Modern and Statistical Physics. September/October 2010

Do 3 of 5 problems

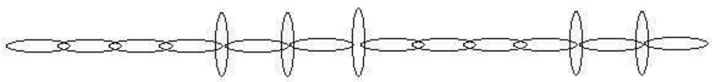
1. A top quark decays to a bottom quark and a W boson. The W boson then decays to a positron and a neutrino. Give your answers below in terms of the masses m_t , m_b , m_W of the top quark, the bottom quark, and the W boson respectively. You may treat the positron and the neutrino as massless. You are encouraged to set c=1.

(a) (14 points) What is the momentum of the bottom quark, in the rest frame of the top quark?

(b) (14 points) What is the maximum possible momentum of the positron, in the rest frame of the top quark?

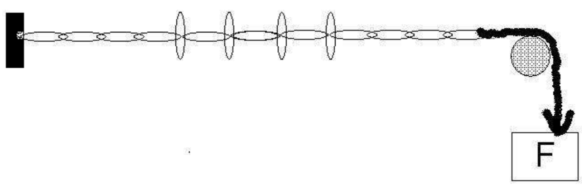
(c) (12 points) Suppose the top quark has speed 0.5c in the lab frame, before it decays. What is the maximum possible momentum of the positron in the lab frame?

2. A simple model of a rubber band is a one-dimensional (horizontal) chain consisting of N (N >> 1) linked segments, as shown schematically in the diagram. Each segment has two possible states: horizontal (H) with length a, or vertical (V), with length zero; with the figure indicating a state of HHHH-VHVH-VHHH-HVHV-H. The segments are linked such that they cannot come apart. The chain is in thermal contact with a reservoir at temperature T.



(a) (10 points) If there is no energy difference between the two states, what is the average length of the chain?

(b) (25 points) Fix the chain at one end and hang weight from the other end, supplying a force F as shown. Determine the average length of the chain at any temperature T and show that it is equivalent to



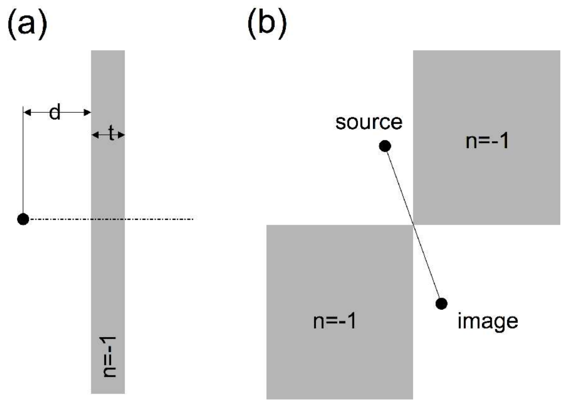
$$\langle L(T)\rangle = \frac{aNe^{Fa/2kT}}{e^{-Fa/2kT}+e^{Fa/2kT}}$$

.

(c) (5 points) In which temperature limit is the extension proportional to F (Hooke's law)? Calculate the constant of proportionality (that is the "k" from Hooke's law).

3. Artificial materials (or metamaterials) can be engineered to provide a negative index of refraction *n<0*. In this problem we explore the amazing properties of two arrangements of metamaterials described in the Figure below.

1. (6 points) Consider an incoming optical ray at the interface from vacuum to a metamaterial. Write down the expression for the refracted angle and draw the incoming and refracted ray for n=-1. Indicate the difference(s) with the result obtained for conventional materials.
2. (17 points) We now consider a slab of metamaterial with thickness t as drawn in the Figure (a) with n=-1.
    a. (6 points) Let a point-like source object be at a distance d=t upstream of the slab, show that there is an image and give the location of this image.
    b. (6 points) Same question if the object is at a distance d=1/2t.
    c. (5 points) Comment on the properties of a simple slab with negative index of refraction compared to a standard (n>0) slab.
3. (17 points) Consider the configuration shown in Figure (b).
    a. (6 points) Via geometrical construction, show that the source is imaged on the lower-right quadrant.
    b. (6 points) Show that rays emitted by the source are also imaged on the source itself.
    c. (5 points) Discuss possible applications of such imaging/recirculating configurations.

