NIU Ph.D. Candidacy Examination Spring 2024 (1/10/2024)

Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. [40 points] A parallel-plate capacitor consists of two circular plates of radius R placed a distance d apart. The voltage across the plates, supplied by lead wires, is given by $V = V_0 cos\omega t$. Ignore fringe fields.
   (a) Determine the electric and magnetic fields $\vec{E}$ and $\vec{B}$ between the plates. [12 points]
   (b) Determine the current in the lead wires and the current density in the plates as a function of time. [14 points]
   (c) Determine the $\vec{B}$ field in the region above the top plate. How does it compare to the field within the plates? [14 points]

2. [40 points] A linear dielectric sphere of radius a and dielectric constant κ carries a uniform charge density ρ, surrounded by vacuum.
   (a) Find $\vec{E}$ and $\vec{B}$ inside and outside the sphere. [15 +10 points]
   (b) Find the energy W of the system. [15 points]

3. [40 points] A cylindrical permanent magnet of length L and radius a has uniform magnetization $\vec{M}$ directed along the axis of the magnet (see Figure).

   (a) Find the magnetic field outside the magnet on the z axis. [20 points]

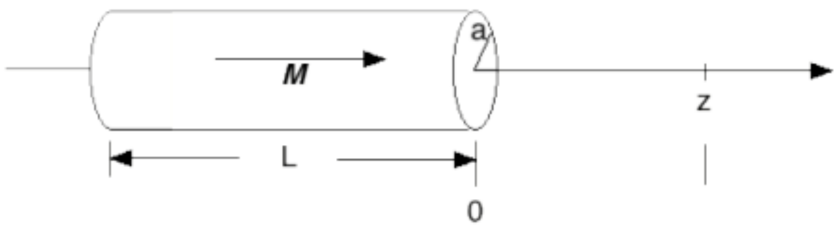
   [hints: While volume current density ($\nabla \times \boldsymbol{M}$) is zero, surface current density ($\boldsymbol{M} \times \hat{\boldsymbol{n}}$) is non-zero. $\hat{\boldsymbol{n}}$ is normal vector of surface.]

   [handy formula: $\int \frac{a^2}{(a^2+x^2)^{3/2}} dx = \frac{x}{\sqrt{a^2+x^2}}$.]

   (b) Analyze the limit z>>a, L and compare the result to the field of a magnetic dipole. [20 points]

   [hints: Vector potential of magnetic dipole with magnetic moment **m** can be written as $\boldsymbol{A}(\boldsymbol{r}) = \frac{\boldsymbol{m}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}$.]

   [handy formula: $\frac{1\pm bx}{\sqrt{(ax)^2+(1\pm bx)^2}} \approx 1 - \frac{a^2}{2}x^2 \pm a^2 bx^3$.]



4. [40 points] An electromagnetic wave with angular frequency ω moves through a material that obeys Ohm's Law with conductivity σ. The permittivity and permeability of the material are the same as that of vacuum.

   (a) Derive the separate, uncoupled, second-order wave equations for the electric and magnetic fields $\vec{E}$ and $\vec{B}$. [12 points]

   (b) Find expressions for the electric and magnetic fields of a wave moving in the $\hat{z}$ direction and polarized in the $\hat{x}$ direction. [16 points]

   (c) Find the distance that the wave travels for which its intensity is decreased by a factor of 10. [12 points]

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Do not just quote a result. Show your work clearly step by step.

1. [40 points] The space between the plates of a parallel-plate capacitor (Fig. 1) is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab l has a dielectric constant of 2.0, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.
   (a) Find the electric displacement $\vec{D}$ in each slab. [6 points]
   (b) Find the electric field $\vec{E}$ in each slab. [6 points]
   (c) Find the polarization $\vec{P}$ in each slab. [4 points]
   (d) Find the potential difference between the plates. [4 point]
   (e) Find the location and amount of all bound charge. [16 points]
   (f) From all the above charge (free and bound), recalculate the field in each slab, and confirm your answer to (b). [4 points]

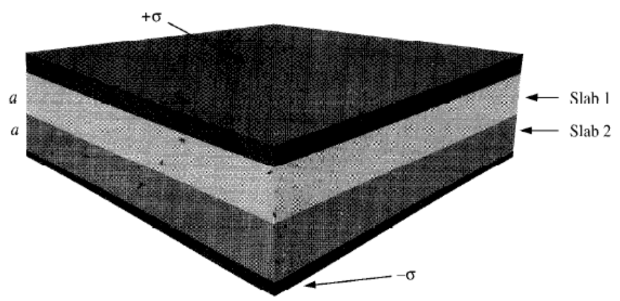

Fig. 1

2. [40 points] A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω.
   (a) Find the vector potential it produces at point $\boldsymbol{r}$ (both $r \leq R$ and $r \geq R$). [20 points]
   (b) Find the magnetic field inside this spherical shell. [10 points]
   (c) Find the magnetic field outside this spherical shell. [10 points]

3. [40 points] An infinitely long solid insulating cylinder has radius a and carries a uniform constant charge density ρ. It is rotating around the $\hat{z}$ axis, which is its axis of symmetry, in the $+\phi$ direction with angular velocity ω. The cylinder has the same permittivity and permeability as vacuum.
   (a) Find the magnetic field $\vec{B}$ everywhere. [12 points]
   Now assume that $\omega = \omega_0 + \alpha t$ is a slowly increasing function of time.
   (b) Find the total electric field $\vec{E}$ everywhere. [18 points]
   (c) Suppose that the cylinder is surrounded by a thin circular loop of wire of radius b and resistance R. Find the current induced in the wire. [10 points]

4. [40 points] Consider an infinitely long transmission line consisting of two perfectly conducting wires running parallel to the x-axis at $y = 0$ and $y = a > 0$, and separated by a dielectric. The wires have capacitance C per unit length and inductance L per unit length. In this problem, the longitudinal components E_x and B_x both vanish everywhere, so we are examining transverse electromagnetic (TEM) waves.

