Ph. D. Qualifying Exam   Sept. 2014

Modern and Statistical Physics

Do 3 out of 5 problems

Problem 1: (40 points)

(a) A laser beam having a diameter D in air strikes a uniform flat piece of glass of index of refraction n at an angle θ. What is the diameter of the beam in the glass?

(b) A narrow beam of white light is incident in air at an angle θ on a uniform flat sheet of glass of thickness L. The index of refraction for red light is n_r and for violet light it is n_v. Determine the diameter of the emerging beam assuming the incident beam is exceedingly narrow.

Problem 2: (40 points)

Find the maximum total energy and maximum transverse momentum for the initally at rest particle after the following elastic scatter reactions:

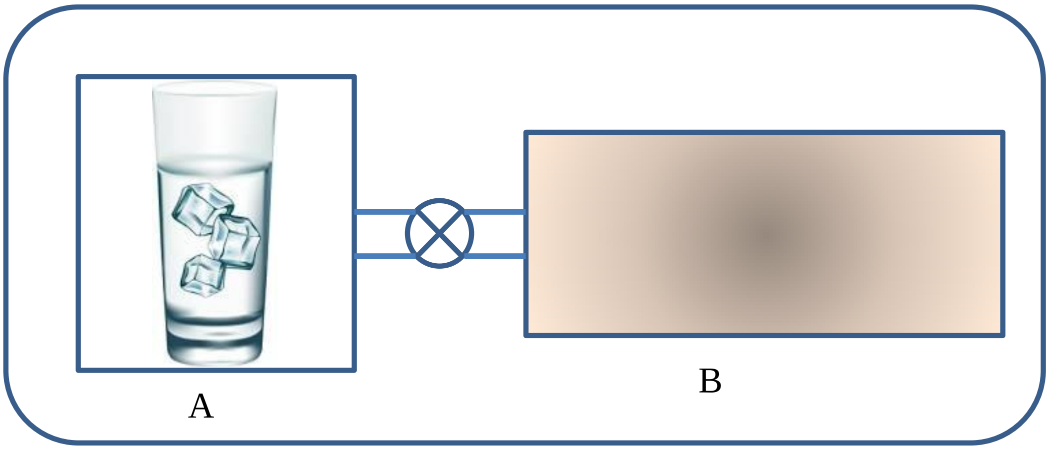
(a) a 10 GeV positron strikes an electron at rest

(b) a 10 GeV proton strikes an electron at rest

(c) a 10 GeV electron strikes a proton at rest

Note: mass electron = 0.5 MeV mass proton=1 GeV (for this problem)

Problem 3: (40 points)



The system above is thermally isolated. It consists of two parts, a bucket (A) containing a mixture of ice and water, and a body (B) with constant heat capacity C. The two parts are connected by a thermodynamic device which can extract heat from part (A) and add heat to part (B).

The entire system is initially at temperature T_0 (the absolute temperature of melting ice). The latent heat of melting of ice is L per unit mass. Ignore all effects due to changing volumes.

(a) How much heat must be removed from (A) to freeze an additional mass m of water? What is the entropy change of (A) in this process? [10 points]

(b) In this process (B) absorbs the heat released by the thermodynamic device, and its temperature increases to T_1. Calculate the heat absorbed ΔQ_b and the entropy change ΔS_b of the body (B) in terms of T_1. [10 points]

(c) What is the minimum possible T_1? What is the minimum possible ΔQ_b? [20 points]

Problem 4: (40 points)

Calculate the radius of the largest atom which may be located interstitially (without exercising stress) in a BCC lattice of a metallic system whose atoms have a diameter of R.

Problem 5: (40 points)

(a) What are the two conditions for Fermi-Dirac statistics that the fermions obey? [5x2 = 10 points]

(b) There is an assembly of N non-interacting fermions under the conservation of particles and energy:

$$\sum_{j=1}^{n} N_j = N$$

$$\sum_{j=1}^{n} N_j \, \varepsilon_j = U$$

where N_j is the number of particles with single-particle energy ε_j; N and U are fixed. There are g_j quantum states at the jth energy level.

