Modern Physics, Optics, Statistical Mechanics and Thermodynamics
September, 2006

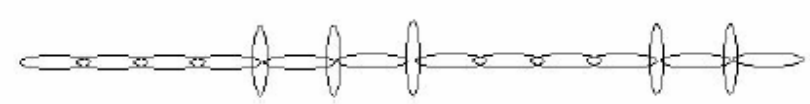
Pick 4 out of 6 problems

1) A flashlight, in its own rest frame, is directed at an angle θ of 45 degrees to the x-axis in the xy plane. What angle will the light beam appear to make to an observer moving towards the flashlight along the x-axis at velocity β?

a. Now consider a relativistic electron traveling in a circular orbit. Explain why the radiation from the electron will be confined to a narrow cone, and calculate the opening angle of the cone.

b.

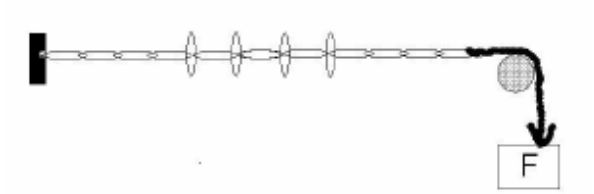
2) A simple model of a rubber band is a one-dimensional (horizontal) chain consisting of N ($N \gg 1$) linked segments, as shown schematically in the diagram.



Each segment has two possible states: horizontal with length a, or vertical, contributing nothing to the length. The segments are linked such that they cannot come apart. The chain is in thermal contact with a reservoir at temperature T.

a. If there is no energy difference between the two states, what is the average length of the chain?

b. Fix chain at one end and hang weight from the other end, supplying a force F as shown. Determine the average length of the chain at any temperature T. Find the length in the limits $T \to 0$ and $T \to \infty$.



c. In which temperature limit is the extension proportional to F (Hooke'law)? Calculate the constant of proportionality.

3) The complete formula for multislit diffraction pattern in Fraunhofer diffraction is given by the expression

$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2$$

where

I_0 is the flux density in the $\theta = 0$ direction for one slit,

$\beta = \frac{kb}{2}\sin\theta$, and b is the slit width,

$\alpha = \frac{ka}{2}\sin\theta$, and a is the slit separation, and

$k = \frac{2\pi}{\lambda}$ is the propagation number.

a. Show that $I(0) = N^2 I_0$, where N is the number of slits.

b. For $N \geq 3$ there are secondary maxima between the principal maxima. Deduce the rule for the number of minima between principal maxima, and, then the number of secondary maxima. Actually deduce this, don't just recite it if you happen to remember it!

c. The result in *a* indicates that the flux density at $\theta = 0$ varies as N^2. Explain how that can be true. After all, e.g., if you have three slits, then there is three times a much light. How can the flux density be nine times as great?

4) Consider a system of two types of charge carriers in the Drude model. The two carriers have different densities (n_1 and n_2) and opposite charge (e and $-e$), and their masses and relaxation times m_1, m_2 and τ_1, τ_2, respectively. Determine the Hall coefficient R_H of this system.

5) Assume that the cross section ratio for the two body reactions given below depends only on phase space of the final state particlesLarine o . Calculate the ratio.

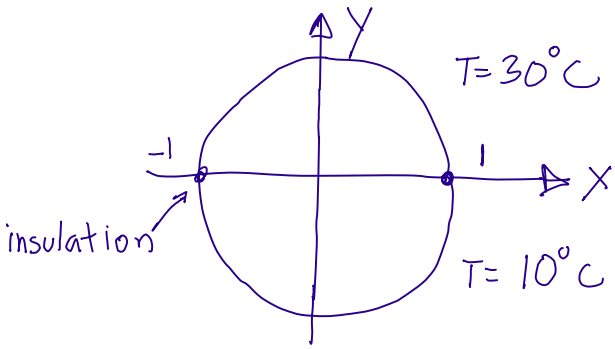
$$\nu_\mu + e \rightarrow \nu_\mu + e$$

$$\nu_\mu + p \rightarrow \nu_\mu + p$$

6). Consider an infinitely long cylinder that has been cut in half, with the two halves thermally insulated from each other (see figure). The top half is held at temperature T=30ºC, and the bottom half at T=10ºC. Find the temperature T(x,y) inside the cylinder. Hint: Use the conformal mapping

