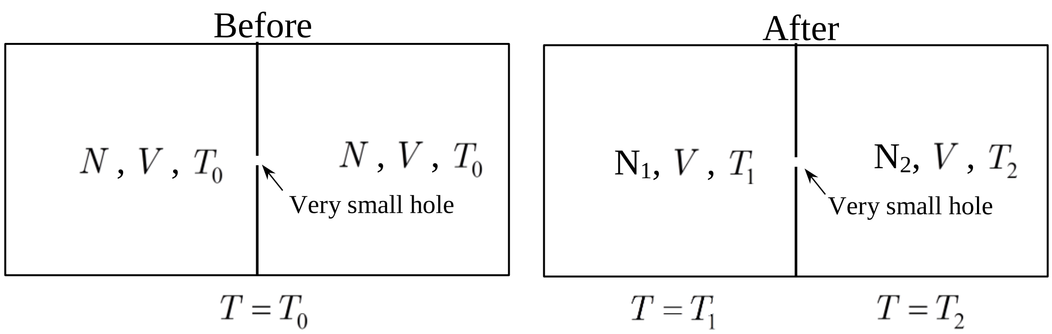
Ph. D. Qualifying Exam　　　　　　　　　　　Aug. 2018

Modern and Statistical Physics

You can do all the problems—the best 3 count towards your total score

Problem 1: (40 points)



Kinetic Theory – Effusion

A vessel is divided into two parts of equal volume by means of a plane partition, in the middle of which is a very small hole. Initially, both parts of the vessel contains an ideal gas at a temperature T_0 and a low pressure P. The temperature of one-half of the vessel is then raised to T_1 while the temperature of the other half becomes T_2.

(a) Derive an expression of the molecular flux Φ from one compartment through this tiny hole for equilibrium (effusion), using the mean speed, $\bar{v}$, number of particles N, and the volume V. [25 points]

(b) Determine the pressure difference ΔP between the two parts of the vessel, using P, T_0, T_1, and T_2 when steady state conditions are achieved. Assume $T_1 > T_2$. [15 points]

Problem 2: (40 points)

Free Electron Gas

Consider a free electron gas in two spatial dimensions with energy dispersion $E(k) = \hbar^2 k^2 / 2m$, assuming temperature $T = 0$.

(a) Express the Fermi wave vector k_F as a function of the electron density n. [10 points]

(b) Express the Fermi energy E_F as a function of the electron density n. [8 points]

(c) Derive the density of states $D(E)$. [12 points]

(d) Evaluate the mean kinetic energy of the electrons as a function of the electron density n. [10 points]

Problem 3: (40 points)

Two photons, with energies E_1 and E_2, annihilate with each other in empty space, with a collision angle θ. The final state is a muon + antimuon pair. In this problem, denote the mass of the muon as M, and its mean lifetime in its rest frame as τ.

(a) Find a necessary inequality for the process to occur, in terms of the given quantities. Discuss the behavior of this requirement for the two limiting cases $\theta = 0$ and $\theta = \pi$. [15 points]

(b) Now suppose that the collision occurs head-on ($\theta = 0$), and that $E_1 > E_2$. What is the maximum energy that one of the resulting muons can have? [17 points]

(c) Now suppose that $\theta = 0$, $E_1 = 9M$, and $E_2 = M$. What will be the maximum mean lifetime of the longer-lived muon? [8 points]

Problem 4: (40 points)

You are given the following information about $\mathrm{U}(238)$ and $\mathrm{U}(235)$:

Fraction $\mathrm{U}(238) = 99.27\%$ Lifetime $\mathrm{U}(238) = 6.52$ billion years
Fraction $\mathrm{U}(235) = 0.72\%$ Lifetime $\mathrm{U}(235) = 1.02$ billion years

ASSUME at production there are equal amounts of U(235) and U(238).

Determine when the uranium was formed (such as in a neutron star or neutron star collision). Give your answer in years before today.

Note: $\ln(0.0072) = -4.9$

Problem 5: (40 points)

Two observers move in opposite directions in a circle of radius R with different constant angular velocities ω_1 and ω_2. When they first meet, they synchronize their clocks. When they meet again, whose clock will be delayed and by how much?

Ph. D. Qualifying Exam Jan. 2018

Modern and Statistical Physics

You can do all the problems—the best 3 count towards your total score

Problem 1: (40 points)

10 liters of an ideal gas at atmospheric pressure is compressed isothermally to a volume of 1 liter and then allowed to expand adiabatically to 10 liters.

