote a result. Show your work clearly step by step.

1. [40 points] A conducting sphere of radius "R", carrying a charge "Q" is placed in a uniform electric field E_0. Find:
   (a) The potential at all points inside and outside the sphere. [14 points]
   (b) Dipole moment of the induced charge on the sphere. [8 points]
   (c) The total electric field everywhere. Use it to find the difference in electrostatic energy between this configuration and the case in which the sphere and charge Q are absent. [18 points]

2. [40 points]
   (a) Derive an expression of the vector potential A at (r, θ, ϕ) of a (perfect) magnetic dipole $\boldsymbol{m}$ pointing in the z direction and placed at the origin. (fig 1a). Also, derive an expression of the magnetic field B of this magnetic dipole $\boldsymbol{m}$. [20 points]
   (b) A thin glass rod of radius R and length L carries a uniform surface charge σ. It is set spinning about its axis at an angular velocity ω. Find the magnetic field at a distance $s \gg R$ from the axis in the xy plane. (Fig 1b). *Hint: Treat it as a stack of magnetic dipoles* [20 points]

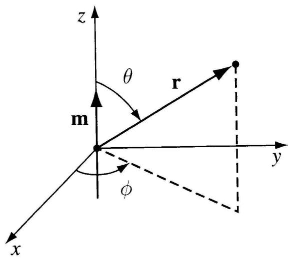

Fig 1a

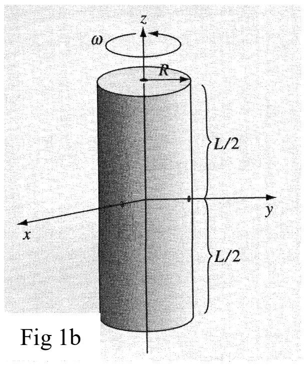

Fig 1b

3. [40 points] Two parallel circular conducting disks each have radius R and are separated by a distance a, forming a capacitor as shown below (Fig 2). Assume that $a \ll R$, so that edge effects can be neglected. The material between the disks has permittivity and permeability the same as vacuum, $\epsilon = \epsilon_0$ and $\mu = \mu_0$, but it does obey Ohm's Law with a non-zero conductivity σ. [Note: this capacitor is not an ideal capacitor, since the material between the disks is not a perfect insulator.] Starting at $t = 0$, a thin wire carries constant current I to the left disk, and another thin wire carries the same constant current I off of the right disk. Assume that the charges are uniformly distributed on the disks at any given time.

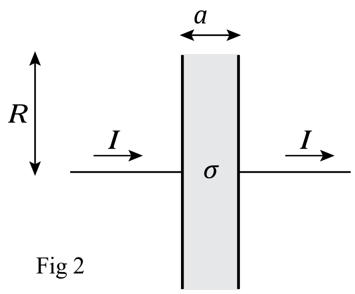

Fig 2

(a) Find a differential equation for the total charge $Q(t)$ on the left disk as a function of time t. [10 points]

(b) Solve the differential equation found in the previous part for $Q(t)$ (with the boundary condition that $Q(0) = 0$). [10 points]

(c) Find the magnetic field $\vec{B}$ between the disks. [20 points]

4. [40 points]

Consider an electromagnetic wave of the form

$$\vec{E}_I(\vec{r},t) = [\hat{z}sin(\theta) + \hat{x}cos(\theta)]E_{0I}e^{i(\vec{k_I}\cdot\vec{r}-\omega t)}. \quad (1)$$

The wave is travelling through the vacuum (dielectric constant ϵ_0), and is incident on a surface at $z = 0$ with dielectric constant ϵ. The coordinate system is defined so that the surface normal points along the $\hat{z}$ direction (See Fig 3). The wave is "p" polarized and makes an incident angle θ with respect to the surface normal, such that:

$$\vec{k_I} = k_0[\sin(\theta)\,\hat{x} - \cos(\theta)\,\hat{z}]. \quad (2)$$

After impinging on the surface, the wave reflects specularly (equal angle). The reflected wave is given by:

$$\vec{E}_R(\vec{r},t) = [\hat{z}sin(\theta) - \hat{x}cos(\theta)]E_{0R}e^{i(\vec{k_R}\cdot\vec{r}-\omega t)}. \quad (3)$$