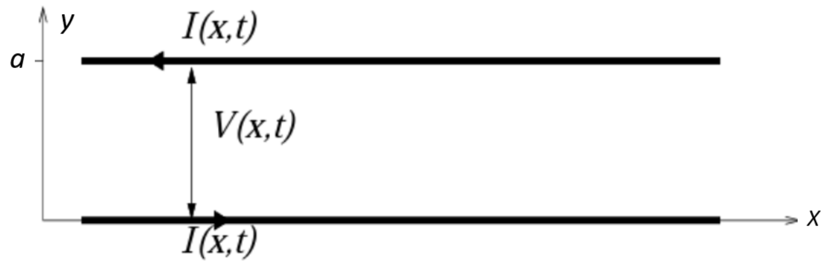

Fig. 2

   (a) Let $V(x,t)$ be the position- and time-dependent EMF between the wires, defined by evaluating $V(x,t) = -\int_0^a E_y(x,y,t)dy$ along a straight path between the wires at fixed x. Show that
$$\frac{\partial V}{\partial x} = \kappa \frac{\partial I}{\partial t}$$
   for some constant κ depending on L and/or C that you should find. Here, $I(x,t)$ is the current at x in the lower wire ($y = 0$). The current at x in the upper wire ($y = a$) has the same magnitude but opposite direction. [10 points]
   (b) By using the charge continuity equation, derive a similar relation between $\frac{\partial V}{\partial t}$ and $\frac{\partial I}{\partial x}$. [10 points]
   (c) Combine the differential equations from parts (a) and (b) to obtain a wave equation for $I(x,t)$ and express the wave velocity c_{TEM} in terms of L and C. [10 points]
   (d) If $I(x,t) = I_0 sin\{k(x - c_{\text{TEM}}t)\}$, what is $V(x,t)$? [10 points]

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Electricity and Magnetism

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Do not just quote a result. Show your work clearly step by step.

1. [40 points] **The electric field inside a dielectric sphere.** A dielectric sphere of radius a has a uniform polarization $\boldsymbol{P}$. Find the electric field inside a sphere $\boldsymbol{E}_{polar}(\boldsymbol{r})$ by answering the following questions (a), (b), and (c). In this problem, we model the dielectric sphere as the superposition of two displaced uniformly charged spheres with opposite charge densities.

   (a) Find the electric field $\boldsymbol{E}_0(\boldsymbol{r})$ inside a sphere of radius a with a uniform charge density ρ (without polarization). Take the center of the sphere as the origin. [20 points]

   (b) Consider two spheres of radius a, one with its center at $+\boldsymbol{u}/2$ from the origin with charge density $+\rho$, the other with its center at $-\boldsymbol{u}/2$ from the origin with charge density $-\rho$. Find the electric field $\boldsymbol{E}_+(\boldsymbol{r})$ and $\boldsymbol{E}_-(\boldsymbol{r})$ for each sphere. [10 points]

   (c) Compute the electric field $\boldsymbol{E}_{polar}(\boldsymbol{r})$ from the above $\boldsymbol{E}_+(\boldsymbol{r})$and $\boldsymbol{E}_-(\boldsymbol{r})$, and then express $\boldsymbol{E}_{polar}(\boldsymbol{r})$ with the polarization vector $\boldsymbol{P}$, assuming $|\boldsymbol{u}| << a$. [10 points]

2. [40 points] A very long solenoid consists of a wire with resistance R, wound N times on a right circular cylinder of length d and radius a, with $d \gg a$.

   (a) Suppose that a time-varying current $I(t) = I_0 cos(\omega t)$ flows in the solenoid wire.
      i) Find the magnetic field vector inside the solenoid. [8 points]
      ii) Find the electric field vector everywhere inside and outside of the solenoid. [16 points]

   (b) A circular ring of wire with radius b is coaxial with the solenoid and outside of it ($b > a$). Suppose now that a time-varying current $I(t) = I_0 cos(\omega t)$ is forced to flow in the ring. What is the current induced in the solenoid? [16 points]

3. [40 points] **Cloud of time-dependent charged particles at rest.** We consider a set of point particles fixed in space with time-dependent charge $q_k(t)$ so that the total charge density given by $\rho(\boldsymbol{r}, t) = \sum_k q_k(t)\delta(\boldsymbol{r} - \boldsymbol{r}_k)$. We assume that $\boldsymbol{E}(\boldsymbol{r}, t = 0) = \boldsymbol{B}(\boldsymbol{r}, t = 0) = 0$, and consider the electric field to be $\boldsymbol{E}(r, t) = \frac{1}{4\pi\varepsilon_0}\sum_k q_k(t)\frac{\boldsymbol{r}-\boldsymbol{r}_k}{|\boldsymbol{r}-\boldsymbol{r}_k|^3}$ for $t > 0$.

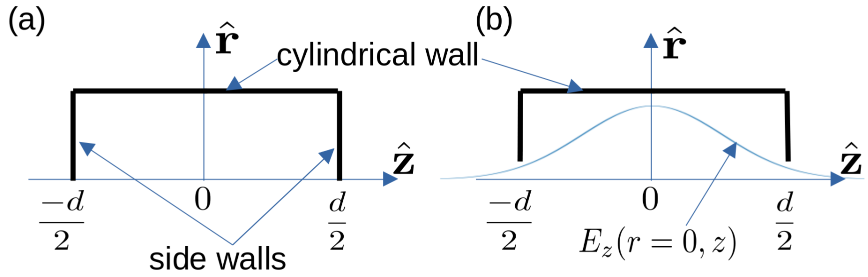
   (a) Construct a simple current density which satisfies the continuity equation [12 points].

   (b) Describe the flow of charge predicted by the current density computed in part (a) [8 points].

   (c) Compute the magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$ and show that, together with $\boldsymbol{E}(r, t)$, the fields satisfy all four Maxwell's equations [20 points].

   Hint: The identity $\nabla^2\left(\frac{1}{|\boldsymbol{r}|}\right) = -\boldsymbol{\nabla}.\left(\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\right) = -4\pi\delta(\boldsymbol{r})$ will be convenient.