(a) Sketch the process in a pV diagram for a monoatomic gas. Label the end points of your graphs quantitatively. [12 points]

(b) Make a similar sketch for a diatomic gas. [10 points]

(c) Is net work done on or by the system? [10 points]

(d) Is the net work greater or less for the diatomic gas? [8 points]

Problem 2: (40 points)

The radius of a neutron star of mass equal to a solar mass is 12.4 km. Find the radius of a neutron star of mass equal to two solar masses. Show all steps needed to derive the answer. Assume no interaction between the neutrons other than the gravitational force and treat the problem non-relativistically.
Hints:
(1) The gravitational energy of a sphere of mass M and radius R is

$$E_{grav} = -\frac{3}{5}G\frac{M^2}{R}$$

(2) The kinetic energy of N spin ½ fermions is

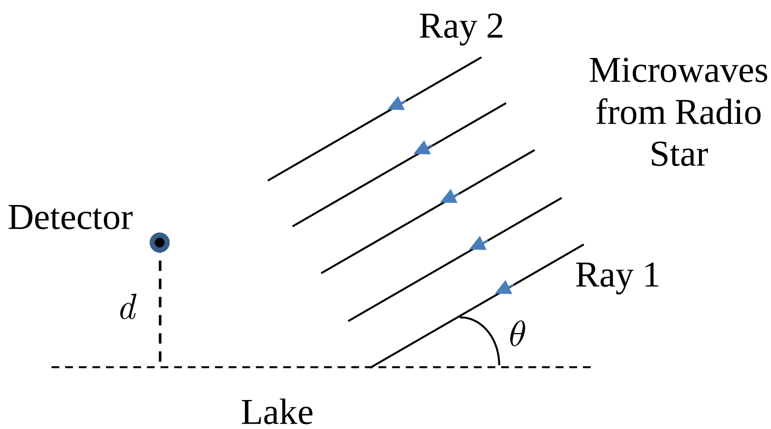
$$E_{kin} = \frac{3}{5}N\varepsilon_F$$

where ε_F is the Fermi energy.

Problem 3: (40 points)
Two particles, each of mass m, collide. The result of the collision is two other particles, each of mass M. In the center-of-momentum frame, each of the two initial-state particles have a very high (relativistic) energy E, and the final-state particles are produced at an angle θ with respect to the collision axis. Now consider what the collision looks like in the "lab" frame, in which one of the initial-state particles is at rest.

(a) What are the energies of the final state particles in the lab frame? [15 points]
(b) At what angles with respect to the collision axis are the final state particles produced, in the lab frame? [15 points]
(c) Now assume that $M = m$, and that $\theta = 60°$, and that the mean lifetime of the final-state particles when they are at rest is τ. What is the smallest that E can be in order for the mean lifetime of one of the final-state particles to be 10τ, as measured in the lab frame of this experiment? [10 points].

Problem 4: (40 points)



Microwave Detector
A microwave detector is located at the shore of a lake at a height d above the water level. As a radio star emitting monochromatic microwaves of wavelength λ rises slowly above the horizon, the detector indicates successive maxima and minima of signal intensity.

(a) What is the difference in phase between Ray 1 and Ray 2 arriving at the detector (in terms of d, λ, and θ)? [20 points]
(b) At what angle θ above the horizon is the radio star when the first maximum is received (in terms of d and λ)? [20 points]

Problem 5: (40 points)

Boltzmann Gas

Here we consider a non-interacting gas of N classical particles in three dimensions. The gas is confined in a container of volume V. The energy of the i^{th} particle is given by $\varepsilon_i = \mathbf{p}_i^2/(2m)$.

(a) Calculate the canonical partition function

$$Z^{(c)}(T,N) = \frac{c_N}{h^f}\int e^{-H\left(\{\mathbf{q}_i,\mathbf{p}_i\}\right)/(k_BT)} d\Gamma$$

where H is the Hamiltonian of the system and $d\Gamma$ is the volume element of the $2f$-dimensional phase-space spanned by the components of the coordinates $\mathbf{q}_i$ and momenta $\mathbf{p}_i$ $(i=1,...,N)$. What is the number of spatial or momentum degrees of freedom, f, of the gas? What is the value and meaning of the constant c_N? Use the thermal de Broglie wavelength, $\lambda_B = h\big/\sqrt{2\pi m k_B T}$, and the average particle distance, $a=(V/N)^{1/3}$, in your expression. [15 points]

(b) Calculate the free energy F from $Z^{(c)}$. F is the thermodynamic potential associated with the canonical ensemble, i.e., [5 points]

$$F = -k_BT\ln Z^{(c)}(T,N)$$

(c) Now, the particle number is not fixed anymore and we go over to the grand canonical ensemble. Calculate the grand canonical partition function by Laplace transformation

$$Z^{(gc)}(T,\mu) = \sum_{N=0}^{\infty} e^{\frac{\mu}{k_BT}N} Z^{(c)}(T,N)$$

What is the meaning of μ? [10 points]

(d) Show that the relation $J = F - \mu N$ for the thermodynamic potential $J(T,\mu)$ (Planck-Massieu function) of the grand canonical ensemble holds in the thermodynamic limit using the fact that the average particle number in the grand canonical ensemble can be obtained as [10 points]

$$\tilde{N} = \frac{\partial}{\partial(\mu/(k_BT))}\ln Z^{(gc)}(T,\mu)$$

Hint: Use Stirling's formula for large N: $N! \approx N^N e^{-N}$

Do not forget to answer the short questions in (a) and (c)!