i. *Derive* the thermodynamic probability ω_j of this assembly of fermions for the jth energy level ε_j. [10 points]

ii. Using the above result, find the total number of microstates corresponding to an allowable configuration ω_{FD}. [5 points]

iii. *Express and Derive* the Fermi-Dirac distribution function for a discrete energy level ε_j. *Hint*: Find the occupation number of each energy level when the thermodynamic probability is a maximum, i.e., the equilibrium macrostate N_i. Apply the method of *Lagrange multipliers* for a function with two variables under the conservation of particles and energy. Set one of the multipliers to be $1/kT$ and the other to be μ/kT, where μ is the chemical potential. [3 + 12 points]

NIU Ph.D. Candidacy Examination Spring 2014 (2/22/2014)

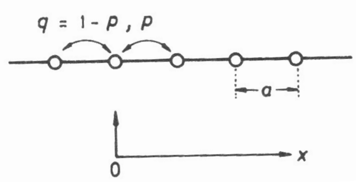
Modern and Statistical Physics

Do 3 out of 5 problems

1. [40 points] ***Classical thermodynamics, Entropy of the van der Waals gas***.
   a) Express the combined first and second law of thermodynamics. [5 points]
   b) Derive a *Tds* equation under the isochoric condition. Use specific heat under the isochoric (constant volume) condition, c_v. [20 points]
   c) Calculate the entropy for a van der Waals gas, using the above *Tds* equation. Here, the equation of state of the van der Waals gas is $\left(P+\frac{a}{v^2}\right)(v-b)=RT$. [15 points]

2. [40 points] ***Statistics of One-dimensional lattice vibration***

There is a one-dimensional lattice with lattice constant a as shown in figure below. An atom transits from a site to a nearest neighbor site every τ seconds. The probabilities of transiting to the right and left are p and $q = 1 - p$, respectively.



   a) Calculate the average position $\bar{x}$ of the atom at the time $t = N\tau$, where $N \gg 1$. [20 points]
   b) Calculate the mean-square value $\overline{(x-\bar{x})^2}$ at the time t. [20 points]

3. [40 points] ***Special relativity, a rocketship travelling back home.***

A rocketship is traveling towards the Earth at 0.6c. It fires a projectile of mass M with a velocity of 0.8c in the rocketship's frame directly at the Earth.

a) What is the energy of the projectile in the Earth's frame?

The rocketship is now traveling perpendicular to a line from the rocketship to the Earth at 0.6c and again fires a projectile at 0.8c in the rocketship's frame and at 90 degrees from the direction of the rocketship's motion (in the spaceship's frame).

b) What is the energy of the projectile in the Earth's frame?
c) What is the tangent of the angle of the projectile relative to the rocketship's motion in the Earth's frame?

You can ignore the effects of gravity as the rocketship is 2 light years away from the Earth.

4. [40 points] ***Higgs boson.***

a) Show that the process $\gamma \rightarrow e^+e^-$ is forbidden in free space. [10 points]

b) Deduce an expression for the energy of a γ from the decay of a Higgs boson: $H \rightarrow \gamma\gamma$ in terms of the Higgs boson mass (m_H), the Higgs boson energy in the lab frame (E), the speed of the Higgs boson (β), and the emission angle (θ^*) of one of the photons in the Higgs boson rest frame. [20 points]

c) The Higgs boson does not couple directly to photons. Draw the most important, lowest order Feynman diagram that illustrates the decay of the Higgs boson into two photons. Remember to label all the particles in your diagram. [10 points]

5. [40 points] ***Optics.***

A blade of grass standing y_{grass} tall is d_A cm in front of a thin positive (convex) lens, A, having a focal length, f_A. A thin negative (concave) lens, b, with a focal length of $-f_B$ is placed d_B behind the first lens B. The blade of grass is illuminated by a monochromatic plane wave parallel to the optic axis of this combined lens system.

a) Draw the ray diagram of imaging the above object by this combined lens system. [10 points]

b) Where does the first lens, A, form the image of the blade of grass? Express it with the above given parameters. What's its magnification? [5 x 3 = 15 points]

c) Where is the final image formed by this combined lens system? Express it with respect to the location of the lens B. [10 points]

d) What is the total magnification of this combined lens system? [5 points]

NIU Ph.D. Candidacy Examination Fall 2013 (9/28/2013)