$$w = \ln\left(\frac{z+1}{z-1}\right)$$

$$z = x + iy$$

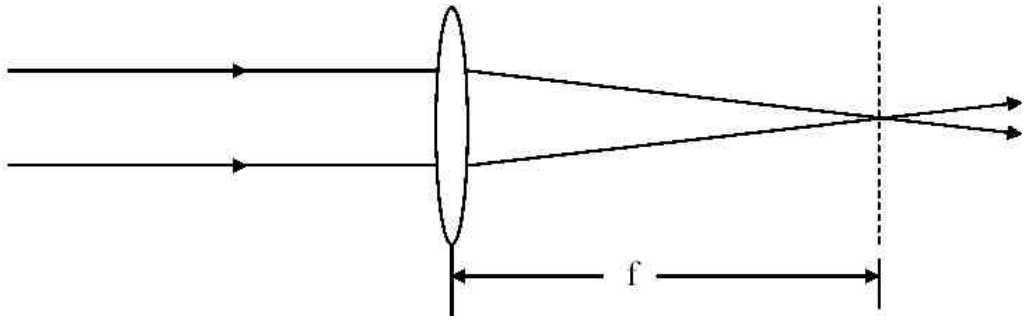


Modern Physics, Optics, Statistical Mechanics and Thermodynamics
February, 2006

Pick 4 out of 6 problems

1. (Bohn) Consider an electron beam of circular cross section moving down the z-axis and passing through a system of focusing magnets. Suppose the beam density remains uniform, and its initial radius is R_0. Suppose further that the transverse components of the electrons' momenta stay uniformly distributed over a circle in the momentum space , and the initial radius of this circle is P_0.
   a. If the focusing system reduces the beam radius from R_0 to R_1, how does the distribution of transverse momenta change?
   b. Part (a) is highly idealized. Suppose, instead, that the electron beam is bunched, and each electron bunch moves non-relativistically through the focusing system. Suppose further that Coulomb self-forces affect the beam, and that the focusing system imparts an external magnetic field B_{ext}. Write down the Vlasov-Poisson kinetic equation for the distribution function of electrons in the six-dimensional phase space of a single electron.
   c. The Vlasov-Poisson equations are, in general, notoriously difficult to solve. Why?
   d. Suppose, now, that the Hamiltonian for the beam bunch is independent of time and that the distribution function is a function only of the Hamiltonian (again, with regards to the six-dimensional phase space of a single electron). Is the beam in equilibrium? Explain your answer.
2. (Ito) The Joule coefficient may be written $h \equiv \left(\frac{\P T}{\P v}\right)_u = \frac{1}{c_v}\left(P - \frac{Tb}{k}\right)$, and the Joule-Thomson coefficient may be written $m \equiv \left(\frac{\P T}{\P P}\right)_h = \frac{v}{c_P}(Tb - 1)$. Here u is the internal energy, b the expansivity, and k the isothermal compressibility. Using these two coefficients,
   a. Find h and μ for a van der Waals gas and
   b. Show that both are zero for an ideal gas.
3. (Ito) Consider a two-level system with an energy $2e$ separating upper and lower states. Assume that the energy splitting is the result of an external magnetic field B. Given that the total magnetic energy is $U_B = -Ne\tanh\left(\frac{e}{kT}\right)$ show that the associated heat capacity is $C_B = Nk\left(\frac{2e}{kT}\right)^2 \frac{e^{2e/kT}}{\left(e^{2e/kT}+1\right)^2}$.
4. (Benbow) Suppose you have large, long-focal length achromatic lens to use as the objective of an astronomical telescope. We are told in geometrical optics that if parallel rays are incident on a lens, the rays will converge to a point at the focal length

of the lens.



If that is so, how can we expect to obtain an image of the moon (certainly not a point source) if we place a screen at the focal point of the lens? Explain, using words, not equations.

5. (Brown) Although a photon has no rest mass, it nevertheless interacts with electrons as though it has the inertial mass

$$m = \frac{p}{u} = \frac{p}{c}$$

where the velocity of the photon is $u = c$.

   a. Compute the photon mass (in units of eV) for photons of wavelengths 5000 Å (visible region) and 1.0 Å (x-ray region). Compare to the mass of an electron (which is 0.511 MeV). (Note: $h = 4.136 \times 10^{-15}$ eV $\cdot$ sec).
   b. When we drop a stone of mass m from a height H near the earth's surface, the gravitational pull of the earth accelerates it as it falls and the stone gains the energy mgH on the way to the ground. A photon of frequency n that falls in a gravitational field gains energy, just as a stone does. Using the photon mass relation, determine the new frequency, n', of the fallen photon, and thus the frequency shift, $n - n'$, suffered by the photon.
   c. A passenger in an airplane flying at 20,000 feet aims a laser beam of red light ($l = 7000$ Å) towards the ground. What is the shift, $l - l'$, in the laser beam wavelength an observer on the ground would measure?