Ph. D. Qualifying Exam   Aug. 2017

Modern and Statistical Physics

You can do all the problems—the best 3 count towards your total score

Problem 1: (40 points)

Chemical potential of an ideal gas:

(a) Show the chemical potential of an ideal gas in terms of the temperature T and the volume V can be expressed as:

$$\mu = c_P T - c_V T \ln T - RT \ln V - s_0 T + const$$

where μ is the chemical potential, c_P is the specific heat at constant pressure, c_V is the specific heat at constant volume, R is the gas constant, and s_0 is an entropy integration constant. [20 points]

(b) Similarly, find μ in terms of T and P. In other words, show that the chemical potential at the fixed temperature T varies with pressure:

$$\mu = \mu_0 + RT \ln\left(\frac{P}{P_0}\right)$$

where μ_0 is the value of μ at the reference point (P_0, T). [20 points]

Problem 2: (40 points)

Black body radiation (Stefan-Boltzmann law):

(a) For black body radiation, the energy density $\underline{\underline{u}} \equiv U / V$ depends on only T. Derive the Stefan-Boltzmann law, $e = \sigma T^4$, where $\sigma = \frac{ca}{4} = \frac{2\pi^5 k^4}{15 h^3 c^2} = const.$ (the Stefan-Boltzmann constant), by applying Bose-Einstein statistics. Here e is the energy flux $(c/4)(U/V)$, c is the wave velocity, $a = (8\pi^5 k^4)/(15 h^3 c^3)$, and k is Boltzmann's constant. [20 points]

(b) Now obtain the above Stefan-Boltzmann law from classical thermodynamics. In classical thermodynamics, it is found that

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

For blackbody radiation, the energy density also depends on the pressure $P = (1/3)\underline{\underline{u}}$. Use the above classical thermodynamics relation to obtain a differential equation for $\underline{\underline{u}} \equiv U / V$ and its solution, which is the Stefan-Boltzmann law. [20 points]

Problem 3: (40 points)

Assume the density of electrons in a white dwarf is $10^{30}/\text{cm}^3$. What are the Fermi energy and average energy in eV of the degenerate electrons assuming

(a) the electrons are non-relativistic? [20 points]

(b) the electrons are ultra-relativistic? [20 points]

(in reality they are somewhere in between *a* and *b*)

Calculate numbers (in eV) using:

electron mass = 0.511 MeV/c^2

$hc = 1240$ eV nm

$\hbar c = 197$ eV nm

Problem 4: (40 points)

Show the energy levels, ordered in energy, for the states in neutral Helium with $n=1$ or 2 for the two electrons. Give the total J, L, and S plus the degeneracy for each state, and explain why the energy ordering occurs. Give a zeroth order estimate for the three primary energy levels (in eV) but do not give numeric values for the shifts of those levels (just explain where the shifts are coming from).

Problem 5: (40 points)

Each problem below has the same weight in points (~ 5.714 points each)

(a) Describe briefly how the following types of detectors work, indicating the corresponding mechanisms that initiate the detection process and the critical conditions for the proper operation of the detectors.

1. Geiger counter
2. Cerenkov counter
3. Bubble chamber
4. Cloud chamber

(b) Describe the dominant processes that give rise to high energy losses (high attenuation) for the following particles (*Note*: there may be more than one dominant process):

1. A 100 keV electron passing through matter.
2. A 100 MeV photon passing through matter.
3. A 100 GeV muon passing through matter.

Ph. D. Qualifying Exam Jan. 2017

Modern and Statistical Physics

You can do all the problems—the best 3 count towards your total score

Problem 1: (40 points)

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(in reality they are somewhere in between *a* and *b*)

Calculate numbers (in eV) using:

electron mass = 0.511 MeV/c^2
$hc=1240$ eV nm
$\hbar c=197$ eV nm

Problem 3: (40 points)

A future colliding beam experiment is proposed to observe the process $e^- e^+ \to HZ$, where H is the Higgs boson, which subsequently decays (in a majority of events) into a bottom quark-antiquark pair: $H \to b\bar{b}$. In some fraction of events, the Z boson can also decay into a bottom quark-antiquark pair: $Z \to b\bar{b}$. Let the e^- and e^+ beams each have energy $E/2$, and the masses of the Higgs boson and the Z boson are M_H and M_Z. You may treat the electron and the bottom quark as massless. The speed of light is $c = 1$.

(a) What are the energies and the magnitudes of the 3-momenta of the Higgs boson and the Z boson in the center-of-momentum frame?
[20 points]

(b) For bottom quark jets produced in such events, what is the maximum possible energy? Do they come from Z decays or H decays?
[20 points]

[You do *not* need to plug in numbers, but the approximate relevant ones might be: $E = 300$ GeV, $M_H = 125$ GeV, $M_Z = 91$ GeV.]

Problem 4: (40 points)

If a degree of freedom q_i contributes a term aq_i^2 to the Hamiltonian, the principle of equipartition says that its average contribution to the energy is $\frac{1}{2}kT$.