with a wavevector given by:

$$\vec{k_R} = k_0[\sin(\theta)\,\hat{x} + \cos(\theta)\,\hat{z}]. \quad (4)$$

The transmitted wave is given by:

$$\vec{E}_T(\vec{r},t) = [\hat{z}sin(\theta') + \hat{x}cos(\theta')]E_{0T}e^{i(\vec{k_T}\cdot\vec{r}-\omega t)}. \quad (5)$$

$$\vec{k_T} = nk_0[\sin(\theta')\,\hat{x} - \cos(\theta')\,\hat{z}]. \qquad \text{Here } n = \sqrt{\epsilon/\epsilon_0}. \quad (6)$$

(a) Find the angle θ' as a function of θ and n, from the boundary condition that the longitudinal component of the electric field, $\vec{E}_{\parallel}$, is continuous across the interface. (Hint: you only need to consider the variation of the phase of the electric field across the interface). [10 points]

(b) Use the two conditions that (1) the normal component of the electric displacement $\left(\vec{D}_{\perp} = \epsilon\vec{E}_{\perp} = n^2\epsilon_0\vec{E}_{\perp}\right)$ is continuous across the interface and (2) the longitudinal component of the electric field $\vec{E}_{\parallel}$ is continuous across the interface, to derive two equations relating the electric field amplitudes E_{0I}, E_{0T}, E_{0R} in terms of θ, θ' and n. [10 points]

(c) Find an equation for reflection amplitude, $a = E_{0R}/E_{0I}$ as a function of $\alpha = \tan(\theta)$, $\alpha' = \tan(\theta')$ and n. [10 points]

(d) Using the relation for θ' from part (a), rewrite the expression for the reflection amplitude entirely in terms of n and θ. [10 points]

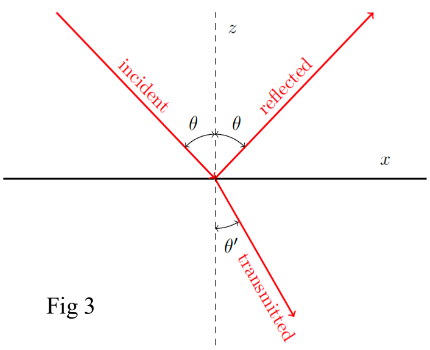

Fig 3

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Electricity and Magnetism

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Do not just quote a result. Show your work clearly step by step.

1. [40 points] A toroidal coil has a rectangular cross section, with inner radius a, outer radius $a + w$, and height h. It carries a total of N tightly wound turns, and *the current is increasing at a constant rate* ($dI/dt = k$).

   (a) In the quasistatic approximation, find the direction and the magnitude of the magnetic field $\vec{B}$ everywhere (for points inside and outside of the toroidal coil). [6 points]

   (b) Calculate the flux of $\vec{B}$ through the cross section the toroid. [14 points]

   (c) Assume w and h are both much less than a. Find the induced electric field $\vec{E}$ at a point z above the center of the toroid. [20 points]

2. [40 points] A small source of electromagnetic radiation is located near the origin. The potentials for large r are given in spherical coordinates by:

$$V(\vec{\boldsymbol{r}}, t) = -V_0 cos\big(\omega(t - r/c)\big)\frac{sin\theta}{r}$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}, t) = A_0 cos\big(\omega(t - r/c)\big)\frac{sin\theta}{r}\left(\hat{\phi} + \hat{r}\right)$$

   where V_0 and A_0 are constants.

   (a) Find the electric and magnetic fields for large r (consistently neglecting contributions that fall off faster than $1/r$). [13 points]

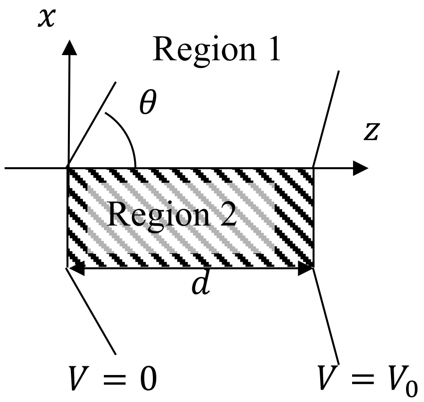
   (b) Solve for V_0 in terms of other quantities, and use the result to simplify your answers for the fields. [13 points]

   (c) Is this electric dipole radiation, magnetic dipole radiation, both, or neither? [5 points]

   (d) A gauge transformation is performed, so that $V = 0$ in the new gauge. What is $\vec{\boldsymbol{A}}$ in the new gauge? [9 points]

3. [40 points] Electron beam with constant current density (J) is coming out of electrode at z=0, and the other electrode at z=d stops the beam. Electrodes outside of the region that electron beam occupies (i.e., region 2) are tilted to shape the electric potential so that electron beam's size in x-direction does not change in region 2.