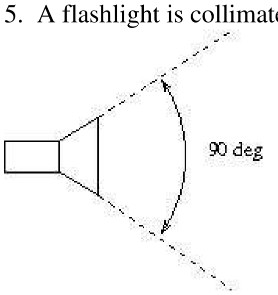


4. In the Big Bang theory of the universe, the radiation energy initially confined in a small region adiabatically expands in a spherically symmetric manner. The radiation cools down as it expands.

a. Derive a relation between the temperature *T* and the radius *R* of the spherical volume of radiation, based purely on thermodynamic considerations. Here the radiation pressure is expressed as $p = U/3V$, and the black body radiation energy density is $u = U/V = aT^4$. [20 points]

b. Find the total entropy of a phonon gas as a function of its temperature T, volume V and the constants $k, \hbar, c$. [20 points]

5. A flashlight is collimated so that its beam goes off in a 90 degree cone in its own rest frame.



a) (20 points) Calculate the opening angle of the flashlight cone when the flashlight is moving forward at velocity v.
b) (20 points) Assuming that $\gamma = 1/\sqrt{1-v^2/c^2} >> 1$ show that the opening angle of the cone is about $1/\gamma$

Modern and Statistical Physics. February 2010
Do 3 of 5 problems

1. Consider the reaction nubar + p → e+ + n.

a. What is the threshold energy for the antineutrino if the reaction occurs off Hydrogen? Give answer in MeV.

b. If the reaction occurs off Carbon 12 (with proton density of 0.1/F^3), the reaction will be suppressed at low antineutrino energies. Explain why. (hint, note the two lowest spectroscopic states in nuclei.)

c. Assuming a Fermi gas model, what is the Fermi energy for protons in C 12? What is the average proton kinetic energy? Give in MeV.

d. Up to what antineutrino energy will suppression in C 12 still be a factor?

mass neutron = 939.6 MeV/c^2 hbar*c = 197 MeV*F hc = 1240 MeV*F
mass proton = 938.3 MeV/c^2
mass electron = 0.5 MeV/c^2

2. For this problem, do not calculate any exponentials or roots but put in all the other numbers for the final answer

a. For a H atom, what is the ratio of the probability to be in the n=2 level compared to the n=1 level at T = 3000 degrees K? E(1S) = -13.6 eV and kT= .025 eV at T = 300 degrees K

b. For a metal with a Fermi energy equal to 4 eV and at T = 300 degrees K determine the following assuming a Fermi gas model:

i. What is the density for conduction electrons? hc = 1240 eV*nm mass electron = 0.5 MeV/c^2
ii. What is the ratio of the density of states between 4.4 eV and 4.0 eV?
iii. What is the ratio of the probability to have the energy be 4.4 eV compared to 4.0 eV?

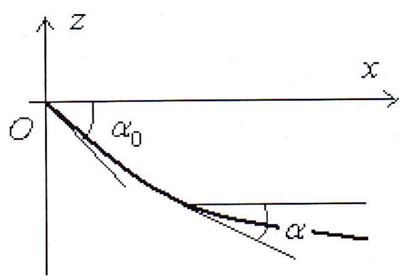
c. For a photon in a Bosonic gas at T = 300 degrees K:

i. What is the ratio of the density of states between 4.4 eV and 4.0 eV?
ii. What is the ratio of the probability to have the energy be 4.4 eV compared to 4.0 eV?

3. A rocketship is traveling towards the Earth at 0.6c. It fires a projectile of mass M with a velocity of 0.8c in the rocketship's frame. What is the energy of the projectile in the Earth's frame? If the rocket ship is traveling perpendicular to a line from the ship to the Earth and again fires a projectile at 0.8c in the ship's frame and at 90 degrees from the direction of the ship's motion (in the rocketship's frame), what is the energy of the projectile in the Earth's frame? What is the tangent of the angle of the projectile relative to the ship's motion (don't calculate but give as the ratio of two components)?

4. **Optical mirage:**

We explore the propagation of an optical ray in the (Oxz) plane; see Figure. We assume the refractive index n obeys the relation $z = an^2 + b$ where a and b are constants.



a. **(10 points)** Show that at a given height z we have $n(z)\cos\alpha = n_0 \cos\alpha_0 \equiv A$ where A is a constant.

b. **(10 points)** Deduce the differential equation for the ray trajectory $\left(\frac{dz}{dx}\right)^2 = \left(\frac{n}{A}\right)^2 - 1$ and show that the resulting ray trajectory is a parabola.

c. **(20 points)** We now wish that the model discussed in (1) and (2) can be used to explain the formation of optical mirages. We consider the (*Oxz*) plane to be filled with an ideal gas. We assume the gas to be in mechanical equilibrium (so that the change of pressure dP depends on the height variation dz as $dP = -\rho g dz$ where g is the gravitational constant and ρ the gas density). We consider a temperature gradient along z of the form $T(z) = T_0(1 - kz)$ where k is a constant and further assume the refractive index and the gas density satisfy the relation $(n^2 - 1)/\rho = const$.