(a) Calculate the rms deviation (or "*energy fluctuations*") of aq_i^2 about its average. [35 points] (*note*: rms = root mean square)

(b) Show that if the Hamiltonian contains N such terms $\sum_{i=1}^{N} aq_i^2$, with average energy $\bar{E} = N \cdot \frac{1}{2}kT$, the rms deviation about this average is then $\sigma = \sqrt{\frac{2}{N}}\bar{E}$ showing that for a system with many degrees of freedom, such as a macroscopic collection of molecules, the average energy fluctuations about the mean are a very small fraction of the mean energy. [5 points]

Problem 5: (40 points)

(a) Using the Maxwell Relations (*see below*), evaluate an expression that will enable you to calculate $\left.\frac{\partial U}{\partial V}\right|_T$ for a gas given its *thermodynamic equation of state* $f(p,T,V)=0$. [25 points]

(b) For the Van der Waals *equation of state* for n moles of gas

$$\left(p+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

derive an expression for $\left.\frac{\partial U}{\partial V}\right|_T$. [15 points]

The Maxwell Relations (assuming the only work done is pdV work) can be conveniently classified according to which pair of variable is chosen to be independent.

1. S and V independent:

$$đQ=TdS=dU+pdV \qquad dU=TdS-pdV$$

$$\left.\frac{\partial U}{\partial S}\right|_V=T, \qquad \left.\frac{\partial U}{\partial V}\right|_S=-p$$

$$\left.\frac{\partial T}{\partial V}\right|_S=-\left.\frac{\partial p}{\partial S}\right|_V$$

--------- **See Next Page** ---------------

2. S and p independent:

$$dH = dU + pdV + Vdp = TdS + Vdp$$

$$\left.\frac{\partial H}{\partial S}\right|_p = T, \qquad \left.\frac{\partial H}{\partial p}\right|_S = V$$

$$\left.\frac{\partial T}{\partial p}\right|_S = \left.\frac{\partial V}{\partial S}\right|_p$$

3. T and V independent:

$$dF = dU - TdS - SdT = -pdV - SdT$$

$$\left.\frac{\partial F}{\partial V}\right|_T = -p \qquad \left.\frac{\partial F}{\partial T}\right|_V = -S$$

$$\left.\frac{\partial p}{\partial T}\right|_V = \left.\frac{\partial S}{\partial V}\right|_T$$

4. T and p independent:

$$dG = dU - TdS - SdT + pdV + Vdp = -SdT + Vdp$$

$$\left.\frac{\partial G}{\partial T}\right|_p = -S \qquad \left.\frac{\partial G}{\partial p}\right|_T = V$$

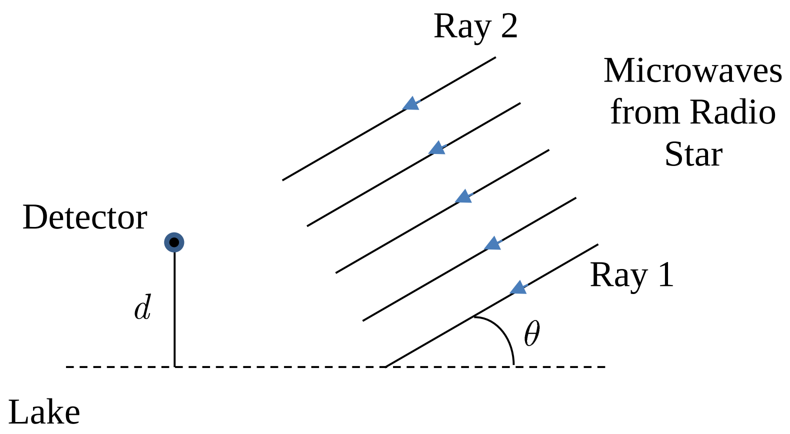
$$\left.\frac{\partial S}{\partial p}\right|_T = -\left.\frac{\partial V}{\partial T}\right|_p$$

Ph. D. Qualifying Exam Aug. 2016

Modern and Statistical Physics

Do 4 out of 4 problems (the best 3 count)

Problem 1: (40 points)



Microwave Detector

A microwave detector is located at the shore of a lake at a height d above the water level. As a radio star emitting monochromatic microwaves of wavelength λ rises slowly above the horizon, the detector indicates successive maxima and minima of signal intensity.

(a) What is the difference in phase between Ray 1 and Ray 2 arriving at the detector (in terms of d, λ, and θ)? [20 points]

(b) At what angle θ above the horizon is the radio star when the first maximum is received (in terms of d and λ)? [20 points]

Problem 2: (40 points)

Classical Thermodynamics

The equation of the sublimation and the vaporization curves for a particular material are given by

$$\ln P = 0.04 - 6/T \quad \text{(sublimation)}$$

$$\ln P = 0.03 - 4/T \quad \text{(vaporization)}$$

where the pressure P is in atmospheres (atm), and the temperature T is in Kelvin. (The following problem involves simple arithmetic calculations.)

For the following subsections, the subscripts 1, 2, and 3 denote solid, liquid, and vapor phases, respectively.

(a) Find the temperature of the triple point T_{TP}. [10 points]

(b) Show that the specific latent heats of vaporization l_{23} and sublimation l_{13} are $4R$ and $6R$, respectively. Here R is the Gas constant. (Assume that the specific volume in the vapor phase is much larger than the specific volume in the liquid and solid phases). [20 points]

(c) Find the latent heat of fusion l_{12}. [10 points]

Problem 3: (40 points)

Boltzmann Gas

Here we consider a non-interacting gas of N classical particles in three dimensions. The gas is confined in a container of volume V. The energy of the i^{th} particle is given by $\varepsilon_i = \mathbf{p}_i^2/(2m)$.