4. [40 points] **Transverse force in a resonant cavity for particle acceleration**. Resonant microwave cavities operated on the TM mode are commonly used to accelerate charged particles. The idealized right-circular cylinder cavity is not useful for accelerating charged particles since it does not include entrance/exit apertures to inject/extract the charged particles; see Fig. 4(a). This problem examines how these apertures impact the electromagnetic field; see Fig. 4(b). We will work in cylindrical coordinate (r, φ, z) and consider the cavity frequency to be ω. We assume an ultra-relativistic particle traveling at the speed of light and focus on a particle moving close to the resonant-cavity axis $\hat{\boldsymbol{z}}$ (paraxial approximation) so that the boundary conditions imposed by the cylindrical wall do not have to be considered in this problem. We also consider the fields to be independent of φ .



(a) Given the cylindrical symmetry of the problem discuss which field components are non-vanishing and specifically give the form of the expected axial electric field $E_z(r = 0, z, t)$ taking its amplitude to be E_0 and considering the boundary condition from the side walls [10 points].

(b) Using Maxwell's equations, give the general relationship between the non-vanishing transverse fields [both the electric $\boldsymbol{E}_\perp(r, z, t)$ and magnetic $\boldsymbol{B}_\perp(r, z, t)$ field components] and the longitudinal electric field $E_z(r = 0, z, t)$. Give the general form the transverse force $\boldsymbol{F}_\perp$. Using the longitudinal field from (a) comment on the transverse force received by a particle as it moves along $\hat{\boldsymbol{z}}$ [13 points].

(c) We now consider the longitudinal electric field to be spatially constant: $E_z(r = 0, z) = E_0$ for $-z \leq \left|\frac{d}{2}\right|$and vanishing elsewhere. Using the results from (b) give the field components and describe how the transverse force is modified [7 points].

(d) We now consider the cavity to have an exit and entrance apertures as shown in Fig. 1(b) so that the fields "leak" outside of the cavity. We approximate the longitudinal electric field dependence on spatial coordinate to be given by $E_z(r = 0, z) = E_0 \exp\left(-\frac{z^2}{2d^2}\right)$. Compute the transverse force experienced by a particle entering the cavity with a radial offset r . Qualitatively discuss the net impact of such a force on a particle being accelerated. [10 points].

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1. [40 points] A particle of charge q and mass m is accelerated from rest by a constant electric field E_0 acting over a length d (Figure 1). While remaining non-relativistic, it then encounters a region of constant magnetic field B_0 perpendicular to its velocity.

   a) Sketch and describe its subsequent motion. [5 points]
   b) For what value of B_0 will the particle reenter the region of constant electric field a distance d from the point at which it left? [15 points]

   This arrangement can be used to measure the ratio q/m of unknown particles (the apparatus is called a mass spectrometer).

   c) Using the results of b), find q/m for a particle whose deflection $2R$ is measured to be D. If $E_0 = 10^5$ N/C, d = 10 cm, B_0 = 0.1 T and D = 9.1 cm, estimate q/m. [10 points]
   d) Compare it with the values for electrons and protons. Which one is the more likely candidate for the unknown particle? (*Hint: Use the formula sheet.*) [10 points]

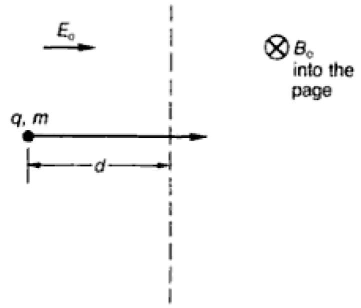

Figure 1 Schematic of A particle of charge q and mass m is accelerated from rest by a constant electric field E_0 acting over a length d, then encounters a region of constant magnetic field B_0 perpendicular to its velocity.

2. [40 points] A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω.

   (a) Find the vector potential it produces at point $\boldsymbol{r}$ (both $r \leq R$ and $r \geq R$). [20 points]
   (b) Find the magnetic field inside this spherical shell. [10 points]
   (c) Find the magnetic field outside this spherical shell. [10 points]

3. [40 points] A thin, perfectly conducting metal disk of radius a rotates with angular velocity ω about its axis of symmetry. There is a constant uniform magnetic field of magnitude B perpendicular to the disk. A wire with resistance R, fixed in space, has sliding contacts at the edge and at the center of the disk.

   (a) Find the current in the wire, in terms of B, ω, a, and R. [25 points]
   (b) Suppose that the disk has mass m, so that the mechanical rotational energy is $\frac{1}{4}ma^2\omega^2$ (you don't need to show this). If the disk starts with angular velocity $\omega = \omega_0$ at time $t = 0$, find ω and the current in the wire, as functions of time $t > 0$, assuming that the disk is not driven. [15 points]

4. [40 points] **Diffraction and Faraday's law** Consider a linearly polarized plane wave (take $E \parallel \hat{x}$) with frequency ω and amplitude E_0 propagating in vacuum along the z direction incident on a perfectly absorbing opaque screen. The screen has a square aperture with side a; see Figure below.

(a) Give the electric and magnetic fields associated with this plane wave as a function of ω and E_0. [10 points]
(b) By a simple application of Faraday's law to a loop parallel to the screen on the side away from the source (see the Figure 2 below), show that the wave has a longitudinal magnetic field component B_z after it has passed through the aperture. [15 points]
(c) Show that the ratio $\frac{B_z}{B_y}$ is a measure of the diffraction angle, i.e., is related to the ratio (wavelength)/(the side length of the square aperture) = $\lambda/$a. [15 points]

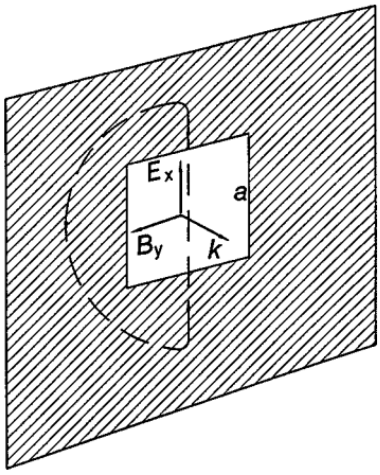

Figure 2: A screen with a square aperture of side a is illuminated by a linearly polarized electromagnetic wave. The imaginary loop shown by the dashed curve lies close to the screen, on the side away from the source, and so is partly in the shadow of the wave.