(15 points) Show that the refractive index is related to z via $z = an^2 + b$ and explicitly determine the constants a and b.

(5 points) Qualitatively describe the formation of a "warm" mirage due to a hot floor.

5. Partition function of an Einstein solid is

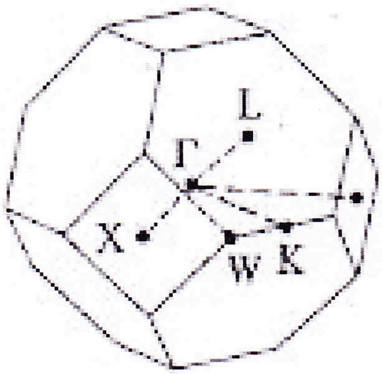
$$Z = \frac{e^{-\theta_E/2T}}{1 - e^{-\theta_E/T}},$$

where θ_E is the Einstein temperature. Treat the crystalline lattice as an assembly of $3N$ distinguishable oscillators.

a. Calculate the Helmholtz function F. [10 points]
b. Calculate the Entropy S. [10 points]
c. Show that the entropy approaches zero as the temperature goes to absolute zero. [10 points]
d. Find the entropy at high temperatures. [10 points]

Modern and Statistical Physics. September 2009
Do 3 of 5 problems

1. The face-centered cubic (fcc) is the most dense and the simple cubic (sc) is the least dense of the three cubic Bravais lattices. Suppose identical solid sphere are distributed through space in such a way that their centers lie on the points of each of these three structures, and spheres on neighboring points just touch, without overlapping. (Such an arrangement of sphere is called a close-packing arrangement)
a). Assuming that the spheres have unit density, show that the density of a set of close-packed spheres on each of the three structures (the "packing fraction") is $\sqrt{2}\pi/6 \approx 0.74$ for fcc, $\sqrt{3}\pi/8 \approx 0.68$ for body-centered cubic (bcc), and $\pi/6 \approx 0.52$ for sc.
b). Show that for a fcc Bravais lattice the free electron Fermi sphere for valence 3 extends beyond point W of the first Brillouin zone (see figure), so that the first Brillouin zone is completely filled [Hint: prove $k_F/\Gamma W = (1296/125\pi^2)^{1/6} = 1.008$]



2. Consider the reaction $\gamma + e \rightarrow \pi^+ + \pi^- + e$ with the electron initially free and at rest (with this frame designated as the lab frame).

a) What is the minimum photon energy for this reaction to proceed?

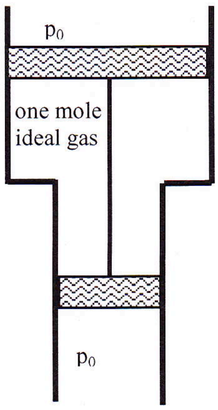
b) Assuming that the photon energy is the minimum determined in a), what is the velocity of either of the pions after the reaction?

c) Assume the pion then decays to a muon and neutrino $\pi \rightarrow \mu + \nu$. In the lab frame, what is the maximum energy of the muon produced in the pion decay assuming the photon energy determined in a).

Give your answers in terms of the pion, muon, and electron masses while setting the photon and neutrino masses to 0.

3. A 55 year old man can focus objects clearly from 100 cm to 300 cm. Representing the eye as a simple lens 2 cm from the retina,
a) what is the focal length of the lens at the far point (focused at 300 cm)?
b) what is the focal length of the lens at the ner point (focused at 100 cm)?
c) what strength lens (focal length) must he wear in the lower part of his bifocal eyeglasses to focus at 25 cm?

4. A smooth vertical tube having two different sections is open from both ends and equipped with two pistons of different areas. Each piston slides within its respective tube section. One mole of ideal gas is enclosed between the pistons. The pistons are connected by a non-stretchable rod. The outside air pressure is 1 atm. The total mass of the pistons is M. The cross sectional area of the larger upper piston A1, and the lower piston A2 are related by $A_1 = A_2 + \Delta A$. Find the rise in the temperature of the gas between the pistons required to lift the piston assembly by a distance L?