(a) Calculate the canonical partition function

$$Z^{(c)}(T,N) = \frac{c_N}{h^f} \int e^{-H\left(\{\mathbf{q}_i,\mathbf{p}_i\}\right)/(k_B T)} d\Gamma$$

where H is the Hamiltonian of the system and $d\Gamma$ is the volume element of the $2f$-dimensional phase-space spanned by the components of the coordinates $\mathbf{q}_i$ and momenta $\mathbf{p}_i$ $(i=1,...,N)$. What is the number of spatial or momentum degrees of freedom, f, of the gas? What is the value and meaning of the constant c_N? Use the thermal de Broglie wavelength, $\lambda_B = h/\sqrt{2\pi m k_B T}$, and the average particle distance, $a = (V/N)^{1/3}$, in your expression. [15 points]

(b) Calculate the free energy F from $Z^{(c)}$. F is the thermodynamic potential associated with the canonical ensemble, i.e., [5 points]

$$F = -k_B T \ln Z^{(c)}(T,N)$$

(c) Now, the particle number is not fixed anymore and we go over to the grand canonical ensemble. Calculate the grand canonical partition function by Laplace transformation

$$Z^{(gc)}(T,\mu) = \sum_{N=0}^{\infty} e^{\frac{\mu}{k_B T}N} Z^{(c)}(T,N)$$

What is the meaning of μ? [10 points]

(d) Show that the relation $J = F - \mu N$ for the thermodynamic potential $J(T,\mu)$ (Planck-Massieu function) of the grand canonical ensemble holds in the thermodynamic limit using the fact that the average particle number in the grand canonical ensemble can be obtained as [10 points]

$$\tilde{N} = \frac{\partial}{\partial(\mu/(k_B T))} \ln Z^{(gc)}(T,\mu)$$

Hint: Use Stirling's formula for large N: $N! \approx N^N e^{-N}$

Do not forget to answer the short questions in (a) and (c)!

Problem 4: (40 points)

Mandelstam Variables

Consider the relativistic two-body scattering $A+B \rightarrow C+D$ with corresponding 4-vectors p_A, p_B, p_C, and p_D. The masses of the corresponding objects are m_A, m_B, m_C, and m_D. You can set the speed of light as $c=1$ if you would like. The Mandelstam variables are useful Lorentz invariants:

$$s=(p_A+p_B)^2$$
$$t=(p_A-p_C)^2$$
$$u=(p_A-p_D)^2$$

(a) Show that $s+t+u=m_A^2+m_B^2+m_C^2+m_D^2$. [10 points]

(b) Show that the total center of mass energy is given by $\sqrt{s}$. [10 points]

(c) The LHC collides protons in the lab frame with an energy of 6.5 TeV against other protons with an energy of 6.5 TeV (so that $\sqrt{s}=13$ TeV). One of the things looked for is the production of new Z' bosons. What is the most massive Z' that the LHC could directly observe?
[10 points]

(d) Why can the photon not decay? [10 points]
Hint 1: Prove that the mass of a decaying particle must exceed the masses of the particles produced in its decay. In other words, if A decays into $B+C$, show that $m_A > m_B + m_C$.
Hint 2: Evaluate in the center of mass frame.

Ph. D. Qualifying Exam   Feb. 2016

Modern and Statistical Physics

Do <u>3</u> out of <u>5</u> problems

Problem 1: (40 points)

(a) The equation of state of a van der Waals gas is

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

where P is pressure, T is temperature, R is gas constant, v specific volume, a and b are characteristic constants for a given gas. Qualitatively explain the meaning of the terms $\frac{a}{v^2}$, and $-b$. [10 points]

(b) A van der Waals gas undergoes an isothermal expansion from specific volume v_1 to specific volume v_2. Calculate the change in the specific Helmholtz function. [15 points]

(c) Calculate the change in the specific internal energy in terms of v_1 and v_2. [15 points]

Problem 2: (40 points)

A negatively charged muon stops in aluminum, which has Z=13, where it is captured and decays down to the 1S state.

(a) What is the energy of the gamma emitted when the muon drops from the $n=2$ to the $n=1$ atomic state? Give your answer in terms of the ground state (1S) energy of hydrogen. [20 points]

If the muon then undergoes the transition: $\mu^- + Al \rightarrow e^- + Al$

(b) What normally conserved quantity is violated? [5 points]

(c) What is the total kinetic energy of the electron in terms of the masses of the muon, electron, and aluminum? [15 points]

masses: muon 105 MeV, electron 0.5 MeV, Aluminum 25000 MeV

Problem 3: (40 points)

One-dimensional relativistic gas: Here we consider a non-interacting gas of N relativistic particles in one dimension. The gas is confined in a container of length L, i.e., the coordinate of each particle is limited to $0 \le q_i \le L$. The energy of the i^{th} particle is given by $\varepsilon_i = c|p_i|$.