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Electricity and Magnetism

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1. [40 points] **A sphere surrounded by a uniform fluid dielectric medium.** A sphere of radius a has a bound charge Q distributed uniformly over its surface. The sphere is surrounded by a uniform fluid dielectric medium with fixed dielectric constant ε, as shown in Fig. 1. The fluid also contains a free charge density given by $\rho(\vec{r}) = -kV(\vec{r})$,
   where k is a constant and V(r) is the electric potential at r relative to infinity.

   (a) Find the potential everywhere, letting $V = 0$ at $\mathbf{r} \rightarrow \infty$. [20 points]

   (b) Find the pressure as a function of $\mathbf{r}$ in the dielectric. [20 points]

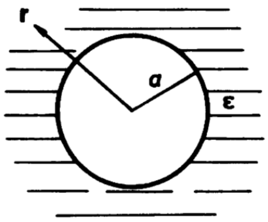

Fig. 1

2. [40 points] **Magnetic dipole and donut magnet toys.** A toy consists of identical donut-shaped permanent magnets (magnetization parallel to the axis), which slide on a vertical rod without friction (Fig.2). Treat the magnets as dipoles with mass m_d and dipole moment $\vec{\mathbf{m}}$. (a) and (b) are questions on a magnetic dipole in general, and (c) and (d) are specific to this magnet toy, using the results of the general dipole.

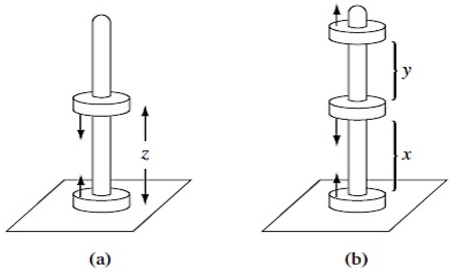

Fig.2 Donut-magnet toys for problem 2(c) and (d)

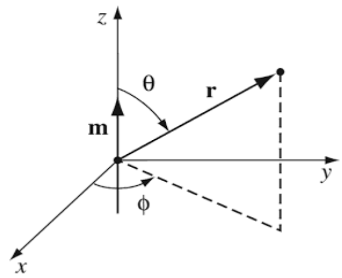

Fig.3. Geometry of magnetic dipole and coordinates for problem 2 (a)

   (a) In general, show the vector potential of a (pure) magnetic dipole $\vec{\mathbf{A}}_{dip}(\mathbf{r})$ is expressed as $\vec{\boldsymbol{A}}_{dip}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{m sin\theta}{r^2}\hat{\phi}$ in the spherical coordinates (r, θ, ϕ) if the magnetic dipole $\vec{\mathbf{m}}$ $(m = |\vec{\boldsymbol{m}}|)$ is at the origin and points in the z-direction shown in Fig.3. [6 points]

   (b) Then, still in general, what is the magnetic field of a (pure) magnetic dipole $\vec{\mathbf{B}}_{dip}(\mathbf{r})$? Express it in the spherical coordinates. [4 points]

**Problem 2 Questions (c) and (d) are in page 2.*

Problem 2 continues: Now let's consider the toy mentioned in page 1.

(c) If one puts two back-to-back magnets on the rod, the upper one will "float" – the magnetic force upward balancing the gravitational force downward (Fig. 1a). At what height (z) does it float? [15 points]

(d) Now one adds a third donut-shaped magnet (parallel to the bottom one) as in Fig. 1b. Show that the ratio of the two heights ($\alpha \equiv x/y$) is expressed as

$2 = \left(1/\alpha^4\right) + \left(1/(\alpha+1)^4\right)$. $\quad * \alpha = 0.850115$ (numerically calculated). [15 points]

3. [40 points] ***Non-magnetic metal disk under time-dependent external magnetic field***.

A non-magnetic metal disk of radius a, thickness ℓ, and conductivity σ is located parallel to the xy plane, and centered at the origin. There is a slowly varying but time-dependent external uniform magnetic field $\vec{B} = B_0 \cos(\omega t)\hat{z}$.

(a) Find the induced current density $\vec{J}$ in the disk. [20 points]
(b) A very small circular wire loop, of radius b, and with resistance R, is located parallel to the disk, and centered above it at the point $(x, y, z) = (0, 0, h)$, with $h \gg a, b, \ell$. Find the current induced in the wire loop due to the magnetic field of the disk. [20 points]

4. [40 points] ***Particle refraction at the boundary between two regions*****:** We consider a charged particle with mass m, charge q, and total energy E, crossing the boundary between two regions *(1)* and *(2)* subjected to different electrostatic potentials Φ_1 and Φ_2 taken to be constants; see Figure. The particle's trajectory is taken to be in the (x, y) plane and crosses the boundary defined by the plane $x = 0$.

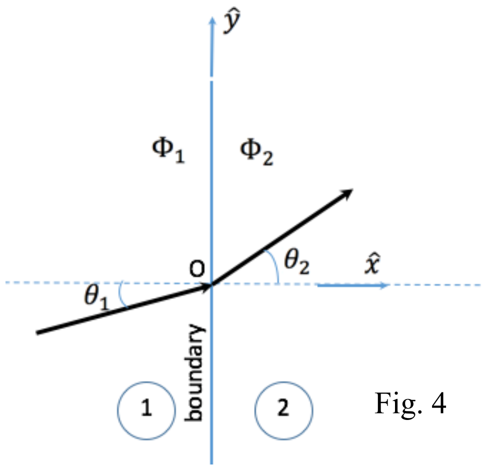

Fig. 4

(a) Given the geometry of the problem give two conserved quantities. [5 points]
(b) From a), show that the particle angles θ_1 and θ_2 are related by a condition similar to Snell's law in Optics. Identify the quantities that play the role of the index of refraction and relate them to the particle properties and potentials Φ_1 and Φ_2. [15 points]
(c) Discuss the evolution of θ_2 (for $\theta_1 \neq 0$) as a function of the potential difference $\Delta\Phi = \Phi_1 - \Phi_2$ identifying the different regime. Especially, derive an expression for the critical angle -- the value of θ_1 corresponding to $\theta_2 = \pi/2$. [13 points]
(d) Discuss how the boundary could be practically implemented and what is the needed surface-charge density to establish the required potential difference. [7 points]