Modern and Statistical Physics

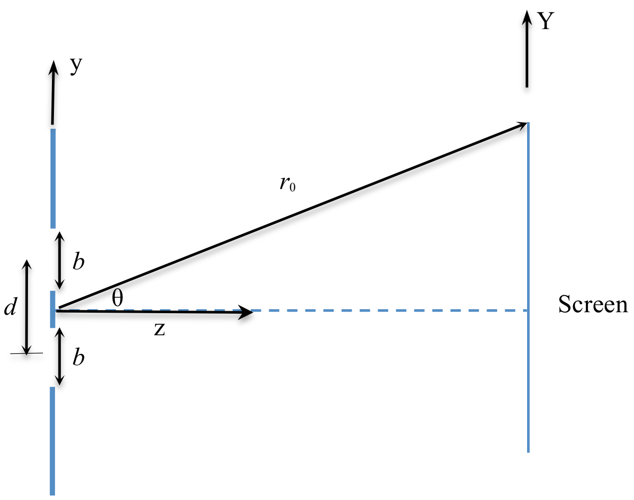
Do 3 out of 5 problems

1. [40 points] ***Classical thermodynamics, Entropy change in mixing***.
Two equal quantities of water, each of mass m and at temperatures T_1 and T_2 are adiabatically mixed together, the pressure remaining constant.

   (a) Calculate the entropy change of the universe (thermodynamic surrounding) including c_P is the specific heat capacity of the water at constant pressure. [25 points]

   (b) Show that $\Delta S > 0$ for any finite temperatures T_1 and T_2. [15 points]

2. [40 points] ***Fraunhofer diffraction of double slits***. Consider an aperture consisting of two long parallel openings of constant width b and separation d in an opaque screen illuminated by a coherent plane wave with a wavelength of λ. The length of the openings is L in the x direction, perpendicular to the paper, and long enough to ignore the diffraction effects of both ends. r_0 lies in the yz-plane, and measures the distance from the geometric center of the grating to a point on the observation screen. The coordinates on the observation screen are (X, Y) and located far away to fulfill the Fraunhofer diffraction condition.



Fraunhofer diffraction geometry for a multiple slit aperture.

   (a) What is the condition for Fraunhofer diffraction? Express it in a mathematical form. [5 x 2 = 10 points]

   (b) Calculate the electric field distribution and intensity (irradiance) along Y-axis on the observation screen for a 2-slits (double slit) case. [20 points]

   (c) Where along the Y-axis do bright fringes occur? Why? [5 points]

   (d) What makes a missing order? [5 points]

3. [40 points] ***Fermi-Dirac gasses.*** We consider an ideal 3D Fermi gas comprising N noninteracting fermions, each of mass m, in a container of volume V held at temperature T.

(a) Express the Fermi function at $T = 0$. [5 points]

(b) Find $g(\varepsilon)d\varepsilon$, the number of quantum states whose energy lies in the range ε to $\varepsilon + d\varepsilon$. [10 points]

(c) Find the average energy per fermion at absolute zero by making a direct calculation of $U(0)/N$, where $U(0)$ is energy at $T = 0$ for N fermions. Express this in terms of the Fermi energy, ε_F. [10 points]

(d) Similarly, calculate the average speed of a fermion gas particle at $T = 0$. Express this in terms of the Fermi velocity, v_F. v_F is defined by $\varepsilon_F = (1/2)mv_F^2$. [15 points]

4. [40 points] A particle with mass M and energy E, collides with another particle of mass M which is initially at rest. The result of the collision is two identical particles, each of mass m.

(a) Find a Lorentz transformation to the center-of-momentum frame, and find the energies of each of the two initial particles in that frame. [20 points]

(b) Find the maximum and minimum possible energies E_{max} and E_{min} of the two final state particle in the lab frame. [10 points]

(c) Find the energies of the two final state particles, if one of them is emitted at a right angle to the initial direction of the incident particle in the lab frame. [10 points]

5. [40 points] ***Statistical mechanics.*** The partition function of an Einstein solid is

$$Z = \frac{e^{-\theta_E/2T}}{1 - e^{-\theta_E/T}},$$

where θ_E is the Einstein temperature. Treat the crystalline lattice as an assembly of $3N$ distinguishable oscillators.