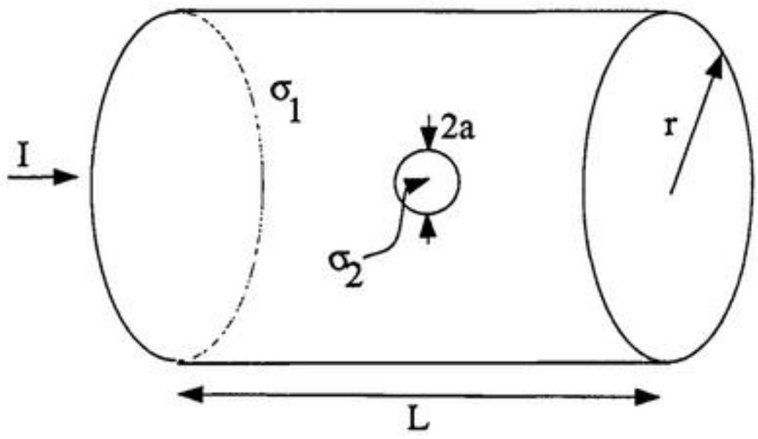
(a) Calculate the single particle partition function $Z_1(T,L)$ for given energy E and particle number N. [12 points]

(b) Calculate the average energy E_1 and the heat capacity C_1 per particle from $Z_1(T,L)$. [12 points]

(c) Calculate the Boltzmann entropy $S_B(E,N)$ of all N particles. Consider them as indistinguishable. [16 points]

Hint: Use Stirling's formula for large N: $N! \approx \sqrt{2\pi N} N^N e^{-N}$.

Problem 4: (40 points)



A cylindrical resistor has a radius b, length L, and conductivity σ_1. At the center of the resistor is a defect consisting of a small sphere of radius a inside which the conductivity is σ_2. The input and output currents are distributed uniformly across the flat ends of the resistor.

(a) What is the resistance of the resistor if $\sigma_1 = \sigma_2$? [5 points]

(b) Approximate the spherical defect as:

- a cylinder of radius a and length a.
- concentric with the cylinder of the physical resistor
- and centered lengthwise on the center of the physical resistor.

What is the resistance of the resistor if $\sigma_1 \neq \sigma_2$? [25 points]

(c) *Estimate* the relative change in the resistance to first order in $\sigma_1 - \sigma_2$ if $\sigma_1 \neq \sigma_2$. (Make any assumptions needed to simplify your method of estimation). [10 points]

Problem 5: (40 points)

Estimate the following quantities, *indicating how you arrived at your estimate* (do not try to calculate exact numbers—make only estimates):

Choose only 4 of the following 7 quantities: [10 points each]

(a) the frequency of radiation used in a household microwave oven,

(b) the energy yield of a fission bomb with a 30 kg uranium core,

(c) the energy of impact of the earth with a meteorite 10 meters in diameter,

(d) the temperature of the sun,

(e) the temperature of a 60-watt light bulb filament,

(f) the speed of sound in helium gas at STP,

(g) the total length of blood capillaries in the human body

You may find the following useful:

Speed of sound in a gas: $v = (\gamma P / \rho)^{1/2}$, where $\gamma = C_P / C_V$

The speed of sound in air is about 330 m/sec

Stefan's Law: $F = \sigma_B T^4$, where $\sigma_B \approx 6 \times 10^{-8}$ W/m^2·K^4

Wein's displacement Law: $\lambda_{\max} T \approx 2.9 \times 10^{-3}$ m·K

The average human has about 4 liters of blood

Typical energy release in a fission process is about 200 MeV

Earth's orbit has radius of about 8 light minutes

The radius of the sun is about 2 light seconds

The earth's orbital velocity is about 30 km/sec

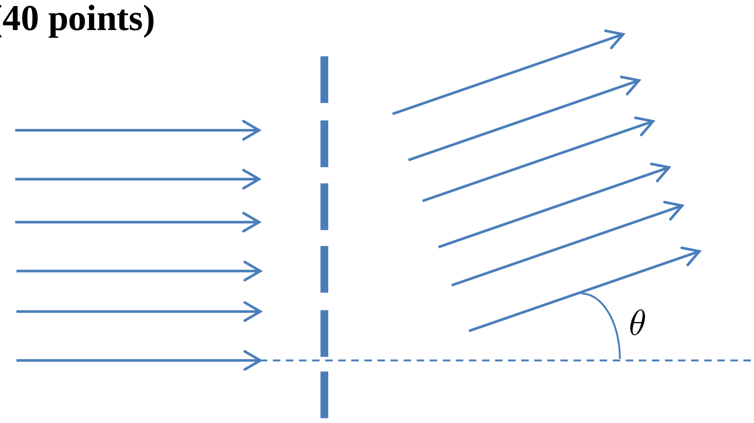
The sun's energy flux at the earth's surface is about 1.3 kW/m^2

Ph. D. Qualifying Exam Sept. 2015

Modern and Statistical Physics

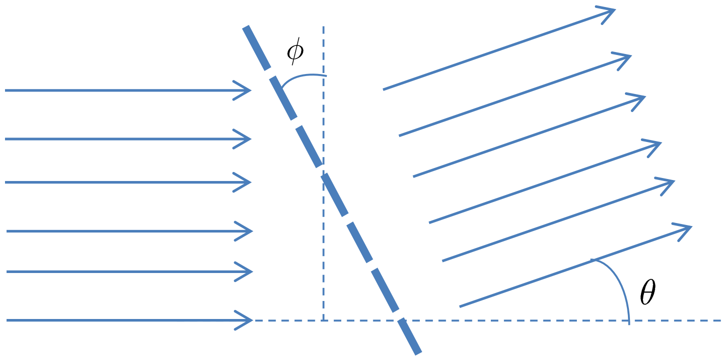
Do 3 out of 5 problems

Problem 1: (40 points)



A parallel, thin, monochromatic laser beam of wavelength λ falls on a diffraction grating at normal incidence. The grating spacing is d.