When the electron beam's density is uniform in x-direction and the voltage applied to each electrode is given as figure below. This emission is under equilibrium state, so there is nothing changing in time. Both electrode and beam are infinitely long in y-direction.



(a) Derive electric potential in region 2 as function of z. (i.e., V(z)=?) [20 points]
[hint: Charge density is not constant along z, but your current density is constant.]
[hint: electrons' kinetic energy at certain z-position will be same as electric potential at this location]

(b) Write current density J as function of V_0 and d. Please assume $dV/dz|_{z=0} = 0$. [10 points]
[hint: $\frac{\partial}{\partial z}\left[\left(\frac{\partial V}{\partial z}\right)^2\right] = 2\left[\frac{\partial V}{\partial z}\frac{\partial^2 V}{\partial z^2}\right]$]

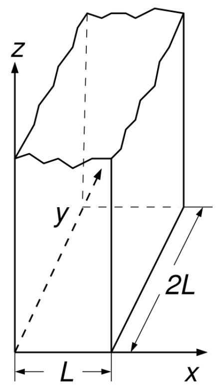
(c) Find what condition electrode angle θ must satisfy to make electron beam's size in x-direction constant as displayed. [10 points]
[hint: $\nabla^2 V = 0$ for region 1, and $V = 0$ at $x = z\tan\theta$.]

4. [40 points] Consider a rectangular waveguide, infinitely long in the z direction, with transverse dimensions $L_x = L$ and $L_y = 2L$ as illustrated in the figure. The walls are a perfect conductor.
   (a) What are the boundary conditions for the components of B and E at the walls? [8 points]

   (b) Write the wave equation which describes the E and B fields of the lowest mode. [10 points]
   [hint: The lowest mode has the electric field in the x direction only.]

   (c) For the lowest mode that can propagate, find the phase velocity and the group velocity.
   [12 points]

   (d) The possible modes of propagation such waveguides separate naturally into two classes.
   What are these two classes and how to they differ physically? [10 points]



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Electricity and Magnetism

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Do not just quote a result. Show your work clearly step by step.

1. [40 points] A parallel-plate capacitor consists of two circular plates of radius R placed a distance d apart. The voltage across the plates, supplied by lead wires, is given by $V = V_0 cos\omega t$. Ignore fringe fields.
   (a) Determine the electric and magnetic fields $\vec{E}$ and $\vec{B}$ between the plates. [12 points]
   (b) Determine the current in the lead wires and the current density in the plates as a function of time. [14 points]
   (c) Determine the $\vec{B}$ field in the region above the top plate. How does it compare to the field within the plates? [14 points]

2. [40 points] A linear dielectric sphere of radius a and dielectric constant κ carries a uniform charge density ρ, surrounded by vacuum.
   (a) Find $\vec{E}$ and $\vec{B}$ inside and outside the sphere. [15 +10 points]
   (b) Find the energy W of the system. [15 points]

3. [40 points] A cylindrical permanent magnet of length L and radius a has uniform magnetization $\vec{M}$ directed along the axis of the magnet (see Figure).

   (a) Find the magnetic field outside the magnet on the z axis. [20 points]

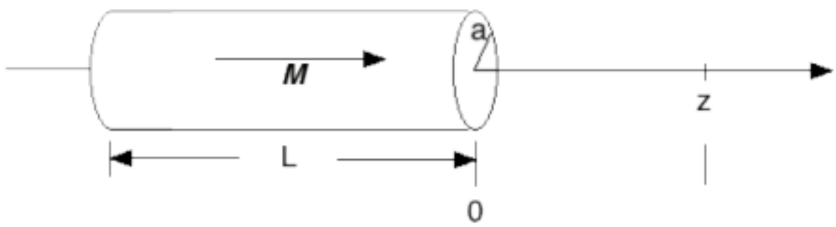
   [hints: While volume current density ($\nabla \times \boldsymbol{M}$) is zero, surface current density ($\boldsymbol{M} \times \hat{\boldsymbol{n}}$) is non-zero. $\hat{\boldsymbol{n}}$ is normal vector of surface.]