6. (Hedin) Consider the two reactions of an elastic scatter of a neutrino off an electron at rest for muon-type and electron-type neutrinos:

   a) $n_m + e \rightarrow n_m + e$
   b) $n_e + e \rightarrow n_e + e$

   a. Which will have the larger cross section for $E_n = 1$ GeV?
   b. Explain (best if you show the first order Feynman diagrams).
   c. For $E_n = 1$ GeV, what is the maximum angle a scattered electron will have with respect to the incoming neutrino?

Modern Physics: Pick 4 of 6

1) The potential energy of gas molecules in a certain central field depends on the distance r from the field's center as $U(r) = \alpha r^2$ where α is a positive constant. The gas temperature is T, the concentration of molecules at the center of the filed is n_0.

a) Find the number of molecules dN, located at the distances between r and $r+dr$ from the center of the field.
b) Find the most probable distance separating the molecules from the center of the field.
c) How many times will the concentration of molecules in the center of the field will change if the temperature decreases by η ($T_{\text{new}} = \eta \times T$). (Hint, how does dN/N, that is, the fraction of molecules located in spherical layer between r and $r+dr$, behave at the center)
d) Find the number of molecules dN, whose potential energy lies within the interval from U to $U+dU$.

FORMULA SHEET for PROBLEM 1

$$\int_0^\infty x^n e^{-x^2} dx = \begin{cases} \sqrt{\pi}/2, & n=0 \\ 1/2, & n=1 \\ \sqrt{\pi}/4, & n=2 \\ 1/2, & n=3 \end{cases} \qquad \int_0^\infty x^n e^{-x} dx = \begin{cases} 1, & n=0 \\ \sqrt{\pi}/2, & n=1/2 \\ 1, & n=1 \\ 2, & n=2 \end{cases}$$

2) Consider a particle of mass m undergoing Brownian motion in one dimension. The particle is under the influence of a viscous friction force $-m\beta v$, an oscillatory driving force $-ma\sin(\omega t)$ and a random force $mA(t)$. Its equations of motion are

$$\dot{x} = v, \quad \dot{v} + \beta v = -a\sin(\omega t) + A(t).$$

a) Suppose $\langle A(t)\rangle = 0$ and $\langle A(t)A(\tau)\rangle = \alpha\delta(t-\tau)$, where α is a constant and $\delta(t)$ is the Dirac delta function. Suppose the initial conditions are $q(0) = q_0$, $v(0) = \alpha/\omega$. Find the mean speed $\langle v(t)\rangle$. Note:

$$\int_0^t d\tau e^{\beta\tau}\sin(\omega\tau) = \frac{e^{\beta t}\left(\beta\sin(\omega t) - \omega\cos(\omega t)\right) + \omega}{\beta^2 + \omega^2}$$

b) Evaluate (by whatever means you chose) $\langle v(t)\rangle$ in the limit $\beta \rightarrow 0$, and provide a physical interpretation of your result.

3) The density of states for a free particle of momentum p confined within a box of volume Ω is given by

$$\frac{dn}{dp} = \frac{\Omega p^2}{2\pi\hbar^3}$$

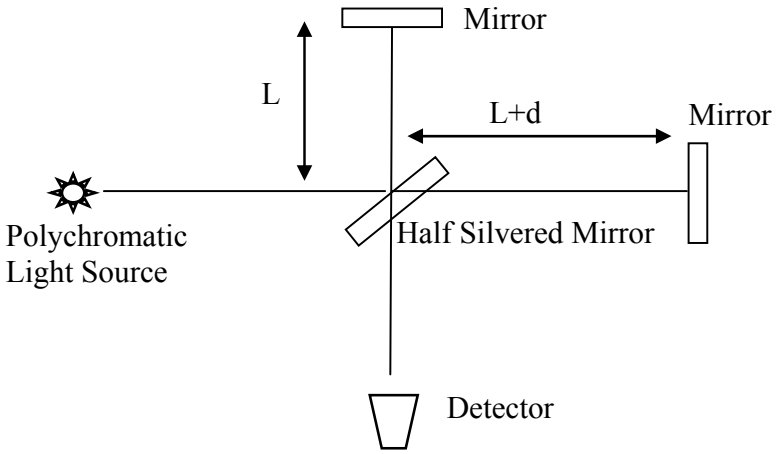
a) Calculate the density of states per unit energy for a non-relativistic electron and for a massless neutrino. Assume each is confined within a box of volume Ω but otherwise free of interactions.

b) Assume that the transition probability for beta decay is dominated by the density of states term. Take the electron to be non-relativistic and the neutrino massless. In terms of the total decay energy, E_{tot}, calculate the most likely energy for the emitted beta particle?