(a) Derive the *grating equation* for normal incidence giving the relation between d, λ, and θ. [10 points]



(b) Next rotate the grating through an angle $\phi < 90°$ around an axis that is parallel to the grating lines. Derive the *grating equation* giving the relation between d, λ, θ, and ϕ. [20 points]

(c) How does the interference pattern produced on a viewing screen differ between parts (a) and (b)? (intuitive guesses are allowed). [10 points]

Problem 2: (40 points)

Classical Thermodynamics

(a) A van der Waals gas undergoes an isothermal expansion from specific volume v_1 to specific volume v_2. Calculate the change in the specific Helmholtz function. [20 points]

Hint: The equation of state of a van der Waals gas is $\left(P+\frac{a}{v^2}\right)(v-b)=RT$. Here *P* is the pressure, *v* the specific volume, *T* is the absolute temperature, and *R* is the gas constant, the constant *a* and *b* are the characteristic constants of the gas.

(b) For the above problem, calculate the change in the specific internal energy in terms of v_1 and v_2. [20 points]

Problem 3: (40 points)

Statistical mechanics

There is a system of two identical particles, which may occupy any of the three energy levels $\varepsilon_0 = 0,\ \ \varepsilon_1 = \varepsilon,\ \ \varepsilon_2 = 3\varepsilon.$

The system is in thermal equilibrium at temperature T, which means that the system is a canonical ensemble. For *each* of the following cases, determine the *partition function* and the *energy* and carefully enumerate the *configurations*.

(a) The particles are Fermions. [10 points]

(b) The particles are Bosons. [10 points]

(c) The particles obey Boltzmann statistics and now they are distinguishable. [10 points]

(d) Discuss the conditions under which Fermions or Bosons may be treated as Boltzmann particles. [10 points]

Problem 4: (40 points)

Electric Conductivity of Copper

The Drude model for a metal assumes that the conduction electrons can be approximated by a gas of free electrons where the only important parameters for the gas are n, the number density of electrons, and τ, the time between collisions.

(a) Show that in this model the electrical conductivity of a metal can be expressed as [24 points]

$$\sigma = \frac{ne^2\tau}{m}$$

(b) Estimate the collision time τ for an electron in copper. The resistivity of copper metal is 2×10^{-6} Ω cm and the atomic density of copper is 9×10^{22} atoms/cm^3. [16 points]

Problem 5: (40 points)

The D^0 meson , a combination of a charm quark and up anti-quark ($c\bar{u}$), can decay to K-mesons and Pi-mesons in several different channels. Three possibilities are

$$D^0 \to K^- + \pi^+ \quad D^0 \to \pi^- + \pi^+ \quad D^0 \to K^+ + \pi^-$$

Where a K^+ meson is composed of $u\bar{s}$, a K^- meson is composed of $\bar{u}s$, a π^+ meson is composed of $u\bar{d}$, and a π^- meson is composed of $\bar{u}d$.

(a) Draw the leading order Feynman diagrams for each of the above decays. [10 points]

(b) Based on the quark flavors present in the initial state and the final state, rank these decay channels from highest branching fraction to lowest branching fraction. Explain your answer. [5 points]

(c) In a general two-body decay, particle A, at rest, decays into particles B and C ($A \to B + C$). Find the energy and the magnitude of the momentum of particles B and C. Express your answer in terms of the particle masses (m_A, m_B, and m_C) and the speed of light (c). [25 points]

Ph. D. Qualifying Exam   Feb. 2015

Modern and Statistical Physics

Do $\underline{3}$ out of $\underline{5}$ problems

Problem 1: (40 points)

Joule-Thompson coefficient of a van der Waals gas
The specific internal energy of a van der Waals gas is given by

$$u = u_0 + c_v T - \frac{a}{v},$$

where u_0 is an initial specific internal energy and c_v the ***isochoric*** specific heat. The constant a is a characteristic constant of the gas, expressed in the equation of state of a van der Waals gas,

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT\ .$$

Here P is the pressure, v the specific volume, T is the absolute temperature, R is the gas constant, and the constant b is another characteristic constant of the gas.

For the following questions, show your work. Don't write just answers.