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1. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b, by linear dielectric material of permittivity epsilon.
   (a) Find the potential at the center (relative to infinity). [15 points]
   (b) Compute the polarization. [9 points]
   (c) Compute the bound charge at the outer surface as well as at the inner surface. [16 points]
2. A toroidal coil has a rectangular cross section, with inner radius a, outer radius $a + w$, and height h. It carries a total of N tightly wound turns, and *the current is increasing at a constant rate* ($dI/dt = k$).
   (a) In the quasistatic approximation, find the direction and the magnitude of the magnetic field $\vec{B}$ everywhere (for points inside and outside of the toroidal coil). [6 points]
   (b) Calculate the flux of $\vec{B}$ through the cross section the toroid. [14 points]
   (c) Assume w and h are both much less than a. Find the induced electric field $\vec{E}$ at a point z above the center of the toroid. [20 points]
3. Consider the propagation of an electromagnetic wave in vacuum between two infinite parallel conducting plates spaced a distance d apart, occupying the planes $z = 0$ and $z = d$. Suppose that the electric field is everywhere parallel to the plates, and is propagating in the x direction.
   (a) Find the electric and magnetic fields between the plates. [30 points]
   (b) Show that there is a minimum angular frequency ω_{min} below which such waves will not propagate, and derive an expression for ω_{min}.[10 points]
4. The angular distribution of radiation emitted by the accelerated charged particle with charge q and velocity $\vec{\boldsymbol{v}} = c\vec{\beta}$ may be determined by

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c} \frac{\left|\hat{\boldsymbol{n}} \times \left[\left(\hat{\boldsymbol{n}} - \vec{\beta}\right) \times \left(d\vec{\beta}/dt'\right)\right]\right|^2}{\left(1 - \vec{\beta} \cdot \hat{\boldsymbol{n}}\right)}$$

   where the unit vector in the direction of the observer.
   (a) What interpretation does t' have? [5 points]
   (b) What approximations, if any, are inherent to this expression? [5 points]
   (c) Suppose the charge undergoes linear acceleration, so that $d\vec{\beta}/dt'$ is parallel to $\vec{\beta}$. Let θ be the angle between $\vec{\beta}$ and $\hat{\boldsymbol{n}}$. Find the locations θ_{max} of the peak radiated intensity. [20 points]
   (d) How do these θ_{max} values depend on the particle's total energy E if the particle is highly relativistic? [10 points]

NIU Ph.D. Candidacy Examination March 2021 (3/12/2021)

Electricity and Magnetism

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Do not just quote a result. Show your work clearly step by step.

1. A parallel-plate capacitor is made of circular plates as shown is Fig. 1. The voltage across the plates (supplied by long resistanceless lead wires) has the time dependence $V = V_0 \cos\omega t$. Assume d << a << c/ω, so that fringing of the electric field and retardation may be ignored (Region (I): between two plates, Region (II): right above the upper plate. These two regions are sufficiently distant from the edges of the plates.).
   (a) Use Maxwell's equation and symmetry arguments to determine the electric and magnetic fields in region I as function of time. [20 points]
   (b) What current flows in the lead wires and what is the current density in the plates as a function of time? [20 points]

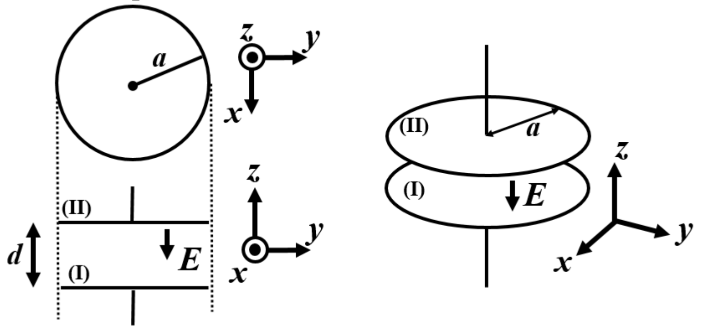


Figure 1

2. A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. (Figure 2)
   (a) Find the vector potential $\vec{A}(r, \theta, \phi)$ for $r \leq R$ and $r \geq R$. [15 x 2 = 30 points]
   (b) Calculate the magnetic field B inside this spherical shell [10 points].
   [*Hint*: Let ***r*** lie on the z-axis so that ω is tilted at an angle Ψ, and also orient the x-axis so that ω lies on the xz-plane (Figure 3). $\mathscr{r}$ is a distance from *da*' to **r**. da' is located at the charged surface of the spherical shell at R.]

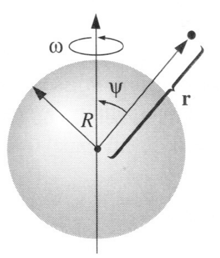


Figure 2

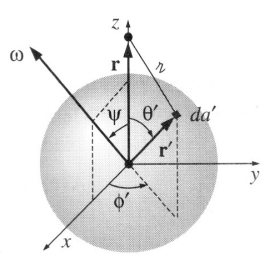


Figure 3

3. Consider a long coaxial cable consisting of an inner cylinder with radius *a* and an outer cylinder of radius b. Suppose that the electric field is given in cylindrical coordinates (r, ϕ, z) in the region $a < r < b$ by

$$\vec{E}(r, \phi, z) = Ar^n cos(kz - \omega t)\hat{r}$$

where A, k, and ω are positive constants and n is a rational number. The region between the cylinders has the same permittivity and permeability as vacuum.