4) An energetic proton strikes a proton at rest. A K+ is produced. Write down a reaction for producing a K+ showing all the final state particles. What is the minimum kinetic energy for the incoming proton to produce this final state (express in terms of the masses of the particles)?

5) An interstellar proton interacts with the 3 degree cosmic microwave background (CMB) and produces an electron-positron pair via p+photon -> p+electron+positron. Assume the CMB is monoenergtic with E=kT= .002 eV. What is the minimum proton energy to produce this reaction? What is the electron energy at this threshold proton energy? The proton mass is 938 MeV/c^2 and the electron mass is 0.5 MeV/c^2.

6)



A Michaelson interferometer has two arms, the first of length L and the second of length $L+d$. The interferometer is illuminated by a polychromatic source of light with a frequency distribution given by

$$I(\nu) = \left(I_0 / \sqrt{2\pi\sigma^2}\right) \exp\left[-\left(\nu - \nu_0\right)^2 / 2\sigma^2\right]$$

a) Describe qualitatively how you expect the interference pattern for the case with polydisperse light to differ from the case where the interferometer is illuminated with a single frequency of light.

b) Calculate the intensity of light observed as a function of the arm offset d. Does this result confirm your prediction from part a?

You may want to use the following integral.

$$\int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{2a^2}} \cos^2(kx)\, dx = \sqrt{\frac{\pi\sigma^2}{2}} \left[1 + \cos(2ky) e^{-2k^2\sigma^2}\right]$$

Statistical Mechanics: Pick 2 of 3. **February, 2005**

1. Consider a large, spherical, self-gravitating system of identical stars for which the distribution function in the phase space of a single star is

$$f(r,v) = C\left[H_0 - H(r,p)\right]^{n-3/2}$$
$$= C_n\left[\Psi(r) - \frac{1}{2}v^2\right]^{n-3/2} \quad \text{for } \Psi > \frac{1}{2}v^2$$
$$= 0 \quad \text{for } \Psi \le \frac{1}{2}v^2$$

Here $\Psi = \Psi(r) = H_0 - \Phi(r)$ denotes a relative potential, with $\Phi(r)$ being the actual potential, and v denotes speed. Using Poisson's equation $\nabla^2\Phi(r) = 4\pi G\rho(r)$, find:

a) The density distribution function $\rho(r)$ corresponding to the index $n = 5$.

b) Are the stellar orbits in this system chaotic? Why or why not? Clearly explain your reasoning.

2. The equation for the distribution of free electrons in a metal in the vicinity of absolute zero is

$$dn = \frac{\sqrt{2}m^{3/2}}{\pi^2\hbar^3}\sqrt{E}dE$$

Making use of this equation, find at T=0K

a) The velocity distribution of free electrons.

b) The ratio of the mean velocity of free electrons to their maximum velocity.

c) What is the functional dependence of the Fermi Energy on the density of electrons?

3. The distribution functions for identical particles (indistinguishable or distinguishable) can be represented by the equation:

$$\frac{N_j}{g_j} = \frac{1}{e^{(\varepsilon_j - \mu)/kT} + a}$$

Where a = 1 for Fermi-Dirac statistics, a= -1 for Bose-Einstein statistics and a=0 for Maxwell-Boltzmann statistics.

a) Sketch and compare N_j / g_j vs. $(\varepsilon_j - \mu)/kT$ for all three distributions.

b) For the Fermi-Dirac distribution, sketch N_j / g_j vs. ε_j for $T = 0$ and T slightly greater than zero.