(a) Find an expression for the Joule coefficient $\eta \equiv \left(\frac{\partial T}{\partial v}\right)_u$. If a = 0, what is η? [5 points]

(b) Find an expression for the specific enthalpy h as a function of v and T. [10 points]

(c) Find the Joule-Thompson coefficient $\mu \equiv \left(\frac{\partial T}{\partial P}\right)_h$, expressed with the isothermal compressibility κ and the ***isobaric*** specific heat c_P. [15 points]

(d) Calculate the isothermal compressibility κ and the Joule-Thompson coefficient μ when $a = b = 0$. [5 x 2 = 10 points]

Problem 2: (40 points)

Fermi-Dirac gases

(a) Show that the average energy per fermion is $\frac{3}{5}\varepsilon_F$ at T = 0 K by making a direct calculation of $\frac{U(0)}{N}$, where ε_F is the Fermi energy, $U(0)$ is the internal energy at 0K, and N is the number of fermions (fermion gas particles). [20 points]

(b) Similarly, calculate the average speed of a fermion gas particle at T = 0 K. Use the Fermi velocity v_F, defined by $\varepsilon_F = \frac{1}{2}mv_F{}^2$. [20 points]

Problem 3: (40 points)

Compton scattering between a photon and moving electron

We consider the head-on collision between a photon with energy $\hbar\omega$ moving along the x direction and an electron with total energy γmc^2 (where $\gamma = [1-\beta^2]^{-1/2}$). The electron propagates along the $-x$ direction; see Fig.1. After the interaction the scattered photon makes an angle θ with the x axis and has a final energy $\hbar\omega'$.

(a) Find the energy of the scattered photon as a function of the angle θ, the electron initial Lorentz factor γ, and $\hbar\omega$. [15 points]

(b) Simplify your formula assuming the involved photon energies are much larger than the electron rest mass. [10 points]

(c) We now specialize to the case of a backscattered photon (i.e. traveling with an angle $\theta = \pi$) and further assume that the electron is ultra-relativistic. Show that the backscattered photon energy is given by the simple formula $\hbar\omega' = 4\gamma^2\hbar\omega$. [7 points]

(d) Discuss how one could use this effect to produce X rays with a final photon energy $\hbar\omega' = 50$ keV using, e.g., an infrared laser. [8 points]

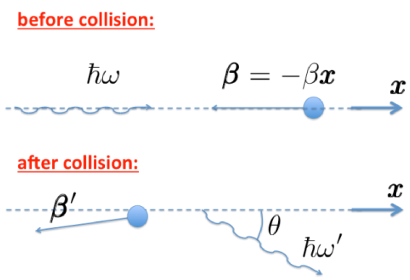
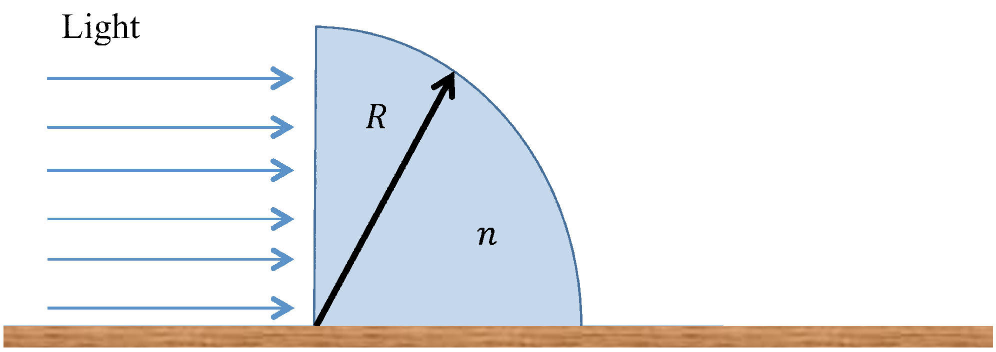


Figure 1

Problem 4: (40 points)



A glass prism in the shape of a quarter-cylinder lies on a horizontal table. A uniform, horizontal light beam falls on its vertical plane surface, as shown above. The radius of the cylinder is R and the refractive index of the glass is $n = 1.5$. A patch of light will form on the table.

(a) What distance from the lens does the patch of light start? (express your result in terms of R) [20 points]

(b) How far from the lens does the patch of light extend? (express your result in terms of R) [20 points]

The Lensmaker's equation might be helpful for this problem:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

Problem 5: (40 points)

On the basis of the experimental results below, try to classify each of the beams 1 through 5 under one of the following headings and *explain your reasoning*:

unpolarized light
partially polarized light
linearly polarized light
circularly polarized light
elliptically polarized light

If the given experiment is not sufficient to determine classification into only one of the above headings, list all the possible polarization states consistent with the given results. [8 points each]

Beam 1: A linear polarizer placed in beam 1 transmits maximum intensity when its transmission axis is at $q = 45°$ to the x-axis and essentially zero intensity when its transmission axis is at $q = -45°$ (note minus sign). The x-axis is transverse to the beam direction.

Beam 2: A linear polarizer placed in beam 2 and rotated about the beam axis transmits the same intensity for all orientations.

Beam 3: A linear polarizer inserted in beam 3 and rotated about the beam axis transmits a maximum intensity in one orientation and a minimum intensity when rotated $90°$ from this maximum, but does not extinguish the beam for any orientation.

Beam 4: A piece of naturally cleaved calcite with one face perpendicular to beam 4 transmits two separated beams for most orientations when the calcite is rotated about the beam axis, but only a single beam for particular orientations that are $90°$ apart.

Beam 5: A quarter-wave plate placed in beam 5 transmits linearly polarized light. As the quarter-wave plate is rotated about the beam axis, the axis of linear polarization of the transmitted light rotates, but the intensity of the transmitted light does not vary.