   (a) Solve for n and determine the magnetic field for $a < r < b$. [16 points]
   (b) Find the charge density and the current density on the inner cylinder. [8 points]
   (c) Find an expression for the vector potential $\vec{A}$ in the region $a < r < b$. [8 points]
   (d) Find the electromagnetic energy density in the region $a < r < b$ and the electromagnetic power transmitted by the cable. [8 points]

4. We consider two pairs of field couple $(\vec{E}, \vec{B})$ given by

$$\vec{E}(\vec{r}, t) = E_0 cos(kx - \omega t)\hat{z}, \quad \vec{B}(\vec{r}, t) = B_0 cos(kx - \omega t)\hat{z} \qquad \text{(I)}$$
$$\vec{E}(\vec{r}, t) = E_0 cos(kx - \omega t)\hat{z}, \quad \vec{B}(\vec{r}, t) = B_0 cos(kx - \omega t)\hat{y} \qquad \text{(II)}$$

   (a) Discuss whether the fields described by the couples (I) and (II) represent an electromagnetic field [8 points]
   (b) For all applicable cases [positive answer in (a)]:
      i. Give the relations between the scalar parameters E_0, B_0, k, and ω? [7 points]
      ii. Find the required charge and current densities. [10 point]
   (c) For (b)ii, how are the charge and current densities modified when $\omega = k/\sqrt{\epsilon_0 \mu_0}$ [7 points]. For this case, compute the Poynting's vector [5 points] and its time-averaged value [3 points]

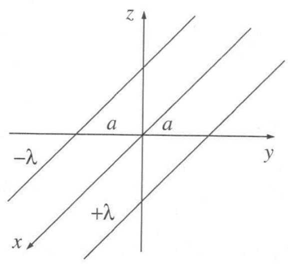
NIU Ph.D. Candidacy Examination Spring 2021 (1/4/2021)

Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

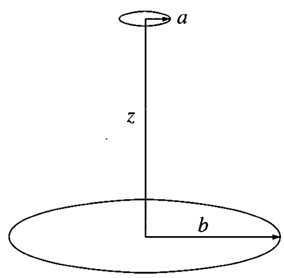
Do not just quote a result. Show your work clearly step by step.

1. Two infinitely long wires running parallel to the x-axis carry uniform charge densities $+\lambda$ and $-\lambda$ as in the figure.
   (a) Find the potential at any point (x, y, z), using the origin as your reference. [20 points]
   (b) Show that the equipotential surfaces are circular cylinders, and then locate the axis and radius of the cylinder corresponding to a given potential V_0. [20 points]



2. Consider a conducting slab of material of uniform conductivity σ, extending infinitely in the x and y directions and from $-b < z < b$. A cylindrical hole is made through the slab with radius a and axis along the z axis. Far from the hole, the slab carries a uniform current density $\mathbf{J} = J_0\hat{\mathbf{x}}$. In the following, assume $a \ll b$ and consider only the region of small $|z| \ll b$, so that the z dependence can be neglected. We neglect the effects of stray fields at the edges of the hole.
   (a) Determine the electric potential distributions in the slab and in the hole, respectively. [10 x 2 = 20 points]
   (b) Find the electric field in the hole. [10 points]
   (c) The space of the hole is filled with a material of conductivity σ'. Determine the current density in this space. [10 points]

3. A small loop of wire (radius a) lies a distance z above the center of a large loop (radius b) as in the figure. The planes of the two loops are parallel, and perpendicular to the common axis.
   a) Suppose that the large loop carries constant current I. Find the flux through the small loop. (The small loop is so tiny that you may consider the field of the large loop to be essentially constant.) [18 points]
   b) Suppose that the small loop carries constant current I. Find the flux through the large loop. (The small loop is so tiny that you may treat is as a magnetic dipole.) [18 points]
   c) Find the mutual inductances, and then confirm that $M_{12} = M_{21}$. [4 points]



4. An infinitely thin flat sheet of charge with density per unit area σ occupies the xy plane. The charges are forced to oscillate along the $\hat{x}$ direction so that their velocity at time t is $\vec{v} = \hat{x}v_0 cos(\omega t)$, resulting in electromagnetic radiation.
   a) Solve for the magnetic and electric fields everywhere as a function of time. [25 points]
   b) How much energy per unit area is radiated away in a long time T? [15 points]

NIU Ph.D. Candidacy Examination Fall 2020 (8/18/2020)

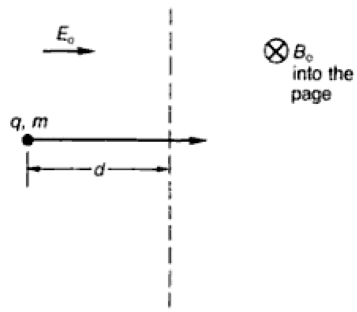
Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

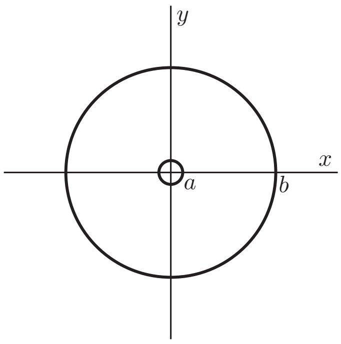
1. A particle of charge q and mass m is accelerated from rest by a constant electric field E_0 acting over a length d (see Figure). While remaining non-relativistic, it then encounters a region of constant magnetic field B_0 perpendicular to its velocity.
   a) Sketch and describe its subsequent motion. [5 points]
   b) For what value of B_0 will the particle reenter the region of constant electric field a distance d from the point at which it left? [15 points]

   This arrangement can be used to measure the ratio q/m of unknown particles (the apparatus is called a mass spectrometer).
   c) Using the results of b), find q/m for a particle whose deflection $2R$ is measured to be D. If $E_0 = 10^5$ N/C, d = 10 cm, B_0 = 0.1 T and D = 9.1 cm, calculate q/m. [10 points]
   d) Compare it with the values for electrons and protons. Which one is the more likely candidate for the unknown particle? (*Hint: Use the formula sheet.*) [10 points]