(a) Calculate the Helmholtz function F. [10 points]

(b) Calculate the entropy S. [15 points]

(c) Show that the entropy approaches zero as the temperature goes to absolute zero. [15 points]

Ph. D. Qualifying Exam Feb. 2013

Modern and Statistical Physics

Do <u>3</u> out of <u>5</u> problems

Problem 1: (40 points)

The specific Gibbs function of a gas is given by

$$g = RT\ln\left(\frac{P}{P_0}\right) - AP$$

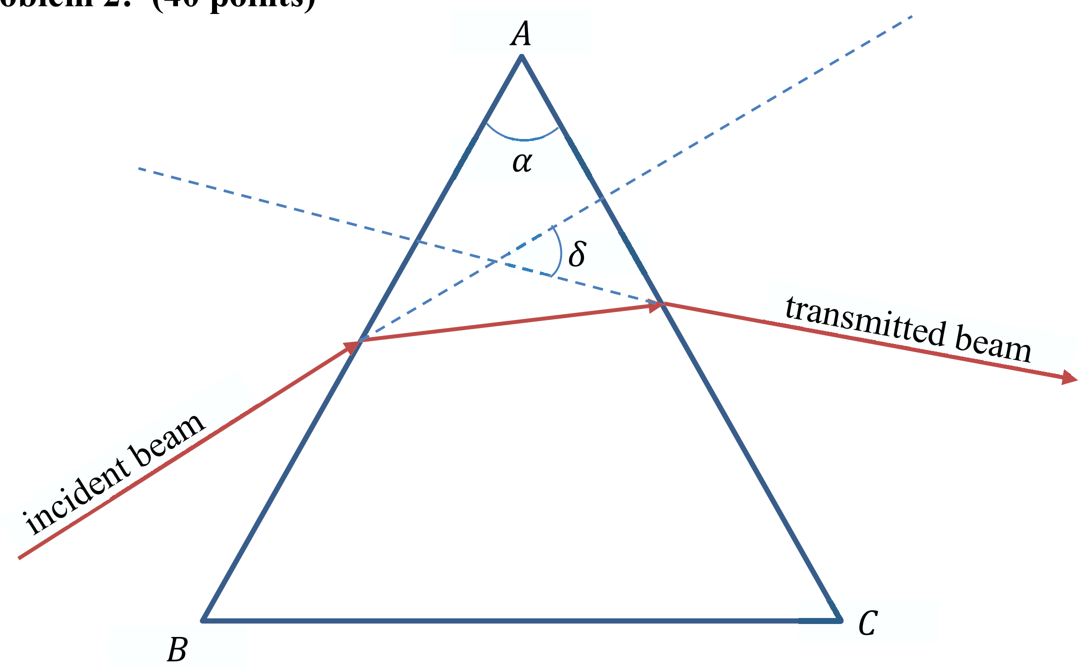
where A is a function of T. Find expressions for:

(a) the equation of state [15 points]

(b) the specific entropy [10 points]

(c) the specific Helmhotz function [15 points]

Problem 2: (40 points)



The angle of deviation, δ, is defined as the angle between the direction of the incident beam and the direction of the beam transmitted by the prism.

(a) When the incident beam strikes the prism at an angle such that the angle of deviation is minimized, this minimum angle of deviation, δ_m, gives a simple relation for the index of refraction of the prism. This occurs when the angle of incidence for the 1st interface ($\overline{AB}$) is equal to the angle of refraction for the 2nd interface ($\overline{AC}$). Find the refractive index of refraction, n, of the prism in terms of the angles α and δ_m for this situation. [25 points]

(b) Find the angle of deviation, δ, for the general case (shown above) when it is not minimized. Find this angle in terms of the index of refraction, n, of the prism, the incident angle of the 1st interface ($\overline{AB}$), and α. [15 points]

Problem 3: (40 points)

(a) Derive an approximate formula for the strength of the magnetic field at the hydrogen nucleus produced by an electron in the first Bohr orbit (using the Bohr model of the atom). [20 points]

(b) Give a *rough order of magnitude estimate* of the strength of the magnetic field (in Teslas). Below are useful physical constants that you can use: [10 points]

permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ N/A^2
permittivity of free space: $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2
charge of the electron: $e = 1.60 \times 10^{-19}$ C
mass of the electron: $m = 9.11 \times 10^{-31}$ kg
Planck's constant: $h = 6.63 \times 10^{-34}$ Jsec

(c) If the nuclear Bohr mageton is $\mu_N = 5.0 \times 10^{-27}$ J/Tesla give an *order of magnitude estimate* of the hyperfine splitting (in eV) of spectral lines. [10 points]

Problem 4: (40 points)

Solve for the threshold energy for the following examples of the production of an electron-positron pair.