5. Assume that the neutron density in a neutron star is $0.1/\text{fm}^3$ (that is 0.1 neutron per cubic Fermi). Assuming T=0, ignoring any gravitational forces, and using a Fermi gas model with uniform density determine

a) the average energy of the neutrons
b) the average energy of electrons if the electron density is 1% of the neutron density
c) show two reactions, one which can convert a neutron to a proton and one which can convert a proton to a neutron
d) determine the neutron to proton ratio. Hint, consider the Fermi energies of the neutrons, protons and electrons at equilibrium

Give answers in a) and b) in MeV using hc = 1240 MeVfm, the mass of the neutron = 1000 MeV/c^2, and the mass of the electron = 0.5 MeV/c^2. You will need to decide if the particles are relativistic or non-relativistic. The answer for d) can be given in terms of the particle masses.

February 2009. Stat/Modern/Thermo PhD. Qualifier. Pick 3 questions out of 5.

1.
   a. (20) Show using conservation of energy and momentum that it is not possible for a free electron moving through vacuum to emit a photon.
   b. (20) Now consider the related problem of an electron moving through superfluid helium. Show that in this case the electron *can* emit a phonon as long as it moves with a velocity exceeding a critical velocity v_c and find v_c. The excitation spectrum of the phonons in superfluid helium is given by $E = up$ where *u* is the velocity of sound and *p* is the momentum of the phonon. You may do this part in the non-relativistic limit.

2.
   a. (10) A C-O white dwarf has a radius = *R*, mass = *M* and is assumed to have constant density. Assuming the electrons are degenerate and non-relativistic find the average energy of the electrons in terms of *M*, *R* and fundamental constants.
   b. (10) Repeat part a) assuming the electrons are relativistic.
   c. (10) Assume the radius shrinks by a factor of 2 so $R' = R/2$ while the mass remains the same. Find the relative change in the gravitational and electron energies (assuming both relativistic and non-relativistic) between *R* and *R'*.
   d. (10) Discuss whether the star is more stable (less likely to collapse) if the electrons are relativistic or if they are non-relativistic.

3. When a large number of atoms come together to produce a solid each atomic level broadens into a band. Draw a picture of simplified band structure, define the valence and conduction bands, and describe the temperature-dependent behavior of conductivity for:
   a. (10) Metals (define the term Fermi energy)
   b. (10) Insulators (define the term band gap)
   c. (10) Intrinsic semiconductors
   d. (10) The n-type and p-type semiconductors

4. One mole of a monatomic ideal gas initially at temperature T_0 expands from volume V_0 to $2V_0$. Calculate the work of expansion and the heat absorbed by the gas for the case of expansion at:
   a. (20) Constant temperature
   b. (20) Constant pressure

5.
   a. (20) Consider a large number of N localized particles in an external magnetic field H (directed along the z direction). Each particle has a spin s = 1/2. Find the number of states accessible to the system as a function of M_s, the z component of the total spin of the system. Determine the value of M_s for which the number of states is maximum.

b. (20) Define the absolute zero of the thermodynamic temperature. Explain the meaning of negative absolute temperature, and give a concrete example to show how the negative absolute temperature can be reached.

September 2008. Stat/Modern/Thermo PhD. Qualifier. Pick 3 questions out of 5.

1. The electronic structure of the atom is described by four quantum numbers.
    a. Define these quantum numbers and describe the values they can acquire.
    b. Write down the electronic configurations for elements with atomic numbers: 8 (Oxygen), 19 (Potassium), 29 (Copper).
    c. Define the term atomic valence and find the valence for the elements O, K, and Cu.
    d. List four main types of bonding found in materials and give examples of the elements and/or compounds for each type of bonding.
2. Consider the reaction $\bar{\nu} + p \rightarrow e^{+} + n$.
    a. Find the threshold energy for the $\bar{\nu}$ if the reaction occurs off Hydrogen.
    b. Explain why the reaction will be suppressed due to Pauli Exclusion if it occurs off ^{12}C (with proton density of 0.1/F^3).
    c. Assuming a Fermi gas model, find the approximate suppression as a function of neutrino energy. ($m_n = 940$MeV/c^2 , $\hbar c = 197$ MEV F, $hc = 1240$ MeV F)

3. A parallel beam of electrons is directed through a narrow slit of width a and a screen is placed at a distance d from the slit. The first diffraction minimum is observed at a distance y from the central maximum.
    a. Find the velocity of the electrons in terms of a, d, y, and m_e assuming the electrons are non-relativistic.
    b. Find the velocity assuming the electrons are relativistic.
    c. Estimate the spread of the electron beam through the slit using the Heisenberg uncertainty principle and show that this is consistent with the spread estimated based on the width of the central maximum.