2. A very long cylinder, of radius a, carries a uniform (not radial) polarization $\vec{\boldsymbol{P}}$ perpendicular to its axis. (*Hint*: Consider it as two cylinders of opposite uniform charge density $\pm\rho$. s is a distance from the axis of a uniformly charge cylinder.)
   a) Find the electric field *inside* the cylinder ($s < a$) and express it with $\vec{\boldsymbol{P}}$. [15 points]
   b) Show that the field *outside* ($s > a$)the cylinder can be expressed as
   $$\vec{\boldsymbol{E}}(s) = \frac{a^2}{2\varepsilon_0}\frac{1}{s^2}\left[2\left(\vec{\boldsymbol{P}}\cdot\hat{\boldsymbol{s}}\right)\hat{\boldsymbol{s}} - \vec{\boldsymbol{P}}\right],$$
   where $\hat{\boldsymbol{s}}$ is a unit vector of the cylindrical coordinates. (*Hint*: Use mathematical approximations.) [25 points]

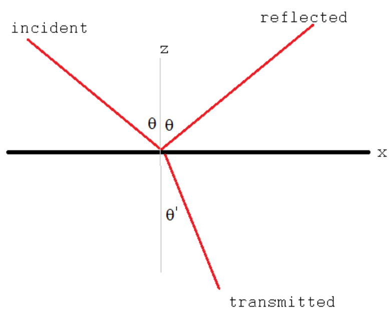
3. Two circular wire loops have very different radii $a \ll b$. Both loops have resistance per unit length K. Their centers are at the origin, and at time $t = 0$ they both lie in the horizontal xy plane as shown.



a) Suppose that the large loop carries constant current I. At time $t = 0$, it is allowed to fall under the influence of gravity with acceleration g along $-\hat{z}$, maintaining its center on the negative z axis and remaining parallel to the xy plane. Meanwhile, the small loop stays fixed. Find the mutual inductance between the two loops as a function of time, and use it to compute the current induced in the small loop. [25 points]

b) Suppose instead that the large loop stays fixed, but the small circular loop carries constant current I, and at time $t = 0$ begins spinning slowly about the x-axis at constant angular frequency ω. Find the current induced in the large circular loop. [15 points]

4. Consider an electromagnetic wave of the form $\vec{E}(\vec{r},t) = \hat{\jmath}E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ travelling through the vacuum (dielectric constant ϵ_0), incident on a surface at $z = 0$ with dielectric constant ϵ. The coordinate system is defined so that the surface normal points along the $\hat{k}$ direction. The wave makes an incident angle θ with respect to the surface normal such that $\vec{k} = k_0(\cos(\theta)\,\hat{\imath} - \sin(\theta)\,\hat{k})$. After impinging on the surface the wave gives rise to a reflected wave with amplitude rE_0 and wavevector $\overrightarrow{k_r} = k_0(\cos(\theta)\,\hat{\imath} + \sin(\theta)\,\hat{k})$and a transmitted wave with amplitude tE_0 and wavevector $\overrightarrow{k_t} = k'(\cos(\theta')\,\hat{\imath} - \sin(\theta')\,\hat{k})$ with $k' = k\sqrt{\epsilon}$.



a) Find the angle θ' as a function of θ, k and k', from the boundary condition that the longitudinal component of the electric field, ($E_{\parallel}$)is continuous across the interface. (Hint: you only need to consider the variation of the phase of the electric field across the interface). [15 points]
b) Derive two equations relating the variables t, r, θ and θ' under the conditions that $E_{\parallel}$ and $D_{\perp}$ (the normal component of the electric displacement) are continuous across the interface. [15 points]
c) Find an equation for r as a function of $\tan(\theta)$ and $\tan(\theta')$, using the two equations from part b. [10 points]

NIU Ph.D. Candidacy Examination Spring 2020 (1/7/2020)

Electricity and Magnetism

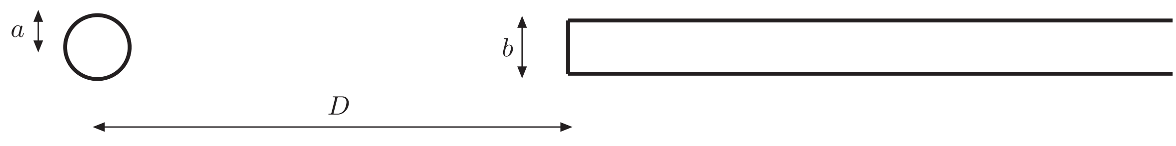
You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. Consider a cylindrical capacitor of length L. The capacitor is comprised of an inner conducting wire of radius a and an outer conducting shell of radius b. The space between ($a < r < b$) is filled with a non-conducting material (dielectric constant ϵ). For the following questions, neglect any fringe fields or edge effects.
   a) What is the value of the electric field as a function of the radial position, r, when the capacitor has a charge Q on it? [10 points]
   b) What is the capacitance of this configuration? [10 points]
   c) Suppose the dielectric is pulled out partially while the capacitor is connected to a battery of constant potential, V. What is the force required to hold the dielectric in this position? [20 points]

2.
   a) Find the vector potential a distance s from an infinite straight wire carrying a current I. [16 points]
   b) Find the magnetic vector potential inside and outside the wire of the radius R. Here, the current is uniformly distributed, i.e., the current density in the wire is constant. [24 points]

3. A small circular wire loop is in the same plane as a very thin rectangular wire loop which extends very far off to the right (where it is closed off somewhere beyond the range shown). The radius of the small circular loop is a, the distance between the parallel wires of the rectangular loop is b, and the distance from the rectangular loop to the center of the circular loop is D. The circular loop has resistance R_1 and the rectangular loop has resistance R_2. You should assume that a and b are both very small compared to D.



   a) Suppose that a current $I(t) = I_0\cos(\omega t)$ flows counterclockwise in the circular loop, where I_0 and ω are constants. Find the current induced in the rectangular loop as a function of time. Be sure to unambiguously specify the direction. [20 points]
   b) Now suppose instead that a current $I(t) = I_0 e^{-kt}$ flows counterclockwise in the rectangular loop, where I_0 and k are constants. Find the current induced in the circular loop as a function of time. Be sure to unambiguously specify the direction. [20 points]