   [handy formula: $\int \frac{a^2}{(a^2+x^2)^{3/2}} dx = \frac{x}{\sqrt{a^2+x^2}}$.]

   (b) Analyze the limit z>>a, L and compare the result to the field of a magnetic dipole. [20 points]

   [hints: Vector potential of magnetic dipole with magnetic moment **m** can be written as $\boldsymbol{A}(\boldsymbol{r}) = \frac{\boldsymbol{m}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}$.]

   [handy formula: $\frac{1 \pm bx}{\sqrt{(ax)^2+(1\pm bx)^2}} \approx 1 - \frac{a^2}{2}x^2 \pm a^2bx^3$.]



4. [40 points] An electromagnetic wave with angular frequency ω moves through a material that obeys Ohm's Law with conductivity σ. The permittivity and permeability of the material are the same as that of vacuum.

   (a) Derive the separate, uncoupled, second-order wave equations for the electric and magnetic fields $\vec{E}$ and $\vec{B}$. [12 points]

   (b) Find expressions for the electric and magnetic fields of a wave moving in the $\hat{z}$ direction and polarized in the $\hat{x}$ direction. [16 points]

   (c) Find the distance that the wave travels for which its intensity is decreased by a factor of 10. [12 points]

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1. [40 points] The space between the plates of a parallel-plate capacitor (Fig. 1) is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of 2.0, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.
   (a) Find the electric displacement $\vec{D}$ in each slab. [6 points]
   (b) Find the electric field $\vec{E}$ in each slab. [6 points]
   (c) Find the polarization $\vec{P}$ in each slab. [4 points]
   (d) Find the potential difference between the plates. [4 point]
   (e) Find the location and amount of all bound charge. [16 points]
   (f) From all the above charge (free and bound), recalculate the field in each slab, and confirm your answer to (b). [4 points]

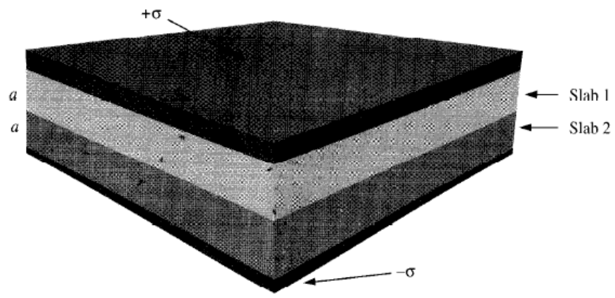

Fig. 1

2. [40 points] A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω.
   (a) Find the vector potential it produces at point $\boldsymbol{r}$ (both $r \leq R$ and $r \geq R$). [20 points]
   (b) Find the magnetic field inside this spherical shell. [10 points]
   (c) Find the magnetic field outside this spherical shell. [10 points]

3. [40 points] An infinitely long solid insulating cylinder has radius *a* and carries a uniform constant charge density ρ. It is rotating around the $\hat{z}$ axis, which is its axis of symmetry, in the $+\phi$ direction with angular velocity ω. The cylinder has the same permittivity and permeability as vacuum.
   (a) Find the magnetic field $\vec{B}$ everywhere. [12 points]
   Now assume that $\omega = \omega_0 + \alpha t$ is a slowly increasing function of time.
   (b) Find the total electric field $\vec{E}$ everywhere. [18 points]
   (c) Suppose that the cylinder is surrounded by a thin circular loop of wire of radius *b* and resistance *R*. Find the current induced in the wire. [10 points]

4. [40 points] Consider an infinitely long transmission line consisting of two perfectly conducting wires running parallel to the x-axis at $y = 0$ and $y = a > 0$, and separated by a dielectric. The wires have capacitance *C* per unit length and inductance *L* per unit length. In this problem, the longitudinal components E_x and B_x both vanish everywhere, so we are examining transverse electromagnetic (TEM) waves.