4. We have an ensemble of N spins that have a Zeeman splitting described by the Hamiltonian

$$H = -\sum_{i=1}^{N} \mu_B B \sigma. \qquad (1)$$

Here B is the magnetic field, μ_B is the Bohr magneton, and $\sigma = \pm 1$ is the direction of the spin.

a. Show that the partition function is given by

$$Z = (2\cosh\frac{\mu_B B}{k_B T})^N. \quad (2)$$

b. Show that the energy in terms of the partition function is given by

$$E = k_B T^2 \frac{\partial \ln Z}{\partial T} \qquad (3)$$

and the magnetization by

$$M = k_B T \frac{\partial \ln Z}{\partial B} \qquad (4)$$

c. Determine E and M for the partition function in Eqn. (2) and sketch them as a function of temperature.

5. Chemical potential of an ideal gas.

a. Starting with $TdS = dU + PdV$ show that the chemical potential of an ideal gas can be written in terms of the temperature T and the volume V as :

$$\mu = c_P T - c_V T \ln(T) - RT \ln(V) - S_0 T + \text{constant}.$$

Here, c_P is the specific heat constant pressure, c_v is the specific heat at constant volume and S_0 is a constant.

b. Starting from $TdS = dH - VdP$ find a similar expression for the chemical potential but now as a function of T and P.

c. Show that the chemical potential at the fixed temperature T varies with pressure:

$$\mu = \mu_0 + RT \ln\left(\frac{P}{P_0}\right).$$

Here, μ_0 is the value of μ at the reference point (P_0, T)

Stat/Modern/Thermo PhD. candidacy examination. March 2008

Pick 3 questions out of 5.

1) Consider a string of length L, mass density μ and string tension τ. The string is maintained at finite temperature T.
    a. Find the average amplitude of a mode of the string of wavelength $\lambda = 2L/n$ in the limit of $k_B T >> hf$, with f the frequency of the mode.
    b. Explain why it is necessary to state that $k_B T >> hf$.

2) Assume that the crystal lattice structure of solid comprising N atoms can be treated as an assembly of $3N$ distinguishable one-dimensional oscillators (Einstein solid).
    a. What is the partition function Z? Use the Einstein temperature θ_E ($\equiv h\nu/k$, where ν is natural frequency).
    b. Calculate the Helmholtz function F.
    c. Calculate the entropy S.
    d. Show that the entropy approaches zero as the temperature goes to absolute zero.

3) The variation of the internal energy U as a function of entropy S is predicted by classical equilibrium thermodynamics to have a functional form of $U - U_0 = \alpha(S - S_0)^n$ where α is a constant for a fixed value of volume.
    a. Show that the temperature as a function of entropy in the case of constant volume is given by $T = \alpha n(S - S_0)^{n-1}$.
    b. Show that $C_v = \dfrac{S - S_0}{n-1} = \dfrac{1}{n-1}\left(\dfrac{T}{\alpha n}\right)^{1/n-1}$.

    At some point it may be useful to derive the relationship $\left(\dfrac{\partial T}{\partial S}\right)_V = \left(\dfrac{\partial^2 U}{\partial S^2}\right)_V$.

4) Silicon has Z=14.
    a. Relative to a hydrogen atom, what are the energy and radius of an electron in the 1s shell?
    b. In the 3p shell?
    c. Assume that there are 2 electrons in the 3p shell. What are the allowed spectroscopic states for this 2-electron system? (Give S, L, J).
    d. What is the energy ordering and why?

5) A charged kaon (at rest) decays by $K^+ \rightarrow \pi^+ + \pi^0$ and then $\pi^0 \rightarrow \gamma + \gamma$.
    a. In terms of the masses of the particles what is the π^0's energy?
    b. What is the maximum photon energy?

Modern/Optics/Thermodynamics September 2007

Pick 4 out of 6 problems.