(a) Show the Feynman diagram for the process [5 points]

$$\gamma + Pb \rightarrow e^{+} + e^{-} + Pb$$

(b) What is the threshold energy for the photon (expressed in terms of electron masses) for the process in Part (a)? [5 points]

(c) What is the threshold energy for the photon (expressed in terms of electron masses) for the process

$$\gamma + e \rightarrow e^{+} + e^{-} + e$$

where e is a free electron? [15 points]

(d) Why are the threshold energies in Parts (b) and (c) different? [5 points]

(e) In intergalactic space, protons can interact with the photons in the cosmic microwave background via

$$\gamma + p \rightarrow e^{+} + e^{-} + p$$

Outline the steps (but don't calculate) needed to obtain the proton threshold energy for this process. Assume the CMB photons all have the same energy $E_{photon} = kT$ ($T = 3$ degrees Kelvin). [10 points]

Problem 5: (40 points)

Measurements of the entropy of a certain paramagnetic salt, as a function of the temperature T (in degrees Kelvin) and of the magnetic field H (in gauss), have led to the following table (in units of R per mole):

T \ H	0	2500	5000	7500	10,000
0.08	0.087	0.022	0.005	0	0
0.16	0.282	0.130	0.087	0.040	0
0.24	0.70	0.298	0.156	0.072	0.003
0.32	1.13	0.50	0.261	0.109	0.022
0.4	1.23	0.75	0.41	0.153	0.033
0.48	1.30	0.93	0.56	0.217	0.043
0.56	1.34	1.05	0.68	0.301	0.068
0.64	1.37	1.13	0.76	0.41	0.109
0.72	1.38	1.17	0.85	0.58	0.190
0.80	1.39	1.19	0.93	0.65	0.281

It is proposed to use this salt to produce very low temperatures by adiabatic demagnetization. The sample can be pre-cooled to 0.8 Kelvin by pumping on liquid He. The biggest available field is 10,000 gauss. What is the lowest temperature which can be reached? Each step in your reasoning should be explained very carefully (this is an *essay question*: numerical computations are not necessary)

Ph. D. Qualifying Exam Sept. 2012

Modern and Statistical Physics

Do 3 out of 5 problems

Problem 1: (40 points)

1. ***Classical thermodynamics, Gay-Lussac-Joule experiment***.
   (a) For any reversible process, calculate the dependence of the specific internal energy on volume, $\left(\frac{\partial u}{\partial v}\right)_T$, from the First law of Thermodynamics. Here, the internal energy $u = u(v, T)$. Here u and v are defined as $u = U/n$ and $v = V/n$, where U, V, and n are the internal energy, volume, and a number of kilomoles. (hint: utilize the condition of an exact differential.) [25 points]

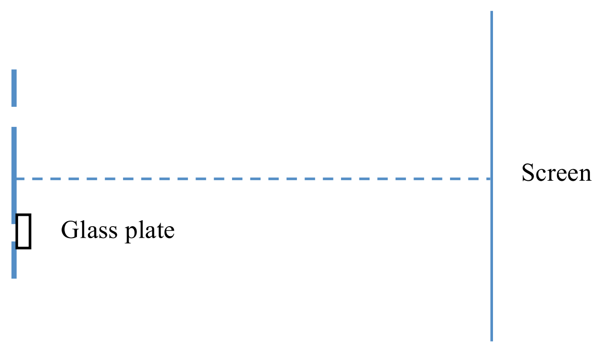
   (b) For an ideal gas, find the dependence of u on v by calculating $\left(\frac{\partial u}{\partial v}\right)_T$. [15 points]

Problem 2: (40 points)

In a two-slit Young interference, the aperture-to-screen distance is L and the wavelength is λ_{air}.

(a) What slit separation, a, is required to produce a fringe spacing of Δy at the screen? [15 points]

(b) Assume a thin glass plate of thickness d and index of refraction n is placed over one of the slits (see figure below). The glass plate causes the entire fringe pattern to laterally shift up or down on the screen. Calculate the lateral fringe displacement. [25 points]



Problem 3: (40 points)

A π *meson* of mass m_π decays at rest into a *muon* (mass m_μ) and a *neutrino* of negligible mass. Express your answers below in terms of the masses m_π and m_μ.