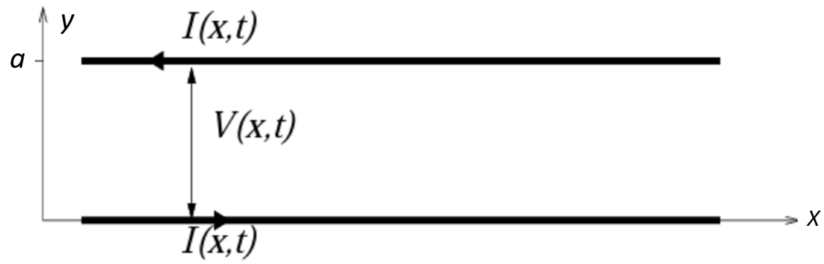

Fig. 2

   (a) Let $V(x,t)$ be the position- and time-dependent EMF between the wires, defined by evaluating $V(x,t) = -\int_0^a E_y(x,y,t)dy$ along a straight path between the wires at fixed *x*. Show that
$$\frac{\partial V}{\partial x} = \kappa \frac{\partial I}{\partial t}$$
   for some constant κ depending on *L* and/or *C* that you should find. Here, $I(x,t)$ is the current at *x* in the lower wire ($y = 0$). The current at *x* in the upper wire ($y = a$) has the same magnitude but opposite direction. [10 points]
   (b) By using the charge continuity equation, derive a similar relation between $\frac{\partial V}{\partial t}$ and $\frac{\partial I}{\partial x}$. [10 points]
   (c) Combine the differential equations from parts (a) and (b) to obtain a wave equation for $I(x,t)$ and express the wave velocity c_{TEM} in terms of *L* and *C*. [10 points]
   (d) If $I(x,t) = I_0 sin\{k(x - c_{\text{TEM}}t)\}$, what is $V(x,t)$? [10 points]

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Electricity and Magnetism

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1. [40 points] **The electric field inside a dielectric sphere.** A dielectric sphere of radius a has a uniform polarization $\boldsymbol{P}$. Find the electric field inside a sphere $\boldsymbol{E}_{polar}(\boldsymbol{r})$ by answering the following questions (a), (b), and (c). In this problem, we model the dielectric sphere as the superposition of two displaced uniformly charged spheres with opposite charge densities.

   (a) Find the electric field $\boldsymbol{E}_0(\boldsymbol{r})$ inside a sphere of radius a with a uniform charge density ρ (without polarization). Take the center of the sphere as the origin. [20 points]

   (b) Consider two spheres of radius a, one with its center at $+\boldsymbol{u}/2$ from the origin with charge density $+\rho$, the other with its center at $-\boldsymbol{u}/2$ from the origin with charge density $-\rho$. Find the electric field $\boldsymbol{E}_+(\boldsymbol{r})$ and $\boldsymbol{E}_-(\boldsymbol{r})$ for each sphere. [10 points]

   (c) Compute the electric field $\boldsymbol{E}_{polar}(\boldsymbol{r})$ from the above $\boldsymbol{E}_+(\boldsymbol{r})$and $\boldsymbol{E}_-(\boldsymbol{r})$, and then express $\boldsymbol{E}_{polar}(\boldsymbol{r})$ with the polarization vector $\boldsymbol{P}$, assuming $|\boldsymbol{u}| << a$. [10 points]

2. [40 points] A very long solenoid consists of a wire with resistance R, wound N times on a right circular cylinder of length d and radius a, with $d \gg a$.

   (a) Suppose that a time-varying current $I(t) = I_0 cos(\omega t)$ flows in the solenoid wire.
      i) Find the magnetic field vector inside the solenoid. [8 points]
      ii) Find the electric field vector everywhere inside and outside of the solenoid. [16 points]

   (b) A circular ring of wire with radius b is coaxial with the solenoid and outside of it ($b > a$). Suppose now that a time-varying current $I(t) = I_0 cos(\omega t)$ is forced to flow in the ring. What is the current induced in the solenoid? [16 points]

3. [40 points] **Cloud of time-dependent charged particles at rest.** We consider a set of point particles fixed in space with time-dependent charge $q_k(t)$ so that the total charge density given by $\rho(\boldsymbol{r}, t) = \sum_k q_k(t)\delta(\boldsymbol{r} - \boldsymbol{r}_k)$. We assume that $\boldsymbol{E}(\boldsymbol{r}, t = 0) = \boldsymbol{B}(\boldsymbol{r}, t = 0) = 0$, and consider the electric field to be $\boldsymbol{E}(r, t) = \frac{1}{4\pi\varepsilon_0} \sum_k q_k(t) \frac{\boldsymbol{r}-\boldsymbol{r}_k}{|\boldsymbol{r}-\boldsymbol{r}_k|^3}$ for $t > 0$.