4. We consider two counter-propagating plane waves with total electric field given by $\vec{E} = E_0[cos(kz - \omega t)\hat{\boldsymbol{x}} + cos(kz + \omega t + \psi)\hat{\boldsymbol{y}}]$.
   a) Compute the associated magnetic field. [5 points]
   b) Find the instantaneous and time averaged electric and magnetic energy densities. [10 points]
   c) Consider the case when $\psi = 0$, what is the wave polarization? [5 points]
   d) Consider the case when $\psi = \pi/2$.
      i. Determine the polarization at $z = 0, \lambda/8, \lambda/4, 3\lambda/8,$ and $\lambda/2$.[10 points]
      ii. Show the field can be written as the sum of two standing waves and give the polarization of these two waves. [10 points]

NIU Ph.D. Candidacy Examination Fall 2019 (8/20/2019)

Electricity and Magnetism

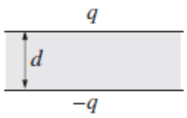
You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. A static electric charge is distributed in a spherical shell of inner radius R_1 and outer radius R_2. The charge density is given by $\rho = \alpha + \beta r$ where α and β are constants and "r" is the radial distance from the center.
    a) Find the electric field everywhere. [20 points]
    b) Find the electric potential for $r < R_1$. [10 points]
    c) Find the energy density for $r < R_1$. [10 points]

    Assume that the potential goes to zero as r tends to infinity.

2. The parallel-plate capacitor shown below is made of two identical conducting plates of area A carrying charges $\pm q$. The capacitor is filled with a compressible dielectric solid with permittivity ε and elastic energy $U_e = \frac{1}{2}k(d - d_0)^2$.



    a) Find the equilibrium separation between the plates $d(q)$. [15 points]
    b) Give the expression of potential difference between the plates $V(q)$ and sketch its dependence on q. [15 points]
    c) Find the capacitance $C(q)$ and comment on any unusual behavior. [10 points]

3.
    a) Calculate the magnetic field of a pure magnetic dipole. Put the magnetic dipole moment $\vec{m}$ at the origin and let it point in the z-direction. [15 points]
    b) A thin glass rod of radius R and length L carries a uniform surface charge σ. It is set spinning about its axis, at an angular velocity ω. Find the magnetic field at a distance $s \gg R$ from the center of the rod. *Hint*: *treat it as a stack of magnetic dipoles*. [25 points]

4. An electric dipole $\vec{p} = p\hat{z}$ moves along the x axis with constant speed v, passing through the origin at time t = 0. Find the vector potential $\vec{A}$ and the magnetic field $\vec{B}$ everywhere as a function of time (Do not assume $v \ll c$.) [40 points]

NIU Ph.D. Candidacy Examination Spring 2019 (1/8/2019)

Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. [40 points] The method of images. Two infinite parallel grounded conducting planes are held a distance a apart. A point charge q is placed in the region between them, a distance x from one plane.
    (a) Find the force F on the point charge q. [20 points]
    (b) What is the force F on q when distance a goes to the infinity? [10 points]
    (c) What is the force F on q at $x = a/2$? [10 points]

2. [40 points] A very long coaxial cylindrical cable of length d has an inner conducting cylinder inner radius a and an outer conducting cylinder with radius b, with $a < b \ll d$. The material between the conducting cylinders is an insulator with a slightly non-uniform electric permittivity $\epsilon(r) = \epsilon_1 + r\epsilon_1'$ and a slightly non-uniform magnetic permeability $\mu(r) = \mu_1 + r\mu_1'$, where $\epsilon_1,\ \epsilon_1',\ \mu_1$ and μ_1' are constants, and r is the cylindrical radial coordinate (the distance from the axis of symmetry of the cylinders).
    (a) Suppose the inner cylinder carries charge Q and the outer cylinder carries charge $-Q$. Find, in whatever order is most convenient: the electric field and the electric potential everywhere between the cylinders, the total energy stored, and the capacitance per unit length of the cable. [20 points]
    (b) Suppose instead that the inner cylinder carries current I and the outer cylinder carries the same current back in the other direction. Find, in whatever order is most convenient: the magnetic field $\vec{B}$ and the vector potential everywhere between the cylinders, the total energy stored, and the inductance per unit length of the cable. [20 points]

3. [40 points] Collection of time-dependent charged particles at rest. We consider a set of point particles fixed in space with time-dependent charge $q_k(t)$ so that the total charge density is given by $\rho(\boldsymbol{r},t) = \sum_k q_k(t)\delta(\boldsymbol{r}-\boldsymbol{r}_k)$. We assume that $\boldsymbol{E}(\boldsymbol{r},t=0) = \boldsymbol{B}(\boldsymbol{r},t=0) = 0$, and suppose that $\boldsymbol{E}(r,t) = \frac{1}{4\pi\varepsilon_0}\sum_k q_k(t)\frac{\boldsymbol{r}-\boldsymbol{r}_k}{|\boldsymbol{r}-\boldsymbol{r}_k|^3}$ for $t > 0$.

   (a) Construct a simple current density which satisfies the continuity equation. [12 points]

   (b) Describe the flow of charge predicted by the current density computed in part (a). [10 points]

   (c) Find $\boldsymbol{B}(\boldsymbol{r},t)$ and check that this field together with $\boldsymbol{E}(r,t)$ do satisfy all four Maxwell's equations. [18 points]

4. [40 points] Transverse-Magnetic (TM) wave guided by a planar conductor. A monochromatic plane wave in vacuum ($x > 0$) with $E_z > 0$ and $E_x < 0$ impinges on a perfect conductor ($x < 0$) at an angle of incidence θ; see Fig. 1.

   (a) Show that, in the steady state (i.e. once the reflected wave is established), a non-uniform TM wave occupies the vacuum space above the conductor. [20 points]

   (b) Calculate the time-averaged Poynting vector everywhere ($x > 0$). [10 points]

   (c) What is the charge density induced on the surface of the conductor? [10 points]

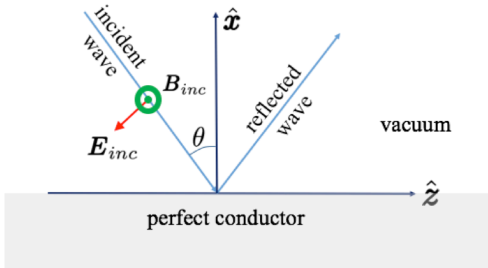


Figure 1: geometry for problem 4