c) Show that for a system of a large number, N, of bosons at very low temperature (such that they are all in nondegenerate lowest energy state $\varepsilon = 0$), the chemical potential varies with temperature according to:

$$\mu \to -kT / N \text{ as } T \to 0.$$

Modern Physics: Pick 2 out of 4

1. Consider an assembly of identical two state atoms in equilibrium with a radiation field. The two states of the atom are labeled 1 and 2 and their energy difference is $h\nu$. State 2 has the higher energy. Let $\rho(\nu) = \frac{8\pi n^3 h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT}-1}\right)$ denote the energy density of the radiation field per unit frequency interval. The probability for stimulated emission from state 2 to 1 and from state 1 to 2 are given by

$$W_{2,1}^{i} = B_{2,1}\rho(\nu)$$
$$W_{1,2}^{i} = B_{1,2}\rho(\nu)$$

The probability of spontaneous emission from state 2 is given by, $W_{2,1}^{s} = A$

a) What is the ratio N_1 / N_2 of the number of atoms in state 1, to the number of atoms in state 2.
b) Show that $B_{2,1} = B_{1,2}$ and use this to find the ratio A/B.
c) Explain briefly why it is necessary to have a non-thermal population distribution between two states in order to achieve laser action.

2. A pion-muon atom (that is the two particles are bound together) can be formed in KL decay via KL → pion-muon atom + neutrino.

a) What are the energy levels of the pion-muon atom relative to the hydrogen atom?

b) What is the lifetime of the pion-muon atom in terms of the pion and muon lifetimes?

c) If the KL is at rest when it decays, what are the momentum and kinetic energy of the pi-mu atom?

Please state all answers in terms of the masses and lifetimes of the particles involved (pion, muon, kaon, proton, or electron).

3. Find the energy Q, expressed in terms of the masses of the atoms M_p, M_a, and the electron m_e, liberated in β- and β+ decays, and in K-electron-capture if the masses of the parent atom is M_p, the daughter atom M_d, and an electron m_e are known. (Assume electron binding energies are negligible).

4. If two waves are to interfere, stipulations are that they must be traveling in the same direction in the same region of space, and that their oscillatory planes be parallel. If this last condition is not met, something else will occur.

a. Suppose

$$\vec{E}_X = E_0\hat{x}\cos(kz - \omega t)$$
$$\vec{E}_Y = E_0\hat{y}\cos(kz - \omega t)$$

What happens here? What do you see if the wave is traveling toward you?

b. Suppose

$$\vec{E}_X = E_0\hat{x}\cos(kz - \omega t)$$
$$\vec{E}_Y = E_0\hat{y}\sin(kz - \omega t)$$

What happens now? What do you see if the wave is traveling toward you?

Ph.D. Qualifying Exam.
Statistical Mechanics, Thermodynamics and Modern Physics.
September 2004

Part I. Statistical Mechanics Thermodynamics. (Pick 2 out of 3) If you answer all three note that only the first two will be graded.

1. A Carnot engine is operated between two heat reservoirs at temperatures of 400 K to 300 K.
   (a) If the engine receives 1200 kilocalories from the reservoir at 400 K in each cycle, how much heat does it reject to the reservoir at 300 K?
   (b) If the engine is operated as a refrigerator (i.e., in reverse) and receives 1200 kilocalories from the reservoir at 300 K, how much heat does it deliver to the reservoir at 400 K?
   (c) How much work is done by the engine in each case?
   (d) What is the efficiency of the engine in (a) and the coefficient of performance in (b)?

2. An ideal monatomic gas consists of N atoms in a volume V. The gas is allowed to expend isothermally to fill a volume 2V. Show that the entropy change is Nkln2.

3. A mercury atom moves in a cubical box whose edge is 1 *m* long. Its kinetic energy is equal to the average kinetic energy of an atom of an ideal gas at 1000 *K*. If the quantum numbers n_x, n_y, and n_z are all equal to *n*, calculate *n*. Note that the atomic mass of mercury is 200.6 *g/mol* (hint: calculate the mass of an individual atom).

Possible useful information
Boltzmann constant, $k = 1.38 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K
Stefan-Boltzmann constant, $\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4 = 5.670 \times 10^{-8}$ $JK^{-4}m^{-2}s^{-1}$
Planck's constant, $h = 6.62 \times 10^{-34}$ Js
Speed of light, $c = 2.99792458 \times 10^8$ ms^{-1}
$R = 8.34 \times 10^3$ J $kmole^{-1}$ K^{-1}
$dU = TdS - PdV$
$H = U + PV$
$F = U - TS$
$G = F + PV$

Values of constants: $h = 6.63 \times 10^{-34}\ J.s$, $k = 1.38 \times 10^{-23}\ J/K$,
$N_A = 6.022 \times 10^{23}$ *particles per mole.*

Part II. Modern Physics and Optics (Pick 2 out of 4)

1. Assume that the mu-neutrino ν_μ and the tau neutrino ν_τ are composed of a mixture of two mass eigenstates ν_1 and ν_2, with masses m_1 and m_2. The mixing ratio is given by:

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

In free space, the states ν_1 and ν_2 evolve according to

$$\begin{pmatrix} |\nu_1(x,t)> \\ |\nu_2(x,t)> \end{pmatrix} = e^{i\mathbf{p}\cdot x/\hbar} \begin{pmatrix} e^{-iE_1 t/\hbar}|\nu_1(0)> \\ e^{-iE_2 t/\hbar}|\nu_2(0)> \end{pmatrix}$$

a) Show that the transition probability for a mu-neutrino to turn into a tau neutrino is given by:

$$P(\mu \to \tau) = \sin^2(2\theta)\sin^2\left[(E_2 - E_1)t/2\hbar\right]$$

b) Show that in the extreme relativistic limit this result becomes:

$$P(\mu \to \tau) = \sin^2(2\theta)\sin^2\left[\frac{(m_2^2 - m_1^2)c^3 L}{4E\hbar}\right]$$

Here, L is the distance traveled by the neutrino, and E is the total energy.

2. A proton is confined to a 2D square well of size

D= 5 F (10^-15 m) such that
V = -50 MeV for 0 < x < D
0 < y < D
and V = 0 everywhere else.

a. How many energy levels are there in this well?
b. What is the degeneracy of each energy level?
c. What is the approximate energy of the lowest energy state? (and at least outline how one can improve estimating the energy).

mass proton = 938 MeV/c^2 Hbar*c = 197 MeV*F

3. An electron with p = 1 GeV/c strikes a proton at rest.
Their masses are 0.51 MeV/c^2 and 938 MeV/c^2

a. What is the maximum momentum transverse to the incoming electron's direction that the exiting proton can have?
b. What is the maximum total energy that the exiting proton can have?
c. What energy would the electron need to have to make a rho meson, with mass of 770 MeV/c^2? That is e+p->e+p+rho.

4. For the waveform traveling in a dense glass

$$\vec{E}(\vec{r},t) = 90(4\hat{x} + 3\hat{y})\left(V/m\right)\cos\left[6.084\pi \times 10^{6} z + 1.110\pi \times 10^{15} t\right]$$

a. Determine the index of refraction of the glass.
b. Determine the wavelength of the light in air.
c. Resolve the "$\vec{E}_0$" into an amplitude times a unit vector.
d. Determine which direction the wave is propagating.

Ph.D. Qualifying Exam.
Statistical Mechanics, Thermodynamics and Modern Physics.
January 2004

Part I. Statistical Mechanics Thermodynamics. (Pick 2 out of 3) If you answer all three note that only the first two will be graded.

1) Calculate the total electromagnetic energy inside an oven of volume 1 m^3 heated to a temperature of 600 degrees Fahrenheit.

2) An ideal monatomic gas undergoes a reversible expansion from specific volume v_1 to specific volume v_2.

   (a) Calculate the change in specific entropy Δs if the expansion is isobaric.
   (b) Calculate Δs if the process is isothermal.
   (c) Which is larger? By how much?

3) For N distinguishable coins the thermodynamic probability is $\omega = \dfrac{N!}{N_1!(N-N_1)!}$, where N_1 is the number of heads and $N-N_1$ the number of tails.

   (a) Assume that N is large enough that Stirling's approximation ($\ln n! \approx n \ln n - n$) is valid. Show that $\ln\omega$ is maximum for $N_1 = N/2$.
   (b) Show that $\omega_{max} \approx e^{N \ln 2}$.

Possible useful information
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Stefan-Boltzmann constant, $\sigma = 5.670 \times 10^{-8}$ $JK^{-4}m^{-2}s^{-1}$
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Speed of light, $c = 2.99792458 \times 10^{8}$ ms^{-1}
$R = 8.34 \times 10^{3}$ J $kmole^{-1}$ K^{-1}
$dU = TdS - PdV$
$H = U + PV$
$F = U - TS$
$G = F + PV$

Part II. Modern Physics and Optics (Pick 2 out of 4)

1) A proton is confined to a 2D square well of size D= 5 F (10^{-15} m) such that

V = -50 MeV	for $0 < x < D$, $0 < y < D$,
and	
V = 0	everywhere else.

a. How many energy levels are there in this well?
b. What is the degeneracy of each energy level?
c. What is the approximate energy of the lowest energy state? Give both the zeroth and first order approximation.

Mass proton = 938 MeV/c^2 $\hbar$*c = 197 MeV*F

2) Consider a neutron star of mass M and radius R. Assume T= 0 K. Ignoring gravitational energy and assuming a simple Fermi gas model, what is the maximum energy a neutron can have? What is the average energy of the neutrons? Do not calculate actual values but leave in the form using the mass of neutron, m_n, and other constants.

3) A photon from the cosmic microwave background with energy 1.2×10^{-3} eV (corresponding to a wavelength of 1 mm) collides head-on with a relativistic electron having kinetic energy 1 GeV. The photon is backscattered, i.e., scattered by 180°, after which the electron continues to move in its initial direction. Find the energy of the photon after the collision. What is the wavelength of the backscattered photon?

4) Consider a plane electromagnetic wave incident in the normal direction on a system of *n* narrow slits each separated by a distance *d*. Derive a formula for the intensity of the scattering as a function of angle and wavelength in the Fraunhoffer approximation.

Graduate Qualifier Fall 2003,

Stat-Mech Pick 2 of 3.

Problem 1:

Consider the Fokker-Planck equation for the distribution function f:

$$\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \nabla f + \underset{\sim}{a} \cdot \underset{\sim}{\partial} f = \underset{\sim}{\partial} \cdot \left(\underset{\sim}{F} f \right) + \underset{\sim}{\partial} \left(\underset{\sim}{\overleftrightarrow{D}} \cdot \underset{\sim}{\partial} f \right),$$

where $\underset{\sim}{\partial}$ denotes a gradient with respect to velocity $\underset{\sim}{v}$. Take a simple model of dynamical friction and diffusion: $\underset{\sim}{F} = \beta \underset{\sim}{v}$ and $\underset{\sim}{D} = D \hat{\underset{\sim}{v}} \hat{\underset{\sim}{v}}$, where β and D are constants and $\hat{\underset{\sim}{v}}$ is the unit vector in the direction of $\underset{\sim}{v}$.

a. Describe the overall effect of the right-hand side on the evolution of the system.
b. How is the coefficient β related to the force fluctuations a constituent particle experiences?
c. Deduce from the Fokker-Planck equation how β and D are related for a system in thermal equilibrium.

Problem 2.

Consider the simplest model for desorption of atoms of mass m from a surface: one that ignored the translational degrees of freedom, and takes the possible energy states of the atom as $-\varepsilon$ (when adsorbed on the surface) and 0 when in the 'vacuum' at temperature T.
a. Use the canonical ensemble for N atoms to obtain an expression for the number of atoms in the vacuum.
b. What is the total energy of the system as a function of temperature?

Problem 3.

Conduction electrons are confined in a metal at T= 300 K. The Fermi energy is 4 eV. What is the relative density of states N(E) for energy of 4.4 eV compared to 3.6 eV? What is the relative probability for an electron to have energy of 4.4 eV compared to 3.6 eV?

Fall 2003

Modern/Optics Pick 2 of 4

Problem 4.

Derive Malus's law: Malus's law relates the *intensity* of light passing through two *ideal* polarizers with their polarizing directions at an angle θ with each other. Polarizers act on the electric field vector (*amplitude*) of the light wave traversing them. An ideal polarizer passes 50% of purely unpolarized light incident on its surface, the remainder being now polarized parallel to the polarizing direction.
Now, armed with Malus's law, consider two ideal polarizers arranged so that a beam of light passing through both finds the second one at an angle of 45° to the first. Then a 3rd ideal polarizer is inserted between them and rotated at angular frequency ω. Derive an expression for the intensity of the light emitted from the system as a function of time.

Problem 5.

Two neutrinos leave a supernova explosion at the same time. They arrive at earth a distance L away separated in time by Δt. The first neutrino to arrive is found to have energy E_1 and the second energy E_2. Find the mass of the neutrino.

Problem 6.

The disintegration constant λ of a radioactive nucleus is defined as the fraction of nuclei that decay per unit time. Let $N(t)$ be the number of nuclei present at time t.

a. Derive the decay law, $N(t) = N(0)\exp(-\lambda t)$

b. What is the relation between half-life $T_{1/2}$ and λ?

c. In an excitation experiment, a parent nucleus p is produced at a rate R, and decays with a disintegration constant λ_p, to a daughter nucleus d, whose disintegration constants is λ_d. At t=0, the number of parent nuclei is $N_p(0)=N_{0p}$ and the number of daughter nuclei is $N_d(0)=0$. Find $N_p(t)$ and $N_d(t)$ for t>0, and show that after a long time, an equilibrium is attained.

Problem 7.

An electron is in the spin state

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

a. Determine the normalization constant A

b. Find the expectation values of S_x, S_y, and S_z.

NIU Ph.D./Master qualifier examination 2003 Spring

Statistical Physics and Modern Physics

Solve 4 problems. Choose 2 problems from I, II and III, and choose 1 problem from V and VI, choose 1 problem from IV and VII.

I Consider a nonrelativistic free particle in a cubical container of edge length L and volume $V = L^3$.

a Each quantum state r of this particle has a corresponding kinetic energy ε_r which depends on V. What is $\varepsilon_r(V)$?

b Find the contribution to the gas pressure $p_r = -(\partial\varepsilon_r/\partial V)$ of a particle in this state in terms of ε_r and V.

c Use this result to show that the mean pressure $\overline{p}$ of any ideal gas of weakly interacting particles is always related to its mean total kinetic energy $\overline{E}$ by $\overline{p} = \frac{2}{3}\overline{E}/V$, irrespective of whether the gas obeys classical, Fermi-Dirac, or Bose-Einstein statistics.

II Consider a system of two atoms, each having only 3 quantum states of energies 0, ε and 2ε. The system is n contact with a heat reservoir at temperature T. Write down the partition function Z for the system if the particles obey

A. Classical statistics and are distinguishable.
B. Classical statistics and are indistinguishable.
C. Fermi-Dirac statistics.
D. Bose-Einstein statistics.

III A certain gas obeys the equation of state $P(V-b) = RT$, where R is the universal gas constant, and b is a constant such that $0 < b < V$ for all V.

a Derive an expression for the work obtained from a reversible isothermal expression of one mole of this gas from an initial volume V_i to a final volume V_f.

b If the gas were ideal, would the same process produce more or less work?

IV Find the threshold energy for the process, $\gamma + p \rightarrow \pi^0 + p$, in which a single π meson is produced when a photon strikes a proton at rest. The rest energy of the pion is 135 MeV and the proton 938 MeV.

V Take an ideal monatomic gas ($\gamma = 5/3$) around the Carnot cycle, where *point 1* at the beginning of the adiabatic compression has pressure $P_1 = P_0$ (atmospheric pressure), volume $V_1 = 13$ liters, and temperature $T_1 = 300$ K. *Point 3* has pressure $P_3 = 2P_0$ and volume $V_3 = 26$ liters. Calculate the values of volume and pressure at all four points of the Carnot cycle.

VI An ideal gas initially at temperature, T_i, pressure, P_i and volume, V_i has its pressure reduced to a final value $P_f < P_i$ via one of the following processes: (1) isochoric; (2) isothermal; (3) adiabatic.

a Sketch each process schematically on a P - V diagram.

b In which process is he work done on the gas zero?

c In which process is the work done on the gas greatest?

d Show that the ratio the absolute magnitudes of the heat transfers, $Q_{isotherermal}$ to $Q_{isochoric}$ is given by

$$\frac{|Q_{isothermal}|}{|Q_{isochoric}|} = \frac{RP_i}{C_v|\Delta P|}\left|\ln\left(\frac{P_i}{P_f}\right)\right|$$

e In which process is the change in internal energy of the gas the greatest? Explain your reasoning.

VII The k-alpha radiation from copper (Z=29) occurs via the following process. An incident high energy electron removes an electron from the n=1 orbital and subsequently an electron from the n=2 orbital falls down to the n=1 orbital releasing an x-ray.

a Ignoring electron-electron interactions calculate, in units of the Rydberg constant R, the energy of the Cu k-alpha x-ray.

b More realistically, the second 1S electron will shield the nuclear charge. Assuming perfect shielding, calculate a better estimate of the Cu k-alpha energy.

c Discuss, in qualitative terms, how inter-electron interactions might modify the Cu k-alpha energy, beyond just the perfect shielding approximation.

d The energy of the Cu k-alpha x-ray is slightly modified by the electron's spin-orbit interaction. This leads to a splitting of the k-alpha line into a k-alpa-1 and k-alpha-2.

i List all possible values of total (orbital + spin) angular momentum for the n=1 and n=2 states.

ii List all transitions from states with n=2 to states with n=1 that are permitted by the selection rules for atomic transitions.

iii Explain the observed ratio of 2:1 for the Cu k-alpha-1 to k-alpha-2 fluorescence yields.