(a) What is the momentum of the *muon*? [15 points]

(b) What is the kinetic energy K_μ of the *muon*? [15 points]

(c) The *muon* decays, too. Let τ_0 be the mean lifetime of the *muon* in its rest frame. What is the mean distance traveled by the *muon* in the rest frame of the π *meson*? [10 points]

Problem 4: (40 points)

A beam of either positively or negatively charged *muons* is stopped in a piece of *Pb*. Answer the following questions separately for both a positive *muon* and a negative *muon*. Hint: consider atom formation and the energy levels of a hydrogen-like atom.

(a) Where is the most likely place for the *muon* to be after it stops in the *Pb* for μ^- and for μ^+? [10 points]

(b) What type of particles are emitted after the *muon* stops for μ^- and for μ^+? [10 points]

(c) What are the energy ranges of the particles emitted in item (b)? Give the answer in terms of eV or MeV for μ^- and for μ^+. [10 points]

(d) What is the lifetime of the *muon* in its own frame compared to it decaying in vacuum? Do not calculate a number but state if it is the same, a shorter, or a longer time and why for μ^- and for μ^+. [10 points]

masses:	*muon* 105 MeV	*electron* 0.5 MeV	*pion* 135 and 140 MeV
	proton 938 MeV	*neutron* 940 MeV	*photon* and *neutrino* 0

Problem 5: (40 points)

Classical Thermodynamics of a Two-state System: A (very small) discrete system has only two states 1 and 2 with energies $E_1 = -\varepsilon_0$ and $E_2 = \varepsilon_0$, respectively. This could, for instance, be a spin $1/2$ particle in an external magnetic field. Since this system contains only one particle, the different thermodynamic ensembles (of the systems described below) do *not* provide equivalent descriptions of the physics. We want to explore this difference for this simplest possible system.

(a) If the system is isolated from the environment which are the possible values for the internal energy of the system? [5 points]

(b) In the following we assume that the system is not isolated any more but instead interacting with a heat bath of temperature T. Using the *canonical distribution* of classical thermodynamics, what are the probabilities p_i to find the system in each of the two states in this case? [10 points]

(c) Find the internal energy as a function of the temperature of the heat bath (express it in terms of a familiar hyperbolic function). [15 points]

(d) Using the limiting values of the expression in (c), what are the possible values for the internal energy of the system when coupled to the heat bath? [10 points]

Ph. D. Qualifying Exam   Feb. 2012

Modern and Statistical Physics

Do <u>3</u> out of <u>5</u> problems

Problem 1: (40 points)

1. ***Phase diagram, the critical point***: The equation of state of van der Waals gas is $\left(P-\frac{a}{v^2}\right)(v-b)=RT$, where a and b are characteristic constants for a given gas, and P, v, T and R are pressure, specific volume, temperature and gas constant, respectively. When we regard the above van der Waals equation as $P = P(v, T)$

   (a) what are the three conditions at the critical point in the critical isotherm ($T = T_C$) on a P-v diagram? [15 points]

   (b) Express the critical specific volume, v_C, the critical temperature, T_C, and the critical pressure, P_C, at the critical point, in terms of the constants a and b. [25 points]

Problem 2: (40 points)

A metal ring is dipped into a soapy solution (index of refraction n_s) and held in a vertical plane so that a wedge-shaped film formed under the influence of gravity. At near-normal illumination with blue-green light (wavelength λ_{bg}) from an argon laser, one can see $\mathcal{N}$ fringes per cm. Determine the wedge angle of the soap film. (*Note:* assume that the wedge angle is very small).

Problem 3: (40 points)

2. A *pi-mu atom* consists of a *pion* and a *muon* bound in a Hydrogen-like atom.

   (a) What are the energy levels for such an atom compared to those for Hydrogen expressed in terms of the *electron*, *pion*, and *muon* masses? [20 points]

   (b) The *pi-mu atoms* are produced in K_L decays

   $$K_L \rightarrow \textit{pi-mu atom} + \textit{neutrino}.$$

   If the K_L has $\beta = 0.8$, what are the minimum and maximum energies of the *pi-mu atom* in the moving frame of the K_L? (express these energies in terms of the particle masses) [20 points]

Problem 4: (40 points)

Briefly explain or describe **4** of the following **6** phenomena (in no more than 200 words for each phenomena): [10 points each]

(a) Electromagnetic structures of the neutron and proton

(b) The laser

(c) C, P, T, and CP symmetry (and any possible *well known* violations)

(d) The transistor

(e) The J/ψ particle

(f) Superconductivity

Problem 5: (40 points)

A system consisting of N (a very large number) identical weakly interacting particles is in equilibrium with a heat bath. The total number of individual states available to each particle is $2N$. Of these, N states are degenerate with energy 0, and N states are degenerate with energy ε.

It is found by observation that the total energy of the system is $\frac{1}{3}N\varepsilon$. Find the temperature of the heat bath under three different assumptions:

(a) That the particles are bosons. [10 points]

(b) That the particles are fermions. [10 points]

(c) That the particles obey the (unphysical) Boltzmann distribution. [10 points]

(d) You should find from Parts (a), (b), and (c) that

$$T(boson) > T(Boltzmann) > T(fermion)$$

Explain why this is so. [10 points]

Modern and Statistical Physics Fall 2011

9/24/2011

Do 3 out of 5 problems

1. ***Classical Thermodynamics*** Ideal Gas [40 points]

   Assume that the earth's atmosphere is an isothermal ideal gas in a gravitational field. Consider a thin layer of the ideal gas at height z and of thickness dz. There is a difference of pressure across this thin layer, dP, which must just balance the gravitational force on the mass. If the pressure at $z = 0$ is P_0 , determine the pressure as a function of height z.

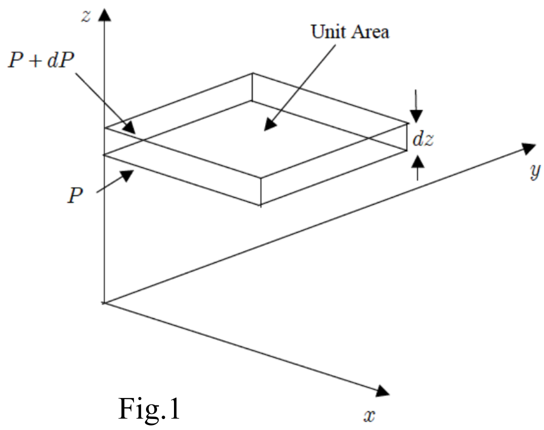


Fig.1

2. ***The Kinetic theory of Gas*** [40 points] The ditribution of particle speeds of a certain hypothetical gas is given by

$$N(v)dv = Ave^{-v/v_0}dv,$$

wherer A and v_0 are constants.

   a. Determine A so that $f(v) \equiv N(v)/N$ is a true probability density function; i.e.,
      $\int_0^\infty f(v)dv = 1$. [10 points]
   b. Find the mean speed, $\bar{v}$, and the root mean square speed, v_{rms}, in terms of v_0. [10 points]
   c. Find the most probable speed v_m. [10 points]
   d. What is the standard deviation of the speeds from the mean, σ, in this case? [10 points]

3. ***Solid state physics, thermodynamics, statistical mechanics*** [40 points] There is a semiconductor with two bands: The lower band is described by $E(\vec{\mathbf{k}})$, the upper band by $E_0 - E(\vec{\mathbf{k}})$. The two bands are separated by an energy gap of 2Δ. (Hint: Use electron-hole symmetry. This is also evident from the way the upper band is defined.)

   a. Calculate the chemical potential as a function of the temperature. [20 points]

b. Examine whether the system can be treated using classical statics at low temperatures. [20 points]

4. ***High energy physics*** [40 points]
   a. A particle of mass m is produced with energy E and decays after traveling a distance l. How long did the particle live in its own rest frame? [8 points]

   b. Use the result of part (a) to determine the spatial separation between the production and decay vertices (a.k.a. "decay length") of a B^0 meson carrying an energy of 65 GeV, if it lived 1.5 ps in its own rest frame. The mass of the B^0 meson is 5.3 GeV. [8 points]

   c. Determine the maximum energy that can be carried off by any one of the decay particles, when a particle of mass m_0 at rest decays into three particles with masses m_1, m_2, and m_3. [24 points]

5. ***Diffraction grating*** **(*Multiple slit diffraction*)** [40 points] Consider diffraction of an array of multiple slits (N equal slits of width d and common spacing of a, numbered from 0 to N - 1) , or diffraction grating, illuminated by monochromatic plane wave normal to the array (Fig 2). We assume that the length of the slit is long enough, and the obervation point P is far enough from the slit.
   a. Obtain the expression for the intensity of the multiple slit diffraction pattern at a point P. [20 points]
   b. Now using the above diffraction grating, we perform an experiment to determine the wavelength of light. When the wavelength of the light λ changes to $\lambda + d\lambda$, the diffraction pattern shifts. If this shifts is smaller than the width of a bright band of the diffraction pattern, the wavelength λ *and* $\lambda + d\lambda$ cannot be distinguished one another. Determine the accuracy of the measurement of the wavelength by this experimental method. [20 points]

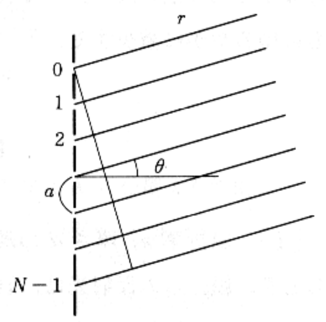


Fig.2

Modern and Statistical Physics Spring 2011

2/26/2011

Do 3 out of 5 problems

1. ***High energy physics*** [40 points]
   Consider the reaction p + gamma --> neutron + pi$^+$.

   a. Assume that the gammas are the cosmic blackbody radiation with average $kT = 3\times10^{-4}$ eV. What is the threshold energy that the proton must have for the reaction to occur? [20 points]

      $m_p \sim m_n \sim 1000$ MeV, $m_\pi = 140$ MeV, $m_\mu = 105$ MeV, $m_\nu = 0$ eV.

   b. The pion then decays to $\mu + \nu$. In the pion rest frame, what is the neutrino energy? What is the neutrino energy in the "lab" frame? This is the source of the highest energy neutrinos in the universe. Give answers in eV units. [20 points]

2. ***Relativity*** C-O white dwarf [40 points]

   a. A C-O white dwarf has radius R and mass M. Assuming the electrons are degenerate, what is the average energy of the electrons in terms of M, R and fundamental constants assuming the electrons are non-relativistic? [15 points]

   b. Repeat assuming the electrons are relativistic. [15 points]

   c. Now assume that the radius shrinks by a factor of 2 so R' = R/2 while the mass remains the same. What is the relative change in the gravitational and electron energies (assuming both relativistic and non-relativistic) between R and R'? [5 points]

   d. Is the star more stable (less likely to collapse) if the electrons are relativistic or non-relativistic? Why? [5 points]

3. ***2D harmonic oscillator*** [40 points]

   Give a 2D harmonic oscillator with Hamiltonian, $H = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega^2\left(x^2+y^2\right) + kmxy$.

   a. For k=0, what are the energies of the ground state and first and second excited states? What are the degeneracies of each state? [20 points]

   b. For k>0, using first order perturbation theory, what are the energy shifts of the ground state and the first excited states? [20 points]

4 ***Classical mechanics*** Van der Waals Gas [40 points]

The Van der Waals equation of state for one mole of a non-ideal gas reads

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT\ .$$

[Note: part (d) of this problem can be done independently of part (a) to (c).]

a. Sketch four isotherms of the Van derWaals gas in the p-V plane (V along the horizontal axis, p along the vertical axis). Identify the critical point. [10 points]

b. Evaluate the dimensionless ratio pV/RT at the critical point. [10 points]

c. In a portion of the p-V plane *below* the critical point, the liquid and gas phases can coexist. In this region the isotherms given by the Van der Waals equation are unphysical and must be modified. The physically correct isotherms in this region are lines of constant pressure, $p_0(T)$. Maxwell proposed that $p_0(T)$ should be chosen so that the area under the modified isotherm should equal the area under the original Van der Waals isotherm. <u>Draw a modified isotherm and explain the idea behind Maxwell's construction. [10 points]</u>

d. Show that the heat capacity at constant volume of a Van der Waals gas is a function of temperature alone (i.e., independent of V). [10 points]

5. ***Statistical mechanics*** [40 points]

There is a system of two identical particles, which may occupy any of the three energy levels $\varepsilon_0=0,\ \ \varepsilon_1=\varepsilon,\ \ \varepsilon_2=3\varepsilon.$

The system is in thermal equilibrium at Temperature T, which means that the system is a canonical ensemble. For *each* of the following cases, determine the *partition function* and the *energy* and carefully enumerate the *configurations*.

a. The particles are Fermions. [10 points]

b. The particles are Bosons. [10 points]

c. The particles obey Boltzmann statistics and now they are distinguishable. [10 points]

d. Discuss the conditions under which Fermions or Bosons may be treated as Boltzmann particles. [10 points]