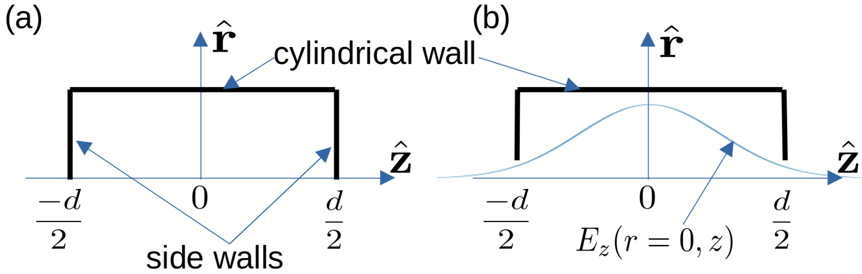
   (a) Construct a simple current density which satisfies the continuity equation [12 points].

   (b) Describe the flow of charge predicted by the current density computed in part (a) [8 points].

   (c) Compute the magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$ and show that, together with $\boldsymbol{E}(r, t)$, the fields satisfy all four Maxwell's equations [20 points].

   <u>Hint:</u> The identity $\nabla^2 \left(\frac{1}{|\boldsymbol{r}|}\right) = -\boldsymbol{\nabla}. \left(\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\right) = -4\pi\delta(\boldsymbol{r})$ will be convenient.

4. [40 points] **Transverse force in a resonant cavity for particle acceleration**. Resonant microwave cavities operated on the TM mode are commonly used to accelerate charged particles. The idealized right-circular cylinder cavity is not useful for accelerating charged particles since it does not include entrance/exit apertures to inject/extract the charged particles; see Fig. 4(a). This problem examines how these apertures impact the electromagnetic field; see Fig. 4(b). We will work in cylindrical coordinate (r, φ, z) and consider the cavity frequency to be ω. We assume an ultra-relativistic particle traveling at the speed of light and focus on a particle moving close to the resonant-cavity axis $\hat{\boldsymbol{z}}$ (paraxial approximation) so that the boundary conditions imposed by the cylindrical wall do not have to be considered in this problem. We also consider the fields to be independent of φ .



(a) Given the cylindrical symmetry of the problem discuss which field components are non-vanishing and specifically give the form of the expected axial electric field $E_z(r = 0, z, t)$ taking its amplitude to be E_0 and considering the boundary condition from the side walls [10 points].

(b) Using Maxwell's equations, give the general relationship between the non-vanishing transverse fields [both the electric $\boldsymbol{E}_\perp(r, z, t)$ and magnetic $\boldsymbol{B}_\perp(r, z, t)$ field components] and the longitudinal electric field $E_z(r = 0, z, t)$. Give the general form the transverse force $\boldsymbol{F}_\perp$. Using the longitudinal field from (a) comment on the transverse force received by a particle as it moves along $\hat{\boldsymbol{z}}$ [13 points].

(c) We now consider the longitudinal electric field to be spatially constant: $E_z(r = 0, z) = E_0$ for $-z \leq \left|\frac{d}{2}\right|$and vanishing elsewhere. Using the results from (b) give the field components and describe how the transverse force is modified [7 points].

(d) We now consider the cavity to have an exit and entrance apertures as shown in Fig. 1(b) so that the fields "leak" outside of the cavity. We approximate the longitudinal electric field dependence on spatial coordinate to be given by $E_z(r = 0, z) = E_0 \exp\left(-\frac{z^2}{2d^2}\right)$. Compute the transverse force experienced by a particle entering the cavity with a radial offset r . Qualitatively discuss the net impact of such a force on a particle being accelerated. [10 points].