1 In inertial frame O a rod of length l is oriented along the x-axis and moving with velocity u in the positive y direction. This rod is then viewed from an inertial reference frame O' moving with velocity v in the positive x direction.

a) What is the length of the rod in O'? [10 points]
b) What angle does the rod make with respect to the x' axis? [30 points]

2 A cubic box (with sides of length L) holds diatomic H_2 gas at temperature T. Each H_2 molecule consists of two hydrogen atoms with mass of m each separated by distance d. Assume that the gas behaves like an ideal gas. Ignore the vibrational degree of freedom.

a. What is the average velocity of the molecules? [10 points]
b. What is the average velocity of rotation of the molecules around an axis which is the perpendicular bisector of the line joining the two atoms (assuming each atom as a point mass)? [10 points]
c. Derive the expressions expected for the molar heat capacities C_p and C_v for such a gas. [10 x 2 points]

3.

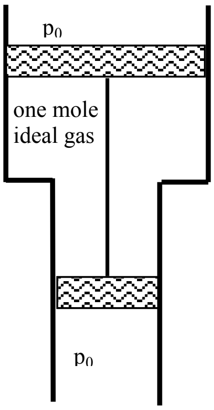
a. What is the threshold kinetic energy for the proton for $p^+ n \rightarrow p^+ p^+ \pi^-$ assuming the neutron is at rest? [20 points]
b. If now the neutron is in a Carbon nuclei of size 60 F^3 (with 6 protons and 6 neutrons), what is the Fermi energy of the neutron, and what is the threshold kinetic energy in this case? [20 points]

$m_n = m_p = 1000$ MeV, $m_\pi = 140$ MeV, $\hbar c = 200 MeVF$

4. A smooth <u>vertical</u> tube having two different sections is open from both ends and equipped with two pistons of different areas. Each piston slides within its respective tube section. One mole of ideal gas is enclosed between the pistons. The pistons are connected by a non-stretchable rod. The outside air pressure is 1 atm.

The total mass of the pistons is M. The cross sectional area of the larger upper piston A_1, and the lower piston A_2 are related by $A_1 = A_2 + \Delta A$.

How much (in Kelvin) must the inner gas (between the pistons) be heated in order to lift the piston assembly by L=5cm?



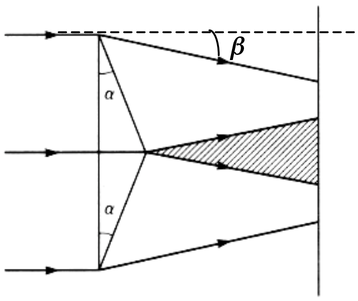
$M=5kg$
$p_0=1atm$
$A_1-A_2=\Delta A=10cm^2$

CONSTANTS:

$g=9.8\ m/s^2$
$R=8.3J/(^oK\cdot mole)$
$k_{boltz}=1.38x10^{-23}J/^oK$

5. Eight non-interacting neutrons are confined to a 3D square well of size D= 5 F (10^{-15} m) such that V = -50 MeV for
   $0 < x < D$,
   $0 < y < D$,
   $0 < z < D$
   and V = 0 everywhere else.
   a. How many energy levels are there in this well? [10 points]
   b. What is the degeneracy of each energy level? [10 points]
   c. What is the approximate Fermi energy for this system? [10 points]
   d. What is the relative probability to be in the lowest energy state to the fourth lowest energy state at kT= 10 MeV? Just write down the ratio (don't calculate the value). [10 points]
   m_n= 1000 MeV, $\hbar c = 200 MeVF$

6. ***Interference by a biprism.*** A plane wave (wavelength λ) enters perpendicular to the biprism (the prism angle α, refractive index *n*) as shown in the figure. The wave transmitted through both sides of the biprism is bent (refracted) and overlap at the viewing screen S (parallel to the biprism) where an interference pattern can be observed.
   a. Find the refracted angle, β. [20 points]
   b. Find the interval of adjacent interference lines. [20 points]



biprism

Statistical and Modern. Spring 2007. Pick 4 out of 6.

1) a) Starting with the first law of thermodynamics and the definition of c_p and c_v, show that

$$c_p - c_v = \left[p + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P .$$

Here c_p and c_v are the specific heat capacities per mole at constant pressure and volume respectively, and U and V are the energy and volume of one mole.

b) Use the above result plus the expression

$$p + \left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_V$$

to find $c_p - c_v$ for a van der Waals gas with equation of state

$$\left(p + \frac{a}{V^2} \right) (V - b) = RT .$$

Here a and b are constants.

c) Use this result to show that as $V \to \infty$ at constant p, you obtain the ideal gas result for $c_p - c_v$.

2) The rotational motion of a diatomic molecule is specified by two angular variables q and f and the corresponding canonical conjugate momenta, p_q, p_f. Assuming the form of the kinetic energy of the rotational motion to be

$$e_{rot} = \frac{1}{2I} p_q{}^2 + \frac{1}{2I \sin^2(q)} p_f{}^2$$

a) Derive the classical formula for the rotational partition function , $r(T)$,

$$r(T) = \frac{2IkT}{\hbar^2}$$

b) Calculate the Helmholtz free energy F_{rot}.

c) Calculate the corresponding entropy and specific heat.

The following may be helpful

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{p}{a}}$$

$$\int \frac{dx}{\sin^2(ax)} = -\cot(ax)/a$$

3) Assume that the neutron density in a neutron star is $0.1/\mathrm{fm}^3$ (that is 0.1 neutron per cubic Fermi). Assuming T=0 and ignoring any gravitational forces calculate the ratio of neutrons to protons to electrons.

Hint: determine their Fermi energy. The electron, neutron and protons masses are .511 MeV/c^2, 939.6 MeV/c^2 and 938.3 MeV/c^2. The constant $hc = 1240\text{MeV fm}$. You should be able to work out "by hand" an approximate value.

4) A $\pi-\mu$ atom consists of a pion and a muon bound in a Hydrogen-like atom.
a) What are the energy levels for such an atom compared to those for Hydrogen?

b) $\pi-\mu$ atoms are produced in K$_L$ decays ($K_L \to \pi-\mu+\nu$). If the K$_L$ has $\beta = 0.8$ what are the minimum and maximum energies of the $\pi-\mu$ atom expressed in terms of the K, π and μ masses with $m_\nu = 0$?

c) Approximately what fraction of K$_L$ decays will produce a $\pi-\mu$ atom (hint: use the Heisenberg uncertainty principle)?

5) a) You are familiar with the quarter-wave thin film coating that acts as a “reflection–reducer”. For the moment, let us look at a simpler thin film—the air gap between two pieces of glass such as you would find in a Newton’s rings experiment. Why do we get constructive interference in the reflected when the thickness is one-fourth of the wavelength of light or some odd multiple of a quarter wavelength? Why isn’t it constructive at one-half wavelength of the light? For assistance, I present two of the Fresnel equations (in two forms) for reflected light.

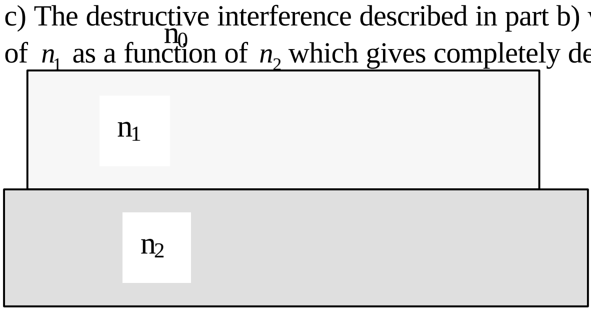
$$r_{\parallel} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}$$

$$r_{\perp} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

Where the parallel and perpendicular symbols refer to the plane of incidence, and *i*,*t* refer to incident and transmitted media, θ ’s are angles of incidence and transmission, and *n*’s are indices of refraction.

b) In light of the previous, to get destructive reflection in a thin film-i.e.-a quarter-wave film, such as the one illustrated below, what condition must prevail among the indices of refraction for the three media (n_0 may be taken as = 1.0 for air.)

c) The destructive interference described in part b) will generally not be complete. Find the value of n_1 as a function of n_2 which gives completely destructive interference at normal incidence.



6) In a big-bang theory of the universe, the radiation energy initially confined in a small region adiabatically expands in a spherically symmetric manner. Here the radiation (photon) pressure is expressed as $p = U/3V$, and the black body radiation energy density is $u = U/V = aT^4$ The radiation cools down as it expands.

a) Derive a relation between the temperature T and the radius R of the spherical volume of radiation, based purely on thermodynamic considerations.

b) For the above problem, show the total entropy of a photon gas is expressed as $S = \frac{4}{3}aT